

Selection Rules of Nuclear Parameters for Neutrinoless Double Beta Decay

M. H. Sidky

Dept. of Eng. Math. And Physics, Faculty of Engineering, Cairo University, EGYPT
Sidkym@yahoo.com

Abstract: Theoretical and experimental nuclear data are collected from different sources to establish selection rules for the probability of the neutrino less double beta decay and neutrino mass. In this work the proposed selection rules have been applied to five double beta decay emitters by using different alterations of the QRPA model within the available range of the strength of the particle-particle interaction $0.8 \leq g_{pp} \leq 1.2$. It is found that acceptable results have been obtained with the emitters ^{100}Mo , ^{130}Te by using full-RQRPA, pn-RQRPA techniques with small basis of Hilbert space. New value for neutrino mass $m_\nu \pm \delta m_\nu = 0.262 \pm 0.009$ eV is determined. Such value agrees with the available experimental determinations and improves the relative uncertainty $\delta m_\nu / m_\nu$ from 12.5% to 3.43%. [M. H. Sidky. **Selection Rules of Nuclear Parameters for Neutrinoless Double Beta Decay.** *Life Sci J* 2013;10(4):3516-3521]. (ISSN:1097-8135). <http://www.lifesciencesite.com>. 468

Keywords: phase space factor, nuclear matrix element, probability of $0\nu\beta\beta$ decay mode, pn-QRPA technique

1. Introduction

Double beta decay may occur with two neutrino emission ($2\nu\beta\beta$ mode) or without neutrinos emerge ($0\nu\beta\beta$ mode). Nuclear physics can contribute to the solution of questions concerning the nature of neutrinos by investigation of $0\nu\beta\beta$ mode. This mode can only occur for massive Majorana neutrinos [1]. The discovery of neutrino mass m_ν in oscillation experiments [2] makes the search for the nature of massive neutrinos [3] particularly relevant and timely. Kinematic measurements restrict the neutrino mass scale to be below 1 eV [4].

The decay rate of the $0\nu\beta\beta$ decay mode $\omega^{0\nu}$ is expressed by [5]:

$$\omega^{0\nu} = \ln 2 G^{0\nu} (M^{0\nu} m_\nu)^2 \quad (1)$$

The ratio between the decay rate of the $0\nu\beta\beta$ decay mode $\omega^{0\nu}$ and the total decay rate ω^T is the probability of occurrence of the $0\nu\beta\beta$ mode $R^{0\nu}$. It can be expressed by using eqn. (1) as follows:

$$R^{0\nu} = \ln 2 G^{0\nu} (M^{0\nu} m_\nu)^2 / \omega^T \quad (2)$$

The phase space factor $G^{0\nu}$ of the $0\nu\beta\beta$ decay mode are calculated by using the integral form of the $0\nu\beta\beta$ decay mode. The results are listed in another work [5] for some double beta decay emitters. Table (1) presents the minimum values of the total decay rate $(\omega^T)_{\min}$

for five $\beta\beta$ emitters: ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe . They are calculated as well as the reported experimental lower limits [6-9] of the half lives for the $0\nu\beta\beta$ decay mode

$(T^{0\nu})_{\min}$ of these emitters. $M^{0\nu}$ is the nuclear matrix element of the $0\nu\beta\beta$ decay mode of an

emitter. It consists of three main factors: (1) Gamow-Teller transition matrix element $(M_{GT})^{0\nu}$ of the $0\nu\beta\beta$ decay mode. (2) Fermi transition matrix element $(M_F)^{0\nu}$ of the same mode. (3) the ratio g_ν / g_A ,

where g_ν , g_A are the vector, axial-vector nuclear currents. They are related to $M^{0\nu}$ by the following relation: $M^{0\nu} = (M_{GT})^{0\nu} - (g_\nu / g_A)^2 (M_F)^{0\nu}$.

Previously [5] the parameter $M^{0\nu}$ has been evaluated by pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small and large basis of the Hilbert space. The proton-neutron quasi particle random phase approximation (pn-QRPA) have clarified that the particle-particle interaction, which is the counterpart of the particle-hole interaction, enhances the spin-isospin correlations in the ground-state wave functions. The SQRPA technique uses the boson expansion for the phonon and β operators associated with pn-QRPA technique [10]. An alternative approach for extending pn-QRPA is based on the idea of partial restoration of the Pauli exclusion principle by taking into account the next terms in the commutator of the like nucleon operator involved in the derivation of the pn-QRPA equations [5]. The commutator is replaced by its expectation value in the RPA (correlated) g.s and this leads to a renormalization of the relevant operators and of the forward and backward going QRPA amplitudes as well. This technique is called pn-RQRPA. It has been extensively used for both $2\nu\beta\beta$ and $0\nu\beta\beta$ decay modes and for transition to g.s. and excited states and for different nuclei [11,12]. The extension of this technique when the proton-neutron pairing interactions, besides the proton-proton and neutron-neutron ones, are also included was called the full-RQRPA[13].

This work presents a procedure to determine m_ν and its uncertainty δm_ν as well as theoretical data [5] of $G^{0\nu}$, $M^{0\nu}$ and experimental determinations of $(\omega^T)_{\min}$ gathered from different laboratories [see table (1)].

Table (1) experimental data for five $\beta\beta$ decay emitters

Emitter	$(\omega^T)_{\min} \times 10^{-24} (y^{-1})$	Experiment
^{82}Se	1.917	NEM – 3 [6]
^{100}Mo	0.627	NEM – 3 [6]
^{116}Cd	4.059	SOLOTVINO [7]
^{130}Te	0.230	CUORICINO [8]
^{136}Xe	0.153	DAMA [9]

Characteristic of $\omega^{0\nu}$ with g_{pp}

The variation of $M^{0\nu}$ with the strength of the particle- particle interaction g_{pp} is well known [5]. According to eqn. (1) the variation of $\omega^{0\nu}$ with g_{pp} should be looks like the change of $M^{0\nu}$ with g_{pp} assuming that m_ν is kept constant. The expected behaviour of $\omega^{0\nu}$ versus g_{pp} is presented in fig. (1). In such figure there are two distinct values: $\omega^{0\nu} = 0.5 \omega^T$ and $\omega^{0\nu} = \omega^T$. They correspond to $R^{0\nu} = 0.5$ and $R^{0\nu} = 1$. These values form three different regions:

- (1) $0 < \omega^{0\nu} < 0.5 \omega^T$.
- (2) $0.5 \omega^T < \omega^{0\nu} < \omega^T$.
- (3) $\omega^T < \omega^{0\nu}$.

In each one of these regions there is a sample for the characteristic of $\omega^{0\nu}$ with g_{pp} [see fig. (1)].

Those samples have different properties as presented in table (2).

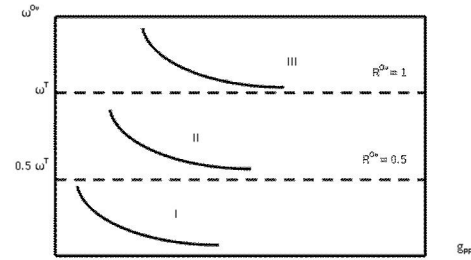


Fig. (1) Characteristics of $\omega^{0\nu}$ with g_{pp} . The two decay modes $2\nu\beta\beta$ and $0\nu\beta\beta$ are equally probable at $R^{0\nu} = 0.5$. They are forbidden for $R^{0\nu} > 1$

Table (2) Comparison between different regions of $\omega^{0\nu}$, $R^{0\nu}$

$\omega^{0\nu}$	$0 < \omega^{0\nu} < 0.5 \omega^T$	$0.5 \omega^T < \omega^{0\nu} < \omega^T$	$\omega^T < \omega^{0\nu}$
$R^{0\nu}$	$0 < R^{0\nu} < 0.5$	$0.5 < R^{0\nu} < 1$	$1 < R^{0\nu}$
Curve	I	II	III
Decay modes	$2\nu\beta\beta$ decay mode is more probable than $0\nu\beta\beta$ decay mode.	$0\nu\beta\beta$ decay mode is more probable than $2\nu\beta\beta$ decay mode.	$2\nu\beta\beta$ and $0\nu\beta\beta$ decay modes are Forbidden.

Limits for $R^{0\nu}$, m_ν

The variation of $M^{0\nu}$ with the strength of the particle- particle interaction g_{pp} has been shown graphically [5] by using pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small and large basis of the Hilbert space. This has been utilized in this work as well as eqn. (2) to generate another variation for $R^{0\nu}$ with g_{pp} such that m_ν is kept constant. A set of distributions of $R^{0\nu}$ with g_{pp} are presented in fig. (2). The limits of the available range of g_{pp} are g_s and g_f . According to eqn. (2) the lowest and highest distributions shown in fig. (2) correspond to the lower and upper neutrino mass limits m_1 , m_2 . The vertical coordinates R_{\min} , R_{m1} , R_{m2} , R_{\max} shown in fig. (2) are related to m_1 , m_2 by [see eqn. (2)]:

$$R_{\max} = C (M_s m_2)^2 \tag{3}$$

$$R_{m2} = C (M_f m_2)^2 \tag{4}$$

$$R_{m1} = C (M_s m_1)^2 \tag{5}$$

$$R_{\min} = C (M_f m_1)^2 \tag{6}$$

In the above equations $C = \ln 2 G^{0\nu} / \omega^T$ and M_s , M_f are the nuclear matrix elements of the $0\nu\beta\beta$ decay mode which correspond to g_s , g_f .

$$R^{0\nu}$$

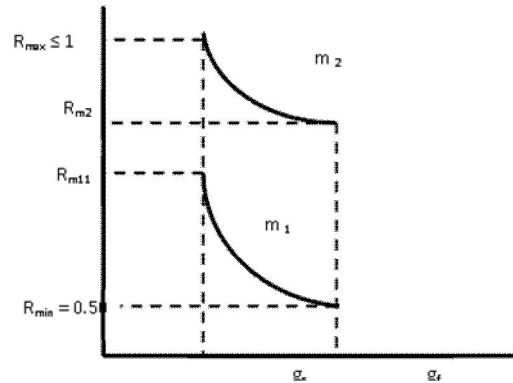


Fig. (2) The lower and upper limits of neutrino mass are m_1 , m_2 . The available range of g_{pp} is: $g_s \leq g_{pp} \leq g_f$. The $0\nu\beta\beta$ decay mode is more probable than the $2\nu\beta\beta$ decay mode within the range $0.5 < R^{0\nu} < 1$.

Selected range for m_1 , m_2

In fig. (2) the lower and upper neutrino mass limits m_1 , m_2 are related to the neutrino mass m_ν and its uncertainty δm_ν by the following relations:

$$m_\nu = (m_2 + m_1) / 2 \tag{7}$$

$$\delta m_\nu = (m_2 - m_1) / 2 \tag{8}$$

The neutrino mass m_ν and its uncertainty δm_ν are determined experimentally to be:

$$m_\nu \pm \delta m_\nu = 0.240 \pm 0.030 \text{ eV}, \delta m_\nu / m_\nu = 12.50 \% \quad [14] \quad (9)$$

$$m_\nu \pm \delta m_\nu = 0.225 \pm 0.075 \text{ eV}, \delta m_\nu / m_\nu = 33.33 \% \quad [15] \quad (10)$$

The neutrino mass limits m_1, m_2 should satisfy the following conditions:

(1) Consistency with the results given by eqns. (9), (10). To do that: $m_1 \leq 0.27 \text{ eV}$, $m_2 \geq 0.21 \text{ eV}$

(2) Improve the experimental relative uncertainty $\delta m_\nu / m_\nu$ given by eqns. (9), (10) to be $\delta m_\nu / m_\nu < 12.5 \%$. According to eqns. (7-10) this means: $m_2 / m_1 < 9 / 7$.

Choice of $R_{\min}, R_{m1}, R_{m2}, R_{\max}$

Eqns. (3), (4), (5), (6) can be written in another form:

$$R_{m1} = R_{\min} (M_s / M_f)^2 \quad (11)$$

$$R_{m2} = R_{\min} (m_2 / m_1)^2 \quad (12)$$

$$R_{m1} = R_{\max} (m_1 / m_2)^2 \quad (13)$$

$$R_{m2} = R_{\max} (M_f / M_s)^2 \quad (14)$$

It should be noticed that $M_s > M_f$ [5] and $m_2 > m_1$ [see fig. (2)]. Thus according to eqns. (11), (12) the lower limits of R_{m1} and R_{m2} should be greater than R_{\min} . Also in eqns. (13), (14) the upper limits of R_{m1} and R_{m2} should be smaller than R_{\max} . In this work the lower and upper limits of R_{m1} and R_{m2} are selected to be: $(R_{\min} + 5\%)$ and $(R_{\max} - 5\%)$. Also the parameter R_{\min} is selected to be 50%. It is the boundary between two regions: $R^{0\nu} > 0.5$ ($0\nu\beta\beta$ decay mode is more probable than $2\nu\beta\beta$ decay mode) and $R^{0\nu} < 0.5$ ($2\nu\beta\beta$ decay mode is more probable than $0\nu\beta\beta$ decay mode).

Selection rules for $m_1, m_2, R_{\min}, R_{m1}, R_{m2}, R_{\max}$

In this work the Selection rules for $m_1, m_2, R_{\min}, R_{m1}, R_{m2}, R_{\max}$ are:

$$m_1 \leq 0.27 \text{ eV} \quad (15)$$

$$m_2 \geq 0.21 \text{ eV} \quad (16)$$

$$m_2 / m_1 < 9 / 7 \quad (17)$$

$$55\% < R_{\max} \leq 100\% \quad (18)$$

$$R_{\min} = 50\% \quad (19)$$

$$55\% \leq R_{m1} < R_{\max} - 5\% \quad (20)$$

$$55\% \leq R_{m2} < R_{\max} - 5\% \quad (21)$$

Determination of m_2, R_{\max} for $^{82}\text{Se}, ^{100}\text{Mo}, ^{116}\text{Cd}, ^{130}\text{Te}, ^{136}\text{Xe}$ at $g_s = 0.8$ [5]

It has been reported [5] that $g_s = 0.8$ for all the double beta decay emitters and nuclear techniques used in this work. The upper limit of $R_{\max} = 100\%$ [see eqn. (18)] has been used in eqn. (3) to calculate m_2 associated with ($0\nu\beta\beta$) decay mode of $^{82}\text{Se}, ^{100}\text{Mo}, ^{116}\text{Cd}, ^{130}\text{Te}, ^{136}\text{Xe}$. This has been done by using the reported values [5] of M_s obtained from pn-QRPA, pn-RQRPA, full-RQRPA and SQRPA techniques with small (S) and large (L) basis of Hilbert space. The results of calculations are presented in table (3).

In table(3) * denotes the cases which verify eqns. (15), (16), (17). For the other cases another trial is carried out in which the upper limit of $m_1 = 0.27 \text{ eV}$ [see eqn. (15)] has been used in eqn. (17) to give the upper limit of $m_2 = 0.347 \text{ eV}$. This limit is used in eqn. (3) to calculate R_{\max} as well as M_s [5]. The results of calculations are listed in tables (4,5). The cases presented in table (4) verify eqn. (18). On the other hand the cases shown in table (5) should be disappointed as they fail to satisfy eqn. (18).

Table (3) m_2 in (eV) and $R_{\max} = 100\%$ for some ($0\nu\beta\beta$) decay emitters by using different nuclear models with small (S) and large (L) basis of Hilbert space. The values of m_2 which verify eqns. (15), (16), (17) are denoted by *

Pn-QRPA		Pn-RQRPA		Full-RQRPA		SQRPA		Emitter
S	L	S	L	S	L	S	L	
0.902	1.090	1.190	1.648	1.123	1.775	1.225	1.429	^{82}Se
0.264 *	0.376	0.309 *	0.299 *	0.299 *	0.362	0.356	0.377	^{100}Mo
1.478	1.887	1.309	1.727	1.964	2.249	1.215	1.236	^{116}Cd
0.302 *	0.349	0.281 *	0.3797	0.271 *	0.362	0.608	0.745	^{130}Te
0.5445	0.664	0.514	0.560	0.495	0.623	0.279 *	0.296 *	^{136}Xe

Table (4) $R_{\max} > 55 \%$ for some $0\nu\beta\beta$ decay emitters with different nuclear techniques

Pn-QRPA		Pn-RQRPA		Full-RQRPA		SQRPA		Emitter
S	L	S	L	S	L	S	L	
	85.17%				91.88%	95.01%	84.72%	^{100}Mo
	98.86%		83.52%		91.88%			^{130}Te

Table (5) $R_{\max} < 55 \%$ for some $0\nu\beta\beta$ decay emitters with different nuclear techniques

Pn-QRPA		Pn-RQRPA		Full-RQRPA		SQRPA		Element
S	L	S	L	S	L	S	L	
14.81%	10.15%	8.51%	4.43%	9.55%	3.82%	8.03%	5.90%	^{82}Se
5.52%	3.38%	7.03%	4.03%	3.12%	2.38%	8.16%	7.89%	^{116}Cd
						32.57%	21.69%	^{130}Te
40.61%	27.31%	45.58%	38.39%	49.14%	31.02%			^{136}Xe

Determination of m_1, R_{m2} for $^{100}\text{Mo}, ^{130}\text{Te}, ^{136}\text{Xe}$ at $g_f = 1.2$

The lower limit of the neutrino mass m_1 is calculated from eqn. (12), (19) to be:

$$m_1 = m_2 (0.5 / R_{m2})^{1/2} \quad (22)$$

Eqn. (4) shows that R_{m2} is calculated as well as M_f . The determination of M_f [5] depends on the choice of g_f . The factor g_f must be carefully selected

to keep the parameter $R_{m2} > 55\%$ [see eqn. (21)]. In this work $g_f = 1.2$ has been used to determine M_f [5]. This is done for pn-QRPA, pn-RQRPA, full-RQRPA and SQRPA techniques with small and large basis of Hilbert space to calculate R_{m2} from eqn. (4) for a set of emitters presented in table (6). This set contains all the cases presented in table (4) and those denoted by * in table (3).

Table (6) R_{m2} for some $0\nu\beta\beta$ decay emitters with different nuclear techniques

Pn-QRPA		Pn-RQRPA		Full-RQRPA		SQRPA		Emitter
S	L	S	L	S	L	S	L	
54.67%	26.94%	45.92%	51.16%	59%	32.08%	9.93%	4.83%	^{100}Mo
30.98%	3.74%	67.38%	39.60%	57.56%	50.82%			^{130}Te
						8.73%	7.94%	^{136}Xe

The values of R_{m2} presented in table (6) indicate that the cases which verify eqn. (21) are

(1) ^{100}Mo with small basis of Hilbert space in full-RQRPA technique.

(2) ^{130}Te with small basis of Hilbert space in pn-RQRPA and full-RQRPA techniques.

The upper and lower limits of $R^{0\nu}, m_\nu$ for these emitters are in the shown distributions of figs. (3-5). A new values of $m_\nu, \delta m_\nu$ are determined and listed in table (7).

All the listed values of $m_\nu \pm \delta m_\nu$ in table (7) agree with the available experimental determinations [see eqns. (9), (10)]. In table (7) the most precise result is $m_\nu \pm \delta m_\nu = 0.262 \pm 0.009$ (eV). Its relative uncertainty $\delta m_\nu / m_\nu = 3.43 \%$ is better than the previous experimental one 12.5 % [see eqn. (9)] by a factor of about 4.

Table (7) new determinations for $m_\nu \pm \delta m_\nu$

Emitter	technique	Base	$m_\nu \pm \delta m_\nu$ (eV)	$\delta m_\nu / m_\nu$ %
^{100}Mo	full-RQRPA	Small	0.287 ± 0.012	4.18
^{130}Te	pn-RQRPA	Small	0.262 ± 0.02	7.46
^{130}Te	full-RQRPA	Small	0.262 ± 0.009	3.43

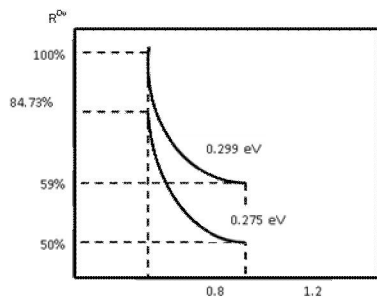


Fig. (3) Limits of $R^{0\nu}, m_\nu$ for ^{100}Mo with small basis of Hilbert space in Full-RQRPA technique.

$50\% \leq R^{0\nu} \leq 84.73\%$ for $m_\nu = 0.275$ eV
 $59\% \leq R^{0\nu} \leq 100\%$ for $m_\nu = 0.299$ eV

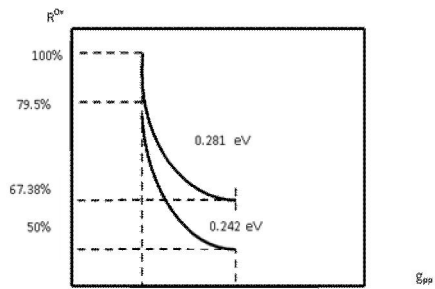


Fig. (4) Limits of $R^{0\nu}, m_\nu$ for ^{130}Te with small basis of Hilbert space used in Pn-RQRPA technique
 $50\% \leq R^{0\nu} \leq 79.5\%$ for $m_\nu = 0.242$ eV
 $67.38\% \leq R^{0\nu} \leq 100\%$ for $m_\nu = 0.281$ eV

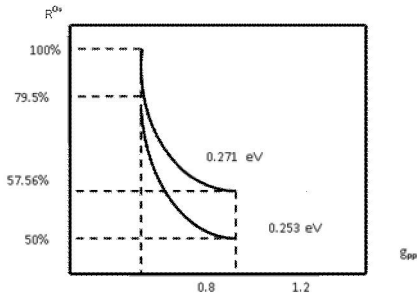


Fig. (5) Limits of $R^{0\nu}, m_\nu$ for ^{130}Te with small basis of Hilbert space used in full-RQRPA technique
 $50\% \leq R^{0\nu} \leq 79.5\%$ for $m_\nu = 0.253$ eV
 $57.56\% \leq R^{0\nu} \leq 100\%$ for $m_\nu = 0.271$ eV

Recommendations

There are some recommendations for the emitters used in this work:

(I) $^{100}\text{Mo}, ^{130}\text{Te}$:

The pn-RQRPA and full-RQRPA techniques with small basis of the Hilbert space have been used successfully with these emitters to give acceptable results for neutrino mass and probability of the $0\nu\beta\beta$ decay mode. This means that renormalization of the

g_{pp}

relevant operators and of the forward and backward going QRPA amplitudes used in pn-RQRPA and full-RQRPA techniques with small basis of the Hilbert space is appropriate to describe the $0\nu\beta\beta$ decay mode of ^{100}Mo , ^{130}Te .

(II) ^{82}Se , ^{116}Cd , ^{136}Xe :

These emitters fail to give an acceptable results for all the techniques used in this work.

It may be possible to improve the unaccepted values of neutrino mass listed in table (3) for the emitters ^{82}Se , ^{116}Cd , ^{136}Xe . This can be done by reducing such values by a factors of about 5, 7, 2 respectively to make consistency with the experimental results given by eqns. (9), (10). According to eqn. (2) this reduction corresponds to 3 different cases:

Case (1):

In this case $R^{0\nu}$, $M^{0\nu}$, ω^T are kept without change and the reported values of $G^{0\nu}$ [5] are multiplied by a factors of about 25, 50, 4. This corresponds to multiply the Q-values of these emitters by another factors: 1.9, 2.2, 1.3 because $G^{0\nu}$ is approximately proportional to Q^5 [16]. It is not possible to verify this case because the Q-values of the emitters ^{82}Se , ^{116}Cd , ^{136}Xe are well determined by the Atomic Mass Evaluation [17].

Case (2):

In this case ω^T , $G^{0\nu}$, $R^{0\nu}$ are kept constant while the values of $M^{0\nu}$ of the emitters ^{82}Se , ^{116}Cd , ^{136}Xe [5] are multiplied by a factors of about 5, 7, 2 respectively. Such multiplication produces a new values of $M^{0\nu}$ which are far from the normal range determined by many groups [5,18,19,20,21,22]. Therefore this case is not acceptable.

Case (3):

In such case $G^{0\nu}$, $M^{0\nu}$, $R^{0\nu}$ are kept constant. The values of ω^T listed in table (1) should be

divided approximately by 25, 50, 4 to be: $0.0767 \times 10^{-24} \text{ y}^{-1}$, $0.0811 \times 10^{-24} \text{ y}^{-1}$, $0.038 \times 10^{-24} \text{ y}^{-1}$ for the emitters ^{82}Se , ^{116}Cd , ^{136}Xe respectively.

Conclusion

This work presents a procedure to determine the probability of the $0\nu\beta\beta$ decay mode and neutrino mass by using two conditions. These conditions are: (1) consistency between the theoretical and experimental determinations of neutrino mass (2) the $0\nu\beta\beta$ mode is more probable than the $2\nu\beta\beta$ mode. These conditions have been applied for five

double beta decay emitters: ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe by using pn-QRPA, pn-RQRPA, full-RQRPA, SQRPA techniques with small and large basis of the Hilbert space. This is carried out within the available range of the strength of the particle-particle interaction $0.8 \leq g_{pp} \leq 1.2$. The emitters ^{100}Mo , ^{130}Te are succeeding in verifying the above

conditions by using pn-RQRPA and full-RQRPA techniques with small basis of the Hilbert space. A new theoretical limits for neutrino mass has been calculated to be: $0.271 \leq m_\nu \leq 0.253 \text{ eV}$. Such limits agree with the available experimental determinations and improve the relative uncertainty $\delta m_\nu / m_\nu$ by a factor of about 4. The probability of the $0\nu\beta\beta$ decay mode varies between 50 % and 100 % within the range $0.271 \leq m_\nu \leq 0.253$.

Acknowledgement

Thanks for Prof. Dr. M.Hesham for his assistance

References

1. III Avignone, Frank T., Steven R et al, "Double Beta Decay, Majorana Neutrinos, and Neutrino Mass " Rev. Mod. Phys. 80 (2008) 481.
2. Alberico W.M., Bilenky S.M., "Neutrino Oscillations, Masses and Mixing" Phys. Part. Nucl. 35 (2004) 297.
3. Petcov S.T., "The Nature of massive Neutrinos" Adv. High Energy phys. Part. 2013 (2013) 852987.
4. Aseev V.N., Belesev A.I., Berlev A.I *et al.*, "An Upper Limit on Electron Antineutrino Mass from Troitsk Experiment" Phys. Rev. D 84 (2011) 112003.
5. Stoica S.and Klapdor-Kleingrothaus H.V. "Critical View On Double-Beta Decay Matrix Elements Within Quasi Random Phase Approximation –based methods" Nucl. Phys. A 694 (2001) 269.
6. Barabash A.S., "Double Beta Decay Experiments" Phys. of Particles and Nuclei, Vol 42, No. 4, (2011) 613.
7. Danevich F.A., Georgadze A.Sh., Kobychov V.V. *et al.*, "Search for 2β Decay of Cadmium and Tungsten isotopes: Final Results of the Solotvina Experiment" Phys. Rev. C68 (2003) 035501.
8. Amaboldi C., Artusa D.R., Avignone III F.T. *et al.* "Results from a Search for the $0\nu\beta\beta$ Decay of ^{130}Te " Phys. Rev. C78 (2008) 035502.
9. Bernabei R., Belli P., Cappella F. *et al.*, "Investigation of $\beta\beta$ Decay Modes in ^{134}Xe , ^{136}Xe " Phys. Lett. 546 (2002) 23.
10. Raduta A.A., Faessler A. and Stoica S, "The Decay Rate Within a Boson Expansion Formalism" Nucl. Phys. A 534 (1991) 149.
11. Schwiieger J., Simkovic F. and Amand Faessler, "Double β Decay to Excited States of Several Medium- Heavy Nuclei Within the Renormalized Quasiparticle Random Phase us Approximation" *et al.* Phys. Rev. C57 (1998)1738.

12. Suhonen J. and O.Civitarese, "Weak Interaction and Nuclear-Structure Aspects of Nuclear Double Beta Decay" Phys. Rep. 300, (1998) 123.
13. Simkovic F., Schwieger, Veselsky *et al.*, "Non Collapsing Renormalized QRPA with Proton-Neutron Pairing for Neutrinoless Double Beta Decay" Phys. Lett. 393 (1997)267
14. Faessler A., "Nuclear Structure, Double Beta Decay and Neutrino mass" Journal of Physics Conference Series 267 (2011) 012059.
15. Ejiri H., "Double Beta Decays and Neutrino Masses" Journal of Physical Society of Japan, Vol 74, No. 8, (Aug. 2005) 2101.
16. Moe M. and P.Vogel," Double Beta Decay" Ann. Rev. Nucl. Part. Sci.,44 (1994) 254.
17. Wapstra A.H., Audi G. and Thibault C., "The AME 2003 Atomic Mass Evaluation" Nucl. Phys. A 729 (2003) 129-336.
18. Haxton W.C. and G.J.Stephenson Jr., " Double Beta Decay" Prog. Part. Nucl. Phys. 12 (1984) 409..
19. Civitarese O., Amand Faessler and Tomoda T., " Suppression of the Two Neutrino $\beta\beta$ Decay" Phys. Lett. B194 (1987)11.
20. Tomoda T. and Amand Faessler, " Suppression of the Neutrinoless $\beta\beta$ Decay" Phys. Lett. B199, No. 4 (1987) 475.
21. Engel J. and Vogel P., " Double Beta Decay in the Generalized – Seneority Scheme" *et al.*, Phys. Lett. B225, No. 1,2 (1989)5.
22. Muto K., Bender E., Klapdor.H.V, " Nuclear Structure Effects on the Neutrinoless Double Beta Decay. *et al.*, Z. Phys A334 (1989)187.

12/23/2013