# SDV-VIKOR: A New Approach for Multi-Criteria Decision Making with No Preference

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**Abstract:** The problem of allocating the weights of criteria when no preference exists has attracted the interest of many scholars. In this paper a new method for allocating weights is presented using the Standard Deviation (SDV) measure. The technique used named *Vlse Kriterijumska Optimizacija I Kompromisno Resenje* in Serbian (VIKOR) is combined to the new method to constitute a new approach called SDV-VIKOR. The new approach can be used when no preference among the criteria considered. Also it is validated and illustrated by ranking the alternatives of a given numerical example.

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## 1. Introduction

People always make decisions in their daily life. However, the most problems are easy to solve, but the more complex problems and more criteria we must have to solve. It produces the multiple criteria decision making problems, when the decisions become more difficult. The merit of MCDM techniques is that they consider both qualitative parameters as well as the quantitative ones, MCDM includes many solution techniques such as Simple Additive Weighting (SAW), Weighting Product (WP) [3], and Analytic Hierarchy Process (AHP) (Saaty,1980). The problem of allocating the weights of criteria when no preference is an open research area. Many scholars tried to tackle this problem by various techniques like Information Entropy Weight method, the weighted average operator (OWA), and other several methods [1].

The Standard Deviation (SDV) is a well known measure of dispersion, which suits the problem of allocating weights in MCDM. In this paper we try to address this problem by employing the Standard Deviation to allocate weights then combining the proposed method to a well-known technique called *Vlse Kriterijumska Optimizacija I Kompromisno Resenje* in Serbian (VIKOR). The new method so-called SDV-VIKOR is applied for ranking alternatives in numerical example given. The rest of this paper is organized as follows: Section 2 is made for the VIKOR approach, the proposed Standard Deviation method is illustrated in section 3, in section 4 a numerical example is given for validation, and finally section 5 is made for conclusion.

## 2. VIKOR

A MCDM problem can be concisely expressed in a matrix format, in which columns indicate criteria (attributes) considered in a given problem; and in which rows list the competing alternatives. Specifically, a MCDM problem with $m$ alternatives ($A_1$, $A_2$, ..., $A_m$) that are evaluated by $n$ criteria ($C_1$, $C_2$, ..., $C_n$) can be viewed as a geometric system with $m$ points in $n$-dimensional space. An element $x_{ij}$ of the matrix indicates the performance rating of the $i^{th}$ alternative $A_i$, with respect to the $j^{th}$ criterion $C_j$, as shown in Eq. (1):

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad (1)$$

The VIKOR method was introduced as an applicable technique to implement within MCDM [4]. It focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. The compromise solution, whose foundation was established by Yu [8] and Zeleny [9] is a feasible solution, which is the closest to the ideal, and here "compromise" means an agreement established by mutual concessions. The VIKOR method determines the compromise ranking list and the compromise solution by introducing the multi-criteria ranking index based on the particular measure of "closeness" to the "ideal" solution. The multi-criteria measure for compromise ranking is developed from the *Lp-metric* used as an aggregating function in a compromise programming method. The levels of regret in VIKOR can be defined as:

$$L_{p,i} = \{\sum_{j=1}^{n} [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)]^p\}^{1/p}, \quad (2)$$

$1 \le p \le \infty$ ,

where $i = 1,2,\ldots,m$. $L_{1,i}$ is defined as the maximum group utility, and $L_{\infty,i}$ is defined as the minimum individual regret of the opponent. The procedure of VIKOR for ranking alternatives can be described as the following steps [2]:

***Step 1*:** Determine that best $x_j^*$ and the worst $x_j^-$ values of all criterion functions, where $j = 1, 2,\ldots, n$ . If the $j^{th}$ criterion represents a benefit then $x_j^* = \max_i f_{ij}$ , $f_j^- = \min_i f_{ij}$ .

***Step 2*:** Compute the $S_i$ (the maximum group utility) and $R_i$ (the minimum individual regret of the opponent) values, $i = 1, 2,\ldots, m$ by the relations

$$S_i = L_{1,i} = \sum_{j=1}^{n} w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-) , \quad (3)$$

$$R_i = L_{\infty,i} = \max_j [\sum_{j=1}^{n} w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)] , \quad (4)$$

where $w_i$ is the weight of the $j^{th}$ criterion which expresses the relative importance of criteria.

***Step 3*:** Compute the value $Q_i$ , $i = 1,2,\ldots,m$ , by the relation

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*) , \quad (5)$$

where $S^* = \min_i S_i$ , $S^- = \max_i S_i$ , $R^* = \min_i R_i$ , $R^- = \max_i R_i$ , and $v$ is introduced weight of the strategy of $S_i$ and $R_i$ .

***Step 4*:** Rank the alternatives, sorting by the *S, R,* and *Q* values in decreasing order. The results are three ranking lists.

***Step 5*:** Propose as a compromise solution the alternative ($A'$) which is ranked the best by the minimum *Q* if the following two conditions are satisfied:

**C1.** "Acceptable advantage":
$Q(A'') - Q(A') \ge DQ$ , where $A''$ is the alternative with second position in the ranking list by *Q*, $DQ = 1/(m - 1)$ and *m* is the number of alternatives.

**C2.** "Acceptable stability in decision making":
Alternative $A'$ must also be the best ranked by *S* or/and *R*. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" ($v \approx 0.5$) , or "with vote" ($v < 0.5$). Here, $v$ is the weight of the decision making strategy "the majority of criteria" (or "the maximum group utility"). $v = 0.5$ is used in this paper. If one of the conditions is not satisfied, then a set of compromise solutions is proposed [2].

Recently, VIKOR has been widely applied for dealing with MCDM problems of various fields, such as environmental policy [6], data envelopment analysis [7], and personnel training selection problem [1].

## 3. Standard Deviation for Allocating Weights

In this paper, the well known standard deviation (*SDV*) is applied to allocate the weights of different criteria. The weight of the criterion reflects its importance in MCDM. Range standardization was done to transform different scales and units among various criteria into common measurable units in order to compare their weights.

$$x'_{ij} = \frac{x_{ij} - \min_{1\le j\le n} x_{ij}}{\max_{1\le j\le n} x_{ij} - \min_{1\le j\le n} x_{ij}} \quad (6)$$

$D'=(x')_{mxn}$ is the matrix after range standardization; max $x_{ij}$, min $x_{ij}$ are the maximum and the minimum values of the criterion ($j$) respectively, all values in $D'$ are ($0 \le x'_{ij} \le 1$). So, according to the normalized matrix $D'= (x')_{mxn}$ the standard deviation is calculated for every criterion independently as shown in Eq. (7):

$$SDV_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(x'_{ij} - \bar{x}'_j)^2} \quad (7)$$

where $\bar{x}'_j$ is the mean of the values of the $j^{th}$ criterion after normalization and $j = 1,2,\ldots,n$.

After calculating (*SDV*) for all criteria, the weight ($W_j$) of the criterion ($j$) can be defined as:

$$W_j = \frac{SDV_j}{\sum_{j=1}^{n} SDV_j} \quad (8)$$

where $j = 1,2,\ldots,n$.

## 4. Numerical Example

In this section, an example of six alternatives to be ranked through comparing five criteria is presented to explain the method proposed. As shown in Table 1, the six alternatives and their performance ratings with respect to all criteria are presented. All criteria to be minimized (the minimum is better).

Table 1. Decision matrix

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| Alternative 1 | 45 | 10 | 6 | 12 | 160 |
| Alternative 2 | 48 | 16 | 1 | 51 | 220 |
| Alternative 3 | 46 | 8 | 5 | 36 | 190 |
| Alternative 4 | 39 | 14 | 8 | 7 | 164 |
| Alternative 5 | 52 | 9 | 7 | 34 | 172 |
| Alternative 6 | 41 | 2 | 4 | 50 | 185 |

In the above example, there is no preference among the criteria, no weights specified for them subjective by the decision maker, so the Standard Deviation method will be applied in this problem. Table 2 illustrates the range standardization done to decision matrix as in Eq.(6).

Table 2. Range standardized decision matrix

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| Alt. 1 | 0.4615 | 0.5714 | 0.7143 | 0.1136 | 0 |
| Alt. 2 | 0.6923 | 1 | 0 | 1 | 1 |
| Alt. 3 | 0.5385 | 0.4286 | 0.5714 | 0.6591 | 0.5 |
| Alt. 4 | 0 | 0.8571 | 1 | 0 | 0.0667 |
| Alt. 5 | 1 | 0.5 | 0.8571 | 0.6136 | 0.2 |
| Alt. 6 | 0.1538 | 0 | 0.4286 | 0.9773 | 0.4167 |

Table 3 shows the values of the Standard Deviation ($SDV_j$), and the weight assigned to each criterion ($W_j$) as shown in Eqs. (7 and 8). The weights' assignment process is very sensitive which will be reflected on the final ranking of the alternatives.

Table 3. Weights assigned to criteria

| | $SDV_j$ | $W_j$ |
|---|---|---|
| $C_1$ | 0.362165 | 0.1949 |
| $C_2$ | 0.35114 | 0.1890 |
| $C_3$ | 0.354754 | 0.1910 |
| $C_4$ | 0.42267 | 0.2275 |
| $C_5$ | 0.367033 | 0.1976 |

By applying the procedure of VIKOR, we can calculate the $S$, $R$ and $Q$ values as shown in Table 4 to derive the preference ranking of the alternatives. The first alternative should be selected because it has the minimum $S$, $R$, and $Q$ values; also, the two conditions mentioned earlier in section 2 are satisfied.

Table 4. Ranking lists and scores

| | $S$ | $R$ | $Q$ | Rank |
|---|---|---|---|---|
| Alt. 1 | 0.360235 | 0.136429 | 0 | **1** |
| Alt. 2 | 0.749031 | 0.2275 | 1 | 6 |
| Alt. 3 | 0.543832 | 0.149943 | 0.310308 | 3 |
| Alt. 4 | 0.366173 | 0.191 | 0.307245 | 2 |
| Alt. 5 | 0.632237 | 0.1949 | 0.67082 | 5 |
| Alt. 6 | 0.416505 | 0.22233 | 0.543978 | 4 |

## 5. Conclusion

In this paper, the SDV-VIKOR proposed method is presented and illustrated. The new method employed the Standard Deviation to allocate the weights in MCDM problems. The standard deviation describes the dispersion of the values of criteria, giving the more dispersed values criteria more importance and much weights. The VIKOR method is combined to the proposed method to rank the alternatives. The proposed approach is illustrated by solving a numerical example.

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