Experimental Validation of Equi-Area Method for Antenna Directivity, Gain and Efficiency Calculations

Sadiq Ullah , Shahbaz Khan, Aziz Ahmed, Babar Kamal

Antennas and Propagation Lab, Telecommunication Engineering Department, University of Engineering and Technology, Peshawar (Mardan Campus), Mardan, 23200, Pakistan.

Abstract: In this paper a detailed analysis of dipole, patch, and array antennas is carried out for 325 sampling points and it is experimentally validated that the equi-area method gives more accurate values of directivity, and gain as compared to the equi-angle method. The equi-area method uses Leopardi's (Leopardi, 2006) algorithm to determine the sampling points where as equi-angle method uses equi-angle steps of 15-degrees to obtain 325 sampling points. It is also verified that the equi-area method is 17 to 18 times computationally efficient than the conventional equi-angle method.

[Sadiq Ullah , Shahbaz Khan, Aziz Ahmed, Babar Kamal. **Experimental Validation of Equi-Area Method for Antenna Directivity, Gain and Efficiency Calculations.** *Life Sci J* 2013;10(7s):1232-1239] (ISSN: 1097-8135). http://www.lifesciencesite.com. 196

Keywords: equi-area method, equi-angle method, sampling points.

1. Introduction

The antenna measurements is the one of the most challenging and crucial component of antenna engineering. Therefore an antenna engineer is required to have adequate understanding of the theory of antenna design and antenna measurements. In the past an extensive contribution to the literature of antenna measurements is given in (Kumer WH, Gillespie, 1978; Kraus, 1988; Yaghjian, 1986; Balanis, 1997). The antenna performance parameters like, gain, directivity, radiation and antenna efficiency may be calculated by analytical expressions as well as approximate numerical methods (Kraus, 1988; Balanis, 1997). The approximate numerical methods are used in the laboratory to analyze the antenna performance parameters. The numerical method uses a grid of sampling points as the measurement locations to gather the antenna radiation information at each point. Therefore the two main factors involved in the numerical analysis of antenna measurements are the sampling criterion and the type of numerical method used. In the past many sampling techniques were developed to get the grid of measurement points as given in (Rahmat-Samti and Cheung, 1987; Chen and Edwards, 1993; Lu et al., 2005; Yaccarino et al., 1996, Wittmann et al., 1998; Wittmann et al., 2004; Oh et al., 2008; Laitinen, 2008).

The accuracy of the measurements depends on the desired sampling technique and type of numerical method. Furthermore the time required to analyze the measured results also depends on the type of numerical method.

The equi-angle sampling and the numerical method given in (Balanis, 1997) gives good results for a range of antennas but its accuracy may be affected for very narrow-beam antennas. This is due to the non-uniform distribution of the sampling points on the spherical surface. The simulation time for the equi-angle method is also affected due to the weighting functions involved in the far-field parameter (i.e. total radiation power) calculations.

Therefore in this paper Leopardi's algorithm is used as a sampling technique to obtain the uniformly distributed grid of sampling points. To analyze the antenna, a novel and computationally efficient method called the equi-area method is used.

Section 2 describes the theory of antenna performance parameters (directivity, gain, radiation efficiency and antenna efficiency) of a dipole, patch and array antennas. The results are presented in section 3; Section 4 concludes the paper.

2. Theory of Antenna Performance Parameters

In this section the theory of antenna performance parameters, such as total radiated power, directivity, gain and efficiency is discussed, using both analytical and numerical approaches. To evaluate these parameters two numerical methods, namely, equi-angle and equi-area methods (Ullah et al., 2007; Ullah et al., 2008), are used for three types of antennas: (1) Cylindrical dipole (2) Rectangular patch and (3) two element array antennas. The total radiated power of an antenna may be calculated using either of the methods. i. e.

$$P_{rad,analytical} = \int_0^{2\pi}\int_0^{\pi} u(\theta,\varphi)\sin\theta d\theta d\phi \qquad (1)$$

$$P_{rad,equi-angle} = (\frac{\pi}{L})^2 \sum_{i=1}^{L} \sum_{j=1}^{M} u(\theta_i, \phi_j) \sin(\theta_i) \quad (2)$$

$$P_{rad,equi-area} = A_{Ri} \sum_{i=1}^{N} u(\theta_{c,i}, \phi_{c,i}) \quad (3)$$

Where u is radiation intensity, B_0 is a constant, L, and $M=2L$ are the number of sampling points along the elevation and azimuth axis. The angles θ_i and ϕ_j represents the value of the ith and jth elevation and azimuth angles in radians respectively. The measurement location is defined by the pair of equi angle spherical coordinates (θ_i, ϕ_j) given by

$$\left.\begin{aligned} \theta_i &= \frac{i\pi}{L}, i = 1,2,.........., L \\ \phi_j &= \frac{2j\pi}{M}, j = 1,2,......, M \end{aligned}\right\} \quad (4)$$

Similarly A_{Ri} represents the beam solid angle of the equi-area regions and is inversely proportional to the total number of radiation pattern sampling points (N), i.e.

$$A_{Ri} = \frac{4\pi}{N} \quad (5)$$

The pair of spherical coordinates ($\theta_{c,i}$, $\phi_{c,j}$) shows the ith measurement location in the equi-area method. The subscript "c" reveals that the sampling point is the centre of these equi-area regions and is obtained using the Leopardi's algorithm.

If the direction of the main beam is not specified then the maximum value of directivity is analytically expressed in the direction of maximum radiation intensity (U_{max}) as given by

$$D = \frac{4\pi U_{max}}{P_{rad}} \quad (6)$$

The total radiation intensity of an antenna is given by

$$u\ (\theta, \phi) = u_\theta + u_\phi = \frac{|E_\theta|^2 + |E_\phi|^2}{2\eta_0} \quad (7)$$

Where, E_θ and E_ϕ are the elevation and azimuth components of the electric field. η_0 is the impedance of the free space (377 ohms). In theory, the numerical radiation intensity patterns are available for the dipole, patch and two-element array antennas (Balanis, 1997), therefore the two numerical methods (equi-angle and equi-area) can be used to find the performance parameters (such as directivity, gain) of these antennas:

2.1. Cylindrical Dipole Antenna

The cylindrical half-wavelength (*7.9 cm*) wired dipole antenna is designed for a frequency of 1900 MHz, with quarter wavelength balloon (*3.95 cm*) for impedance matching as shown in Figure 1.

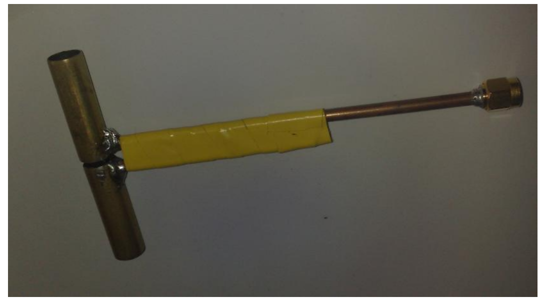

Figure 1. 1900 MHz cylindrical wire dipole antenna

Numerical pattern for a dipole antenna is given in (Balanis, 1997); therefore the total radiated power by dipole can be found using the equi-area method:

$$u_{dipole}(\theta, \phi) = U_{max} \sin^3(\theta) \quad (8)$$

$$P_{rad,dipole,equi-area} = \frac{4\pi}{N} \sum_{i=1}^{N} U_{max} \sin^3(\theta_{c,i}) \quad (9)$$

2.2 Rectangular Patch Antenna

The patch antenna (Figure 2) is designed for a resonance frequency of *2 GHz* on a substrate of height *1 mm* and dielectric constant *2.2* (RT duroid). The length and width are equal to *4.704 cm* (half wavelength). Using the two-slot transmission line model of the rectangular patch antenna, the radiation intensity of the patch is given by:

$$u_{patch}(\theta) = \frac{|V_0|^2}{2\eta_0 \pi^2} \tan^2(\theta) \sin^2\left(\frac{k_0 W}{2} \cos\theta\right) \quad (10)$$

Figure 2. 2 GHz rectangular patch antenna

Where k_0 is the wave number in free space and W is the width of the patch antenna. V_0 is a constant which depend on the thickness of the substrate. The total radiation power of the patch antenna is obtained by using the equi-area method:

$$P_{rad,patch,equi-area} = \frac{4\pi}{N}\sum_{i=1}^{N} u_{patch}(\theta_{c,i}) \quad (11)$$

2.3 Two-Element Array Antenna

The normalized radiation intensity pattern of the two-element dipole array (Figure 3) is proportional to the multiplication of the squares of the element factor and the array factor (Gross, 2005). i. e.

$$u_{array}(\theta) = [\sin(\theta)]^2[\cos(0.5kd\sin\theta)]^2 \quad (12)$$

$$P_{rad,array,equi-area} = \frac{4\pi}{N}\sum_{i=1}^{N}[\sin(\theta_{c,i})]^2[\cos(\frac{\pi f d}{c}\sin(\theta_{c,i}))]^2 \quad (13)$$

Where $k = 2\pi/\lambda = 2\pi f/c$ the wave number, d is the spacing between the two dipole elements, λ is the free space wavelength. The resonant frequency of the single array element (i.e. dipole) is 1900 MHz.

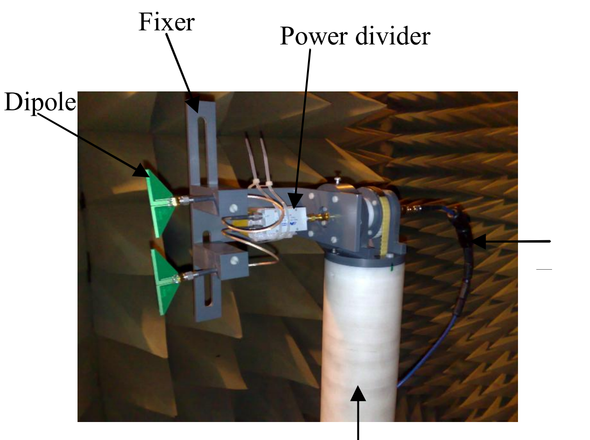

Figure 3. 1900 MHz Array antenna in the anechoic chamber

Once the total radiated power is obtained, no matter, which numerical method is used, the maximum directivity of a given antenna can be calculated using (6). The antenna gain can be calculated using the formula:

$$G = \frac{4\pi U_{\max}}{P_{in}} \quad (14)$$

If we assume that there are no cable losses in the measurement system, then the input power is equal to the sum of radiation power and reflected power (due to mismatch), i.e.

$$P_{in} = P_{rad} + P_r = P_{rad} + \Gamma_{11}^{\ 2} P_{in} \quad (15)$$

$$P_{in} = P_{rad}/(1-\Gamma_{11}^{\ 2}) \quad (16)$$

Where P_r is the reflected power, Γ_{11} is the reflection coefficient of the antenna under the test (AUT), given by;

$$\Gamma_{11} = 10^{S_{11}/20} \quad (17)$$

Where, S_{11} is the return loss of the AUT in *dB* and the factor (1- Γ_{11}^2) is called the reflection loss factor. Once the gain and directivity are known, the radiation efficiency can be calculated using the relation

$$\eta_{rad} = G/D \quad (18)$$

The antenna total efficiency is obtained from the product of radiation and reflection efficiencies:

$$\eta_{ant} = (1-\Gamma_{11}^{\ 2})\eta_{rad} \quad (19)$$

3. Results

The antenna measurements are carried out in the anechoic chamber and magnitude and phases of the elevation and azimuth component of the electric field are recorded. Matlab ® 7.4.0.is used to analyse this recorded data. The antennas are analysed using both equi-angle as well as equi-area methods for *325* sampling points. The *325* sampling points are obtained by using *15*-degrees angular steps in the equi-angle method. Leopardi's algorithm is used to obtain the *325* sampling points in the equi-area method. The grid of the sampling points alone and on a spherical surface is shown in Figure 4. The antennas are analysed one by one as follows:

The cylindrical dipole antenna is analyzed in the frequency range *1600-2000 MHz.* The resonance frequency of the dipole is *1900 MHz.* The antenna directivity and gain are obtained from the measured pattern using both the methods and are compared with those obtained via Numerical Electromagnetic Code (NEC) software. The NEC software uses the Method of Moments (MoM) to calculate the gain and directivity of the antenna.

The variations in gain and directivity curves are approximately the same for the dipole antenna for both the methods as shown in Figure 5, 6, 7 and 8. It is worth noting that the values of directivity acquired by the equi-angle method are slightly higher than that acquired by equi-area method except in the ranges 1650-1740 MHz, and *1970-2000 MHz*. This is due to the fact that the density of the equi-angle points is non-uniform on the spherical surface. While measuring the radiation pattern at certain frequency; regions of distinct radiation intensity are sampled by non-uniformly dense grid of sampling points contained in those regions. This non-uniformity may results in an increase or decrease in the directivity obtained by the equi-angle method w. r. t that obtained by equi-area method, throughout the frequency range.

The deviation in directivity and gain w .r .t the NEC based directivity and gain are plotted in Figure 6 and Figure 8, and it is apparent that the equi-area method shows better accuracy w .r .t the equi-angle method throughout the antenna bandwidth.

The patch antenna has a resonance frequency of *2000 MHz*. The patch antenna directivity and deviation in directivity are illustrated in Figure 9 and Figure 10 whereas the gain and deviation in gain are depicted in Figure 11 and Figure 12 respectively. In this case the measured directivity and gain are compared with the analytical directivity and gain of the patch. The deviation in directivity and gain w. r. t the analytical values are less for the equi-area method as shown in Figure 10 and Figure 12.

The directivity and deviation in directivity for the two-element dipole array (with *d*=12 cm separation between the elements) are illustrated in Figure 13, Figure 14, whereas the corresponding values of gain and deviation in gain are shown in Figure 15 and Figure 16 respectively. The array configuration undergoes resonance at 1900 MHz. The values of directivity and gain are compared with the values obtained from the NEC software and it is proved that the equi-area method gives less deviation throughout the frequency range. Therefore the equi-area method is more accurate as compared to the equi-angle counterpart.

The two-element array with *d=20 cm* separation between the elements is analysed and the results are illustrated as shown in Figs. 17-20. This array configuration resonates at the same frequency, i.e. *1900 MHz*. It is observed that the deviation in gain and directivity for the equi-area method are less than that for equi-angle method. It is worth noting that at resonance frequency (i.e. *1900 MHz*) the equi-area method is significantly accurate.

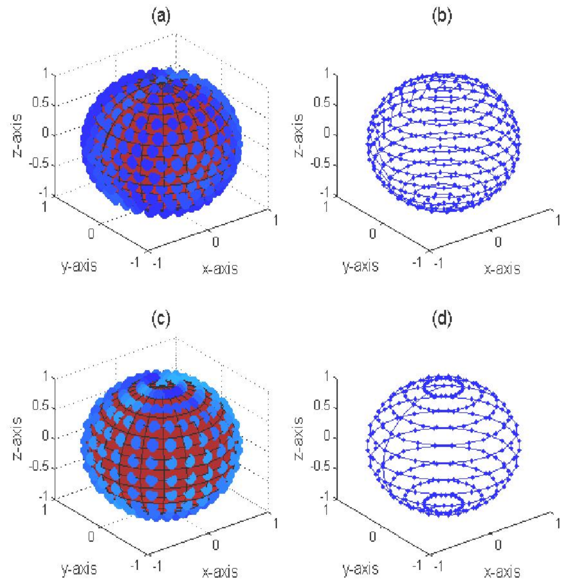

Figure 4. Grid of 325 sampling points; Equi area: (a) On sphere (b) Line-connected; Equi angle: (c) On sphere (d) Line-connected

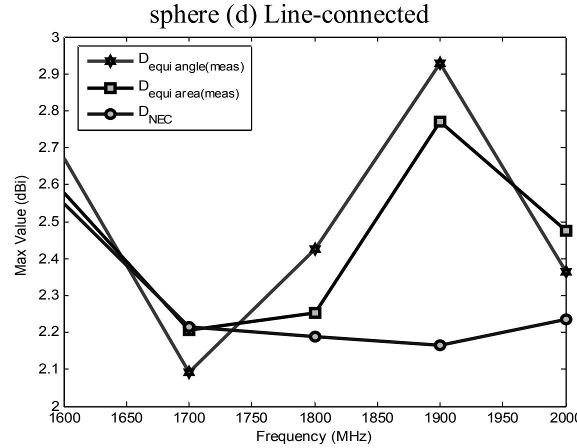

Figure 5. Dipole: directivity analysis

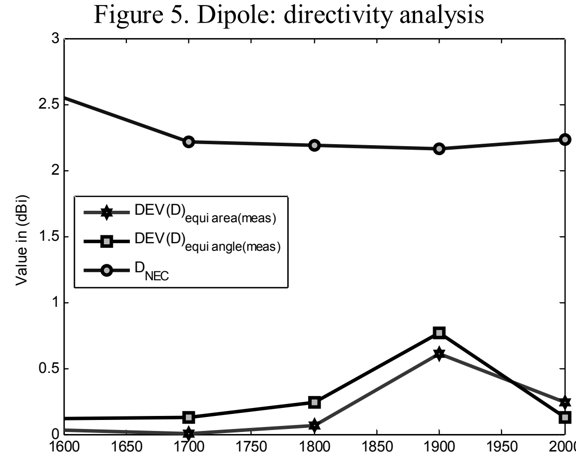

Figure 6. Dipole: Deviation in directivity

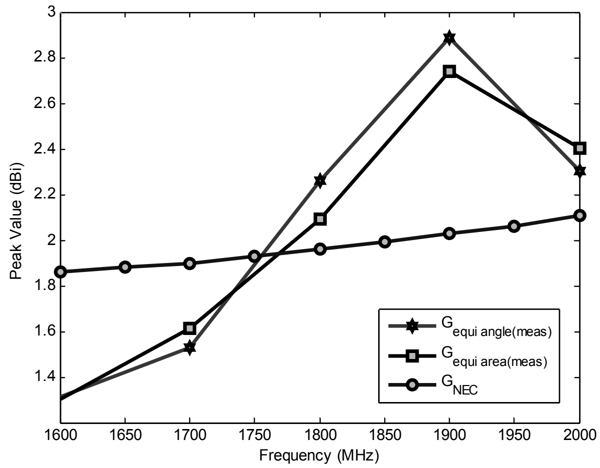

Figure 7. Dipole: gain analysis

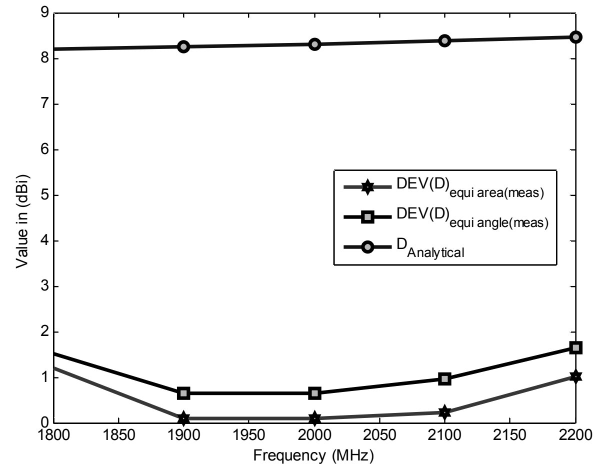

Figure 10. Patch: Deviation in directivity

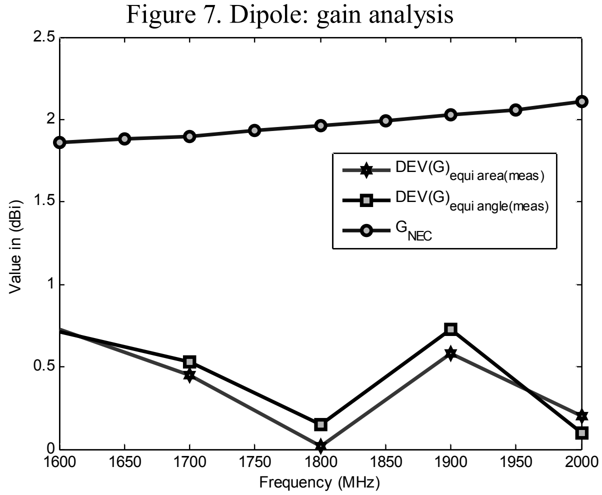

Figure 8. Dipole: Deviation in gain

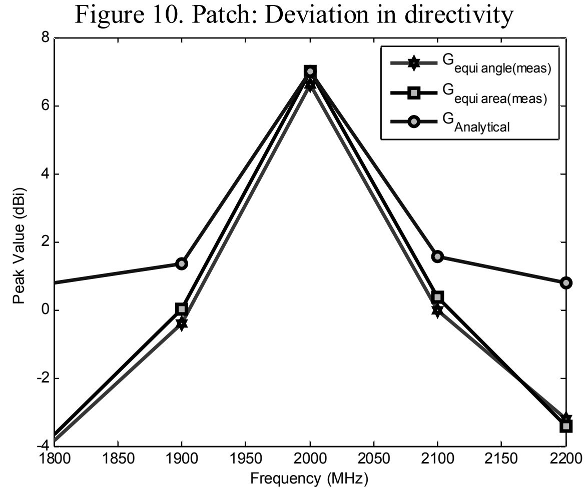

Figure 11. Patch: gain analysis

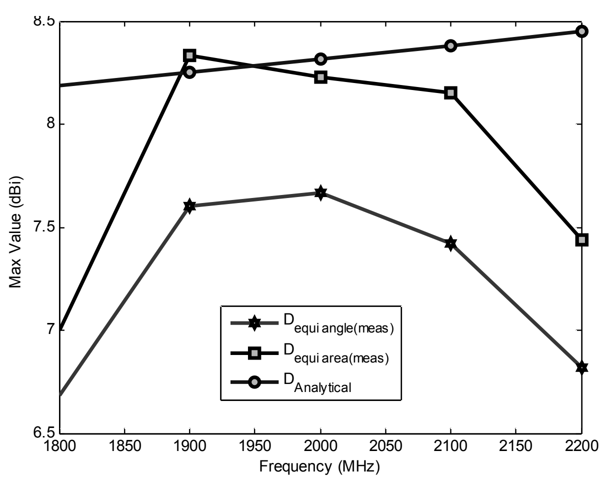

Figure 9. Patch: directivity analysis

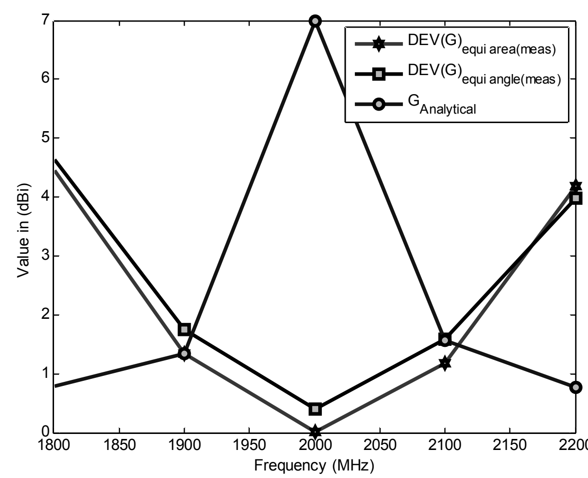

Figure 12. Patch: Deviation in gain

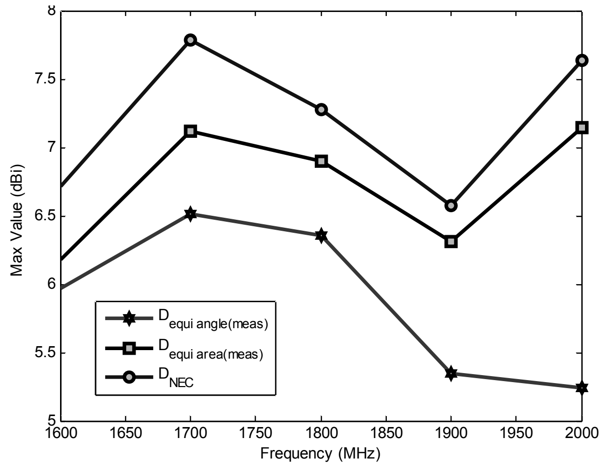

Figure 13. Array (d=12 cm): directivity analysis

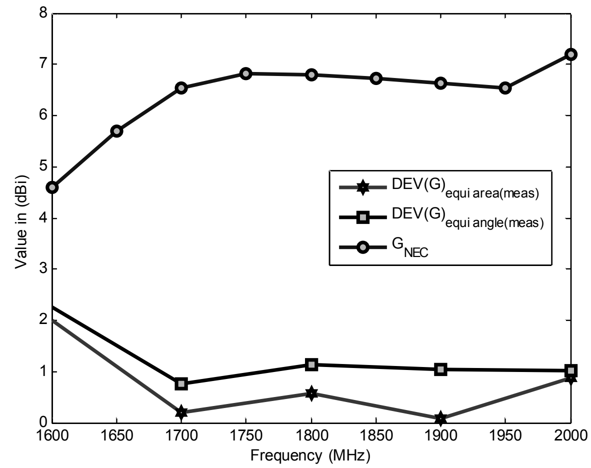

Figure 16. Array (d= 12 cm): Deviation in gain

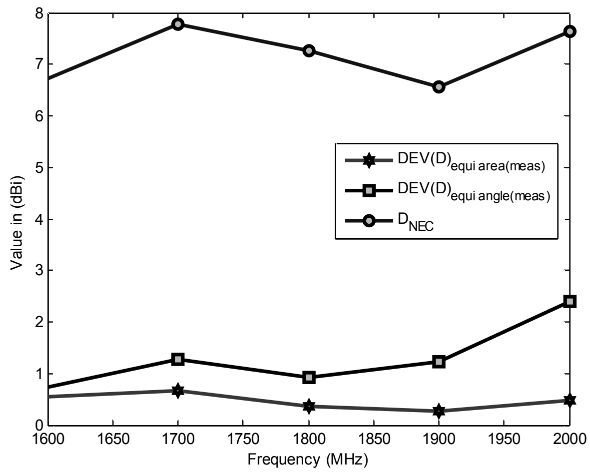

Figure 14. Array (d= 12 cm): Deviation in directivity

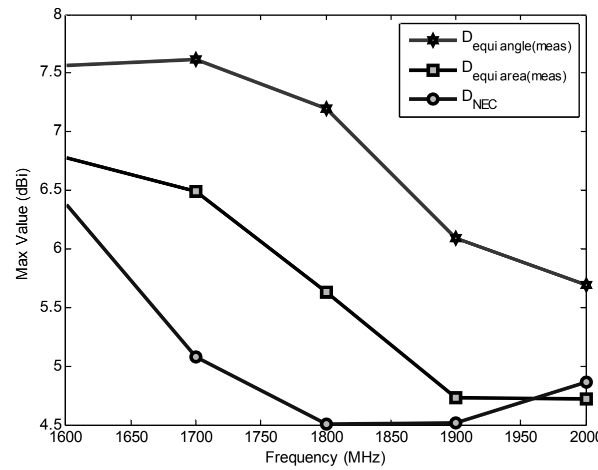

Figure 17. Array (d=20 cm): directivity analysis

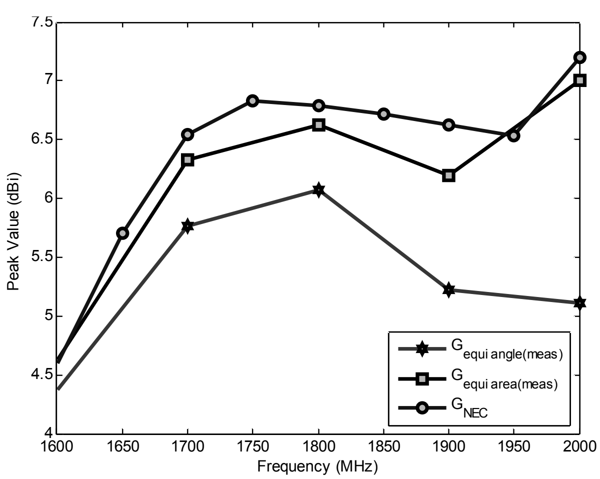

Figure 15. Array (d=12 cm) gain analysis

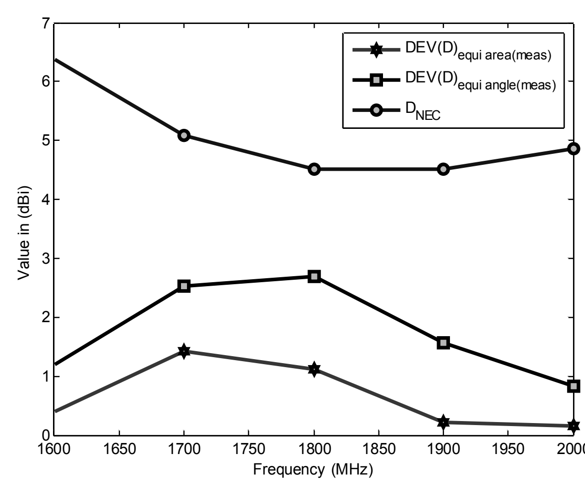

Figure 18. Array (d= 20 cm): Deviation in directivity

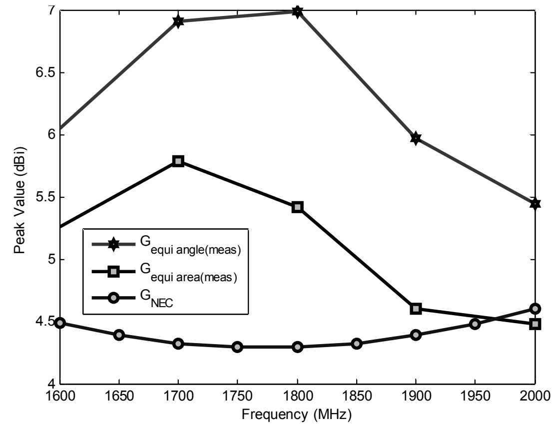

Figure 19. Array (d=20 cm): gain analysis

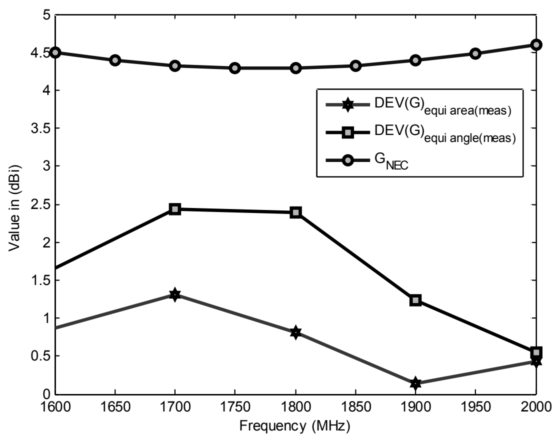

Figure 20. Array (d= 20 cm): Deviation in gain

The equi-angle method is computationally slow as compared to equi-area method because of the weighting function, ‘sinθ’. This claim is verified by calculating the absolute number of seconds in real time that have elapsed in order to execute the equation for antenna directivity. The built-in function in Mathematica®7 was used to determine this absolute time in order to study the computational complexity of both methods. The two methods were tested for different values of sampling points N (Figure 21). It was found that the equi-area method is approximately 17 to 18 times faster than the equi-angle method. The computation time represents the time spent by the CPU to execute the equation for directivity. This study was conducted on a ―*32-bit, 1.60 GHz, Intel Pentium Dual CPU, T2330, with a 2.0GB of RAM”*.

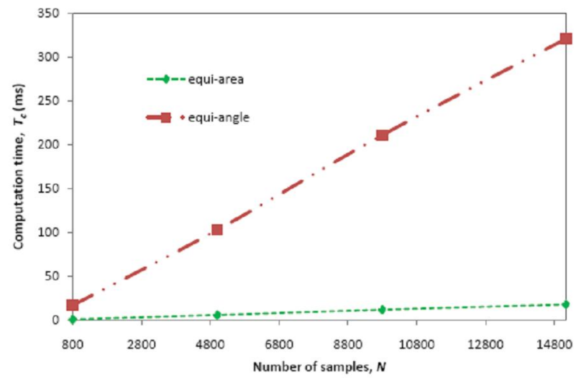

Figure 21. Computational complexity of equi-angle and equi-area methods as a function of sampling points

4. Discussions

In this paper the equi-area based sampling points were used to measure the antenna radiation characteristics. Three types of antennas, were experimentally tested, and numerically validated using the well-known equi-angle method and the novel equi-area method (based on Leopardi algorithm. The two methods were compared on the basis of degree of accuracy in gain, directivity and efficiency calculations, throughout the frequency range. It was experimentally validated that the equi-area method gives more accurate results at the resonance (design) frequency for the given antennas. Furthermore the computational complexity of the equi-area method was low as compared to the equi-angle method for a given specifications of the system (i.e. computer) and number of sampling points. As a result the equi-area method can give 17 to 18 time faster response than the equi-angle method, to analyse the far-field parameters of antennas.

Acknowledgements:

The authors would like to specially thank the Higher Education Commission (HEC) of Pakistan and Loughorough University, United Kingdom for conducting this research

Corresponding Author:
Dr. Sadiq Ullah
Telecommunication Engineering Department, University of Engineering and Technology, Peshawar (Mardan Campus), Mardan, 23200, Pakistan
E-mail: sadiqullah@nwfpuet.edu.pk

References

1. Leopardi P. A partition of the unit sphere into regions of equal area and small diameter. Electronic Transactions on Numerical Analysis. 2006; 25: 309-327.

2. Kumer WH, Gillespie ES. Antenna Measurements- 1978. Proceedings of the IEEE 1978; 66(4):483-507.
3. Kraus JD. Antennas. McGraw-Hill, inc., 1988.
4. Yaghjian AD. An Overview of Near-Field Antenna Measurements. IEEE Transactions on Antennas and Propagation 1986; 34(1):30-45.
5. Balanis CA. Antenna theory: analysis and design. Wily, 1997.
6. Rahmat-Samti Y, Cheung RLT. Nonuniform Sampling Techniques for Antenna Applications, IEEE Transactions on Antennas and Propagation 1987; 35(3): 268-279.
7. Chen, Edwards. Applications of non-uniform sampling techniques and fast Fourier transform in plane near-field antenna measurements. Eighth International Conference on Antennas and Propagation 1993; 1: 311-314.
8. Lu M, Shanker B, Michielssen E. On uniform versus non-uniform sampling of far-field patterns in the plane wave time domain algorithm. IEEE Antennas and Propagation Society International Symposium 2005; 3A:163-166.
9. Yaccarino RG, Williams LI, Rahmat-Samii, Y. Linear Spiral Sampling for the Bipolar Planar Near-Field Antenna Measurement Technique. IEEE Transactions on Antennas and Propagation 1996; 44 (1): 1049-1051.
10. Wittmann RC, Alpert, Francis. Near-field antenna measurement using Nonideal measurement locations. IEEE transactions on antennas and propagation 1998; 46(5):716-722.
11. Wittmann RC, Bradley Alpert K, Francis MH. Near-Field, Spherical-Scanning Antenna Measurements With Nonideal Probe Locations. IEEE Transactions on Antennas and Propagation 2004; 52(8):2184-2187.
12. Oh SS, Kim JM, Yun JH, Kang SY. Antenna Measurement Method in Fresnel Region by φ-Variation Scanning. IEEE Antennas and Wireless Propagation Letters 2008; 7(1): 206-209.
13. Laitinen T. Double φ-Step θ-Scanning Technique for Spherical Near-Field Antenna Measurements. IEEE Transactions on Antennas and Propagation 2008; 56(6): 1633-1639.
14. Ullah S. Flint JA, Seager RD. Equi-Spaced-Points-Method (ESPM) for Antenna Radiation Pattern (ARP) Measurements with 3D- Rotation. Second European Conference on Antennas and Propagation EuCap 2007; 1-6.
15. Ullah S, Flint JA, Seager RD. Patch antenna directivity analysis using an equi-area method. Loughborough Antennas & Propagation Conference 2008; 349-352.
16. Gross F. Smart antennas for wireless communications with MATLAB. McGraw-Hill, 2005.

7/2/2013