Analytical Analysis of the Hydraulic Jump Roller Length in Open Channel

Abolfazl Nazari Giglou[1], Taher Nazari Giglou[2], Afshar Minaei[3]

[1,2,3] Department of Civil Engineering, Parsabad Moghan Branch, Islamic Azad University, Parsabad Moghan, Iran
a.nazari.g@iaupmogan.ac.ir

Abstract: An analytical analysis of the roller length of a hydraulic jump is presented in this article. An important feature of a hydraulic jump is its roller (or recirculation zone), which plays an important role in dissipating of the energy of a supercritical flow in an open channel. When the flow routine changes in the downward flow, conjugate depths are created and jetting happens at this time. The beginning of the jump or the toe of the jump can easily be fixed as the mean position of the oscillation at the abrupt rise of the water surface. But there has not been any general accord to the location of the end of the jump and it has become a controversial issue. According to experiments and different slope values, some empirical formulas have been offered to specify length of the roller. Whereas there is not much investigation about the determining the roller length of a hydraulic jump, we have done an experimental study to represent the formula to estimate the roller length. Computed roller length is compared with calculated roller length of other researchers gained by different formulas. The roller Length depends on the critical depth which had been determined by using the Buckingham's theorem (pi theorem) in the represented article.
[Giglou AN. **Analytical Analysis of the Hydraulic Jump Roller Length in Open Channel**. *Life Sci J* 2013;10(6s):669-673] (ISSN:1097-8135). http://www.lifesciencesite.com. 105

Keywords: Open Channel Flow; Hydraulic jump; Roller Length; Energy dissipation; Sloping channel

1. Introduction

The specific energy of the flow always decreases during the flow. So the specific energy at any discharge ratio of the flow either decreases or increases. The specific energy of the discharge ratio is depends on the flow depth. If we want to compute the E=f(h), so we have two cases. If $h \to 0$ or $h \to \infty$, the specific energy value, in both cases, will be finite ($E \to \infty$). So there is specific depth between these two depths and the specific energy value in specific depth is minimum. The relating depth to this minimum energy is called Critical depth. According to the experiment, if the flow depth is more than the critical depth the flow will be slow, vice versa the flow will be turbulent. If it is built a hydraulic structure in front of the channel and rivers flow, it will be divided into two parts. One of these parts is approaching to the hydraulic structure and the other one is moving off the hydraulic structure, these flow respectively are called upstream and downstream flow. The depths of flow regime at the upstream and downstream flows are called conjugate depths. Conjugate depths can be at the changing part of the inclination or at the rising part of water level. In open channel flows, whenever the flow profile changes from supercritical to subcritical, hydraulic jumps will occur. A hydraulic jump represents a significant head loss that manifests in available energy for scour and creation of turbulence. Hydraulic jumps are one of the three occurrences of Rapidly Varied Flow. In a hydraulic jump the supercritical flow changes to subcritical flow with a rapid rise of flow depth associated with surface air entrainment. The air-water interaction begins at the toe of the jump that produces surface rollers on upper fluid surface, as displayed in Fig 1. Knowledge of the surface profile of a jump is desirable in designing the freeboard for the retaining walls of the stilling basin where the jump takes place. The hydraulic jump is a sudden transition from a high-velocity, supercritical flow into a slow-moving, subcritical flow. It is characterized by a sharp rise of the free-surface elevation associated with strong energy dissipation, large-scale turbulence, air entrainment, and spray. The turbulent hydraulic jump is one of the oldest and hardest problems in hydraulic engineering. The hydraulic jump in open channel flow is analogous to a shock wave in a compressible fluid in several respects, such as the role of parameter (Froude/Mach number), flow invariants jump conditions, passive role of bottom wall and significance of normal stress in jump structure profile. The length of jump, measured from the toe to some tailwater zone. The ratio of sequent depths was correctly predicted by Belanger (1838) by using the momentum equation. Further, theoretical and experimental studies were conducted by the Frenchmen Bresse (1860), Bazin and Darcy (1865), and Boussinesq (1877); Forchheimer (1914, 1925) gave an excellent summary of these studies. Additional experimental data were provided by Gibson (1914) of which the maximum Froude number is 8.60. Further, contributions to the determination of the length of jump (Sarma and Newnham, 1973; Mehrotra, 1976; Gioia, et al. , 1976, 1982; Ewers, 1987; Hager, Bremen and Kawagoshi, 1990). The sequent depth ratio , the surface profile including the internal flow features (Resh et al, 1976; Gioia, et al., 1977; Swamee and Prasad, 1977; Gill, 1980; Pavlov, 1987; Voinich-Syanozhentskii, 1988; Hager and Bremen, 1989; Hoyt and Sellin, 1989) were also presented. Safranez (1933) reanalysed the length of the roller, based on his own, Einwachter's (1930) and Pietrkowski's (1932) data. Rehbock (1933) presented a Merriman-type equation for the sequent depths ratio. Einwachter (1935) contributed to the length of jump and particularly to the mechanics of roller flow and energy dissipation.

2. General Form of Hydraulic Jump

The hydraulic jump is a natural phenomenon that occurs when supercritical flow is forced to change into subcritical flow because of an obstruction in the flow path. This abrupt change in flow condition is accompanied by considerable turbulence and loss of energy. When the water flow in the channel changes from the supercritical flow ($h_1<h_c$) into subcritical flow ($h_2>h_c$), physical phenomenon of hydraulic jump happens. Hydraulic jump is generated at the limit area of the flow and the flow movement is changing quickly at the same area. The phenomenon is valued as a local phenomenon. Hydraulic jump is generated with a turbulent or standing wave surface. In the first case hydraulic jump is finished and, in second case it is not yet finished. The free level (surface) of the flow is above the normal depth line, between the normal depth (h_n) and critical depth (h_c) or below the critical depth. But experiments were shown that the free surface (level) of the flow sometimes is lesser than the critical depth ($h_1<h_c$, $h_2>h_c$). If the flow would have the supercritical condition at some part of the path and It is nearly to change its condition due to the characteristic and special situation of the channel, flow depth increases considerably at a rarely short path and at the result the speed rate decreases considerably by loss of energy. This is a most important water flow phenomenon in open channels which there are turbulence and horizontal vorticity from its beginning to end, This phenomenon is called hydraulic jump. In this case according to the jumping intensity it can be seen turbulences on the water surface. Jump intensity decreases gradually by the end of the jump. Accordingly water energy decreases because of the change of the energy into heat. Also due to the turbulence and disturbance and water impact with weather, some weather is mixed with the water at the surface part and they have been transferred to the downstream and they have been released in the form of the bubbles. The bottom surface of the channel does not play a predominant role, except that the skin friction at the bottom wall affects the physical location of the hydraulic jump. The flow in the hydraulic jump, above the bottom surface, has been regarded as a turbulent shear layer having an air-water interaction on the upper surface, thus forming the rollers in the mixing layer, the extent of which depend on the magnitude of the upstream Froude number. The analysis of the hydraulic jump in a rectangular channel employed the standard turbulent boundary layer equations subject to hydrostatic pressure distribution. The depth averaged equations were obtained, which failed to account for the loss of specific energy. During the jump the related depth to the contracted depth is called divided depth and its value is computed as follow:

$$h_{div.} = \frac{h_s}{2}\left[\sqrt{1+8\left(\frac{h_c}{h_s}\right)^3}-1\right]$$

Where

h_c is critical depth of the flow;

h_s is the contracted depth and its value is computed at spillway downstream and gate at the end of the chute by the following formula:

$$q = h_s\varphi\sqrt{2g(T_o - h_s)}$$

Where

q is the special discharge;

φ is the velocity coefficient;

and T_o is velocity head.

In engineering practice the hydraulic jump frequently appears downstream from overflow structures (spillways) or underflow structures (sluice gates) where velocities are high. It may be used to effectively dissipate kinetic energy and thus prevent scour of the channel bottom, or to mix chemicals in a water or sewage treatment plant. In design calculations the engineer is concerned mainly with prediction of existence, size, and location of the jump. A hydraulic jump is formed when liquid at high velocity discharges into a zone of lower velocity, creating a rather abrupt rise in the liquid surface (a standing wave) accompanied by violent turbulence, eddying, air entrainment, and surface undulations. To have a jump, there must be a flow impediment downstream. The downstream impediment could be a weir, a bridge abutment, a dam, or simply channel friction. Water depth increases during a hydraulic jump and energy is dissipated as turbulence. Often, engineers will purposely install impediments in channels in order to force jumps to occur. Mixing of coagulant chemicals in water treatment plants is often aided by hydraulic jumps. Concrete blocks may be installed in a channel downstream of a spillway in order to force a jump to occur thereby reducing the velocity and energy of the water. Flow will go from supercritical (F_r>1) to subcritical (F_r<1) over a jump. Hydraulic jump can occur in channel with larger bed slop that the gravitational forces acting on the flow must be included. Some experiments carried out by Bazin (1865), Beebe and Riegel (1917), Ellms (1927), Yarnell (1934), Rindlaub(1935), Bakhmeteff and matzke (1936), Bradley and Peterka (1957), Argyropoulo (1962) and Rajarathnam (1966). The jump length is measured to the downstream section at which the mean water surface attains the maximum depth and becomes reasonable level. Errors may be introduced in determining length since the water surface is rather flat near the end of the jump. This is undoubtedly one of the reasons that so many empirical formulas for determining jump length are found in the literature. The jump length for rectangular basins has been extensively studied. According to the different values of the slopes jump length can be computed by the Bakhmeteff formula and in other conditions by the received formulas of the experiments. Of these experiments we can point to the followings:

N.N.Pavlovski

$$L_s = 2.5(1.9h_2 - h_1);$$

M.D.Getausoff

$$L_s = 10.3h_1 \left\{ \sqrt{\left(\frac{h_c}{h_1}\right)^3 - 1} \right\}^{0.81};$$

Bakhmeteff and Matzke

$$L_s = 5(h_2 - h_1) \text{ and etc.}$$

Where

L_s: is the roller length measured horizontally up to the end of the roller (the horizontal distance between the front face of the roller, toe of the jump, and the end of the recirculation zone);

h_1: is the supercritical depth on horizontal flow;

h_2: is the subcritical sequent depth corresponding to h_1.

Also Chow (1973) proposed some guidelines to estimate the length of the roller of hydraulic jump as a function of the upstream flow conditions. Hager et al. (1990) reviewed a broader range of data and correlations. For wide channel (i.e. h_1/B<0.10), they proposed the following correlation:

$$\frac{L_s}{h_1} = 160 \tanh\left(\frac{Fr_1}{20}\right) - 12 \qquad 2<Fr_1<16$$

where L_s is the length of the roller and B is channel wide. This equation is valid for rectangular horizontal channels with $2<Fr_1<16$. Such a correlation can be used when designing energy dissipation basins.

3. Hydraulic Jump Length

The basic geometric elements of the hydraulic jump (Fig. 1 and 2) are as follow:

h_1- First conjugate depth;

h_2- Second conjugate depth;

$\Delta h= h_2-h_1$ – Jump height;

L_s- length of the roller which is equal to the distance of the jump beginning to a point on the water surface immediately after the last roller wave.

In this situation the height of this point is equal to the tailwater height. In all of the presented experimental formulas to compute the length of hydraulic jump (L_s=f(h_s)), there is a big difference in the comparing of their values. Hydraulic jump length can be computed by using the Buckingham's theorem to critical depth. It is clear that jump length depends on the first and second compressing depth (conjugate depth) correspondent of the most shearing areas, critical depth, v_1 and v_2 velocities, mass density (ρ), dynamic viscosity. The length of the jump is an important factor in the design of stilling basins. The beginning of the jump or the toe of the jump can easily be fixed as the mean position of the oscillation at the abrupt rise of the water surface. But there has not been any general accord as to the location of the end of the jump and has become a controversial issue. Bakhmetoff and Metzke who were the first to investigate systematically the longitudinal elements of the jump, took the end of the jump as the section of maximum surface elevation before the drop off caused by the channel conditions downstream. In fact, because of the flat nature of the water surface, they could only mark out a region in which the end of the jump could be arbitrarily fixed. The jump lengths as given by Bakhmeteff and Matzke are somewhat shorter than the jump lengths produced in wider channels probably because they are affected by the friction of the narrow width of the flume. Stevens while discussing the paper by Bakhmeteff and Matzke proposed that the length of the jump is a result of two motions: first the translatory motion of the water prism downward and secondly the vertical motion due to the rate of conversion of kinetic to potential energy. Another definition which seems to have found favour with earlier investigators is that the end of the jump may be taken as the end of the surface roller. But it has been confirmed, firstly by the experimental results reported by Mavis and Luksch (1936) and later by Rouse et al. that the length of the jump is always greater than the length of the roller. Behera and Qureshy, and Qureshy (1947) defined the length of the jump as the distance between the well defined toe and the section at which a cylinder placed in the flow on the floor of the channel will just topple. At first the cylinder should be placed far downstream and moved back upstream until it is toppled by the flow. The shape and size and weight of the cylinder influences the fixing of the length in addition to the forces exerted on the cylinder being affected by the boundary layer near the channel. This definition is from personal error but of little use for designing purposes. The end of the jump as the section at which the high velocity jet begins to leave the floor or immediately downstream of the roller, whichever is farther away from the toe of the jump. Instead of defining the end of the jump as the section at which the high velocity jet begins to leave the floor it would have been better if the bed velocity at the end in the downstream channel had been chosen as a certain percentage of the approach velocity. The length of the jump as the distance from the toe of the jump to the section where the flow depth reached a value of 98% of the tailwater level. The length of the jump can be defined as the distance at which the pressure fluctuation subsidizes and the free surface becomes constant. However, this cannot be utilized for design purposes and is arbitrary. This length depends on the turbulence levels in the approaching flow and in the roller zone.

4. Analysis Method

According to Buckingham theorem (pi theorem), equation of function can be written as follow:

$$f(h_1, h_2, h_c, v_1, v_2, l_s, \rho, \mu) = 0$$
$$\varphi(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \qquad (1)$$

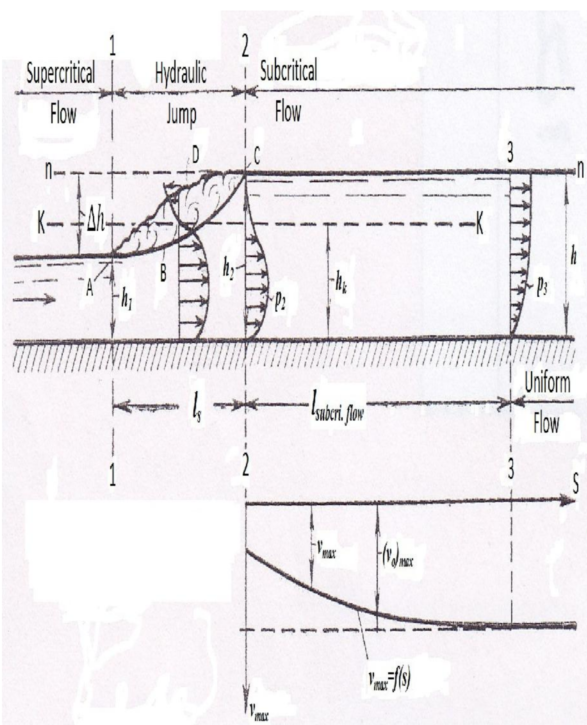

Figure 1. Hydraulic jump in open-channel

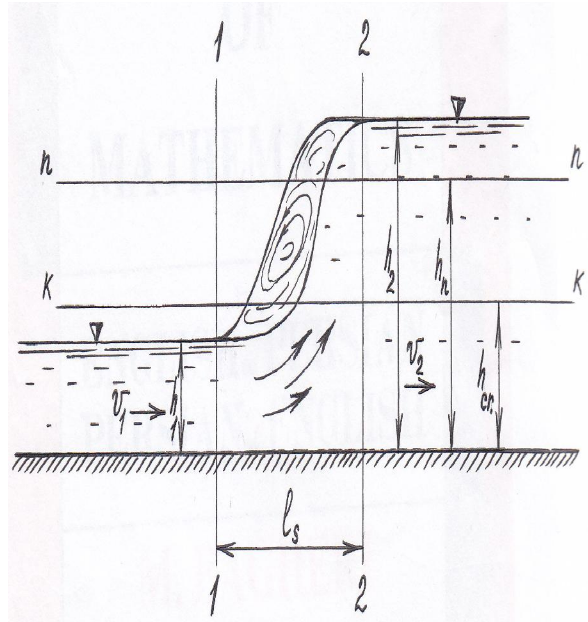

Figure 2. Scheme of hydraulic jump

In this equation π is received from the several physical values and it is dimensionless parameter. It is needed several physical values (n=8) and number of major measured units (m=3) to compute the exact value of π. So the number of dimensionless components will be n-m=5. To compute the value of all π_i, the π components should be selected in way that the three major measurement units can be presented there. For example the mass density of water (ρ), critical depth (h_c) and velocity in first compressing depth of hydraulic jump location (v_1) and the fourth parameter in all of the π-'s will be the remaining physical values. So regarding to these condition dimensionless components equations can be presented as follow:

$$\left.\begin{aligned} \pi_1 &= p^{x_1} h_c^{y_1} v_1^{z_1} l_s \\ \pi_2 &= p^{x_2} h_c^{y_2} v_1^{z_2} v_s \\ \pi_3 &= p^{x_3} h_c^{y_3} v_1^{z_3} \mu \\ \pi_4 &= p^{x_4} h_c^{y_4} v_1^{z_4} h_2 \\ \pi_5 &= p^{x_5} h_c^{y_5} v_1^{z_5} h_1 \end{aligned}\right\} \qquad (2)$$

At the above equations x_i,y_i and z_i are the power values of physical components and their value should be computed in a way that the values of π-'s would be dimensionless. For example the value of $\pi_1$by considering the dimensions of p , h_c andv_1 has been computed by the following method:

$$L^0M^0T^0 = M^{x_1}T^{x_1}L^{-4x_1}L^{y_1}T^{-z_1}L$$

From here

L: $-4x_1+y_1+z_1+1=0$

M: $x_1=0$

T: x_1-z_1=0

So x_1=0, z_1=0 and y_1=-1.

If we replace the values of x_1=0, z_1=0 and y_1=-1 in the π_1 equation, for getting the dimensionless π_1, the following equation will be resulted:

$$\pi_2=\frac{v_2}{v_1};\ \pi_3=\frac{\mu}{\rho h_c^2};\ \pi_4=\frac{h_2}{h_c};\ \pi_s=\frac{h_1}{h_c}$$

We shall write the following equation to compute the hydraulic jump length:

$$\frac{l_s}{h_c}=\varphi\left(\frac{v_2}{v_1};\frac{\mu}{\rho h_c^2};\frac{h_2}{h_c};\frac{h_1}{h_c}\right) \qquad (3)$$

So, the use of the Buckingham's theorem (pi theorem) and statistical method of experimental design theory makes it possible to create a mathematical model for determining the length of the roller in a hydraulic jump in a prismatic rectangular horizontal channel in the form of a regression equation (on the basis of an analysis of the available data)

$$l_s=kh_c \qquad (4)$$

where *k* is the constant and experimental coefficient and its value depends on the flow velocity, conjugate depths, critical depth and dynamic viscosity to compute the hydraulic jump length. The value of *k* is obtained by comparing of Equation (4) with other several experimental equations. The value of k also can be computed by the experimental methods. At least the value of *k* in this research has been 11. For preliminary calculations Equation (4) is reduced to a simplified form

$$l_s=11h_c. \qquad (5)$$

5. Conclusion

A comparison of the results of the calculation with the given formula, with the data obtained by the known relations in hydraulics [1-6], shows that they are satisfactorily accord. A minimum difference is observed with the data obtained by Aivazyan's formula (extreme deviation 3.13% and average in absolute value 1.45%). Calculations by Chertousov's formula give an extreme deviation 7.97% and average 4.48%, and by Pavlovskii's formula the deviations are nearly the same. So the value of length of the roller of hydraulic jump in prismatic horizontal open channels can be computed easily on the condition that the critical depth value will be clear.

Corresponding Author:
Mr. Abolfazl Nazari Giglou
Department of Civil Engineering
Parsabad Moghan Branch, Islamic Azad University,
Parsabad Moghan, Iran
E-mail: a.nazari.g@iaupmogan.ac.ir

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3/12/2013