

## Convergence of Intuitionistic Fuzzy Filters in Syntopogenous Intuitionistic fuzzy Structures

Tantawy O., F. M. Sleim, Z. Abueldahb

Mathematics Department, Faculty of science, Zagazig University, Egypt.  
[drosamat@yahoo.com](mailto:drosamat@yahoo.com)

**Abstract:** In this paper we introduce the intuitionistic fuzzy filter, convergence of intuitionistic fuzzy filter in syntopogenous intuitionistic fuzzy spaces and their properties are also studied.

[Tantawy O., F. M. Sleim, Z. Abueldahb. **Convergence of Intuitionistic Fuzzy Filters in Syntopogenous Intuitionistic fuzzy Structures.** *Life Sci J* 2013;10(4):2938-2945]. (ISSN:1097-8135). <http://www.lifesciencesite.com>. 392

**Key words:** intuitionistic fuzzy set, intuitionistic fuzzy filters, convergence of intuitionistic fuzzy filter, syntopogenous intuitionistic fuzzy spaces, and convergence of intuitionistic fuzzy filter in syntopogenous intuitionistic fuzzy spaces.

### Introduction

The concept of a fuzzy set was introduced by Zadeh [18], and many topics in mathematics was introduced using the fuzzy sets. In 1983 Atanassov [1, 2, 3, 4] generalized the fuzzy sets to intuitionistic fuzzy sets, and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [5, 6]. Kandi et al., [10] redefined the concept of intuitionistic fuzzy set which was defined by Atanassov, in more simple form.

In 1963 Császár [7] introduced the syntopogenous structures which are a unified theory of topologies, proximities and uniformities. In 1983 Katsars and Petalas [8, 9] used the ideas of Császár and the concept of fuzzy set to introduce the fuzzy syntopogenous structures. Kandil et al., [11] used the ideas of Császár and Katsars and Petalas [7, 8, 9] and the concept of intuitionistic fuzzy set redefined [10] to introduce the intuitionistic fuzzy syntopogenous structures. Recently Tantawy et al., [17] introduced the convergence of fuzzy filter in fuzzy syntopogenous spaces. Also in [16] they defined some separation axioms on syntopogenous intuitionistic fuzzy structures.

Mondal and Samanta [12] introduced the definition of generalized intuitionistic fuzzy sets.

Park and Park [13] used the generalized intuitionistic fuzzy sets given by Mondal and Samanta to introduce the concept of generalized intuitionistic fuzzy filter.

By using the intuitionistic fuzzy set modified by **Kandil** and the notion of generalized intuitionistic fuzzy filter given by Park and Park we will define the intuitionistic fuzzy filter. Also we will introduce the convergence of intuitionistic fuzzy filter in syntopogenous intuitionistic fuzzy spaces using [16, 17] and study their properties.

### Preliminaries

In this section we recall many of the concepts and properties which are needed in the sequel and studied by others.

**Definition 2.1:** [9] An intuitionistic fuzzy set  $\underline{A}$  (IFS for short) is an ordered pair

$\langle A^1, A^2 \rangle \in I^X \times I^X$  where  $I^X$  is the family of all fuzzy sets on a given non-empty set  $X$  such that  $A^1 \subseteq A^2$  and denoted by  $\underline{A} = \langle A^1, A^2 \rangle$ .

The family of IFSs on  $X$ , will be denoted by  $\Pi^X$ . The IFS  $\underline{1} = \langle 1, 1 \rangle$  is called the universal IFS, and the IFS  $\underline{0} = \langle 0, 0 \rangle$  is called the empty IFS.

The binary operations on  $\Pi^X$  are given in the following:

**Definition 2.2:** [9] Let  $\underline{A} = \langle A^1, A^2 \rangle$  and  $\underline{B} = \langle B^1, B^2 \rangle$  be two IFSs on a non-empty set  $X$ . then :

(1)  $\underline{A} \subseteq \underline{B}$  iff  $A^1 \subseteq B^1$  and  $A^2 \subseteq B^2$ .

(2)  $\underline{A} = \underline{B}$  iff  $A^1 = B^1$  and  $A^2 = B^2$ .

(3)  $\underline{A} \cup \underline{B} = \langle A^1 \cup B^1, A^2 \cup B^2 \rangle$ .

(4)  $\underline{A} \cap \underline{B} = \langle A^1 \cap B^1, A^2 \cap B^2 \rangle$ .

(5)  $\underline{A}^c = \langle A^2^c, A^1^c \rangle$ .

According to above definitions we can generalized the operations of intersection and union

to arbitrary family  $\{ \underline{A}_i : i \in J \}$  of IFSs as follows:

$$\left\langle \bigcup_{i \in J} \underline{A}_i, \bigcup_{i \in J} A_i^1, \bigcup_{i \in J} A_i^2 \right\rangle \text{ and } \bigcap_{i \in J} \underline{A}_i = \left\langle \bigcap_{i \in J} A_i^1, \bigcap_{i \in J} A_i^2 \right\rangle.$$

**Theorem 2.3:**[9]( $\Pi^X, \cap, \cup, ^c$ ) is a Morgan Algebra i.e. it satisfies the following axioms :

(1)Commutative laws

$$i) \underline{A} \cup \underline{B} = \underline{B} \cup \underline{A} \quad ii) \underline{A} \cap \underline{B} = \underline{B} \cap \underline{A}$$

(2) Associative Laws

$$i) (\underline{A} \cup \underline{B}) \cup \underline{C} = \underline{A} \cup (\underline{B} \cup \underline{C})$$

$$ii) (\underline{A} \cap \underline{B}) \cap \underline{C} = \underline{A} \cap (\underline{B} \cap \underline{C})$$

(3) Identity laws

$$i) \underline{A} \cup \underline{0} = \underline{A} \quad ii) \underline{A} \cup \underline{1} = \underline{1}$$

$$iii) \underline{A} \cap \underline{0} = \underline{0} \quad iv) \underline{A} \cap \underline{1} = \underline{A}$$

(4) Distributive laws

$$i) \underline{A} \cup (\underline{B} \cap \underline{C}) = (\underline{A} \cup \underline{B}) \cap (\underline{A} \cup \underline{C})$$

$$ii) \underline{A} \cap (\underline{B} \cup \underline{C}) = (\underline{A} \cap \underline{B}) \cup (\underline{A} \cap \underline{C})$$

(5) De Morgan's laws

$$i) (\underline{A} \cup \underline{B})^c = \underline{A}^c \cap \underline{B}^c$$

$$ii) (\underline{A} \cap \underline{B})^c = \underline{A}^c \cup \underline{B}^c$$

**Definition 2.4:**[9] Let X and Y be two non-empty sets,  $f : X \rightarrow Y$  be a function;

(1) If  $\underline{A} = \langle A^1, A^2 \rangle$  is an IFS in X, then the image of  $\underline{A}$  under  $f$ , denoted by

(2)  $f(\underline{A})$  is an IFS in Y, defined by  $f(\underline{A}) = \langle f(A^1), f(A^2) \rangle$ ; where

$$f(\underline{A}^i)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} A_i(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \quad \forall y \in Y, \text{ and } i=1, 2.$$

(3) If  $\underline{B} = \langle B^1, B^2 \rangle$  is an IFS in Y, then the preimage of  $\underline{B}$  under  $f$ , denoted by  $f^{-1}(\underline{B})$  is the IFS in X defined by  $f^{-1}(\underline{B}) = \langle f^{-1}(B^1), f^{-1}(B^2) \rangle$ , where  $f^{-1}(B^1)(x) = B^1(f(x))$  and  $f^{-1}(B^2) = B^2(f(x)) \quad \forall x \in X$ .

The properties of the image and the preimage of intuitionistic fuzzy set is the same as the fuzzy case as follows:

**Theorem 2.5:**[9] Let  $f : X \rightarrow Y$  be a function,  $\underline{A}, \underline{B} \in \Pi^X$  and  $\{ \underline{A}_i : i \in J \} \subseteq \Pi^X$ . Let  $\underline{C}, \underline{D} \in \Pi^Y$  and  $\{ \underline{C}_i : i \in J \} \subseteq \Pi^Y$ . Then:

$$(1) \underline{A} \subseteq \underline{B} \Rightarrow f(\underline{A}) \subseteq f(\underline{B}).$$

$$(2) \underline{C} \subseteq \underline{D} \Rightarrow f^{-1}(\underline{C}) \subseteq f^{-1}(\underline{D}).$$

(3)  $\underline{A} \subseteq f^{-1}(f(\underline{A}))$ , and the equality holds if  $f$  is injective.

(4)  $f(f^{-1}(\underline{C})) \subseteq \underline{C}$ , and the equality holds if  $f$  is surjective.

$$(5) (f^{-1}(\underline{C}))^c = f^{-1}(\underline{C}^c).$$

$$(6) f\left(\bigcup_{i \in J} \underline{A}_i\right) = \bigcup_{i \in J} f(\underline{A}_i)$$

$$(7) f^{-1}\left(\bigcup_{i \in J} \underline{C}_i\right) = \bigcup_{i \in J} f^{-1}(\underline{C}_i)$$

(8)  $f\left(\bigcap_{i \in J} \underline{A}_i\right) \subseteq \bigcap_{h \in J} f(\underline{A}_h)$ , and the equality holds if  $f$  is injective,

$$(9) \bigcap_{i \in J} f^{-1}(\underline{C}_i) = f^{-1}\left(\bigcap_{i \in J} \underline{C}_i\right)$$

(10)  $f(\underline{0}) = \underline{0}$ , if  $f$  is surjective, then  $f\left(\frac{1}{2}\right) = \frac{1}{2}$ .

(11)  $f^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}$  and  $f^{-1}(\underline{0}) = \underline{0}$ .

Now we introduce the concept of intuitionistic fuzzy point as follows.

**Definition 2.6:**[9] Let X be a nonempty set and  $x \in X$  be a fixed element,  $\alpha, \beta \in I = [0,1]$  such that  $\alpha \leq \beta, \beta > 0$ . An intuitionistic fuzzy set  $x^{(\alpha, \beta)} = (x^{(\alpha)}, x^{(\beta)}) \in \Pi^X$  is called an intuitionistic fuzzy point (IFp, for short) i.e. the IFp  $x^{(\alpha, \beta)}$  is an ordered pair of two fuzzy points  $x^{(\alpha)}, x^{(\beta)}$  where  $x^{(\alpha)} \leq x^{(\beta)}$ .

When  $\alpha = 0$ . Then  $x^{(\alpha)} = 0$  and the intuitionistic fuzzy point  $x^{(\alpha, \beta)}$  is called a vanishing intuitionistic fuzzy point (VIPs, for short), i.e.  $x^{(0, \beta)} = (0, x^{(\beta)})$  where  $\beta \in I^0 = (0, 1]$ .

**Theorem 2.7:**[9] Let  $\underline{A}, \underline{B} \in \Pi^X$  and  $\{\underline{A}_i : i \in J\} \subseteq \Pi^X$ , where  $\underline{A}_i = \langle A_i^1, A_i^2 \rangle$ . Then:

(1)  $\underline{A} \subseteq \underline{B}$  iff  $x^{(\alpha,\beta)} \in \underline{A}$  implies  $x^{(\alpha,\beta)} \in \underline{B} \forall x^{(\alpha,\beta)}$  in  $X$ .

(2)  $x^{(\alpha,\beta)} \in \bigcap_{i \in J} \underline{A}_i$  iff  $x^{(\alpha,\beta)} \in \underline{A}_i \forall i \in J$ .

(3) If there exist  $i \in J$  such that  $x^{(\alpha,\beta)} \in \underline{A}_i$ , then  $x^{(\alpha,\beta)} \in \bigcup_{i \in J} \underline{A}_i$ .

(4)  $\underline{A} = \bigcup \{x^{(\alpha,\beta)} : x^{(\alpha,\beta)} \in \underline{A}\}$ .

**Remark 2.8:**[9]

The union of an IFS and its complement needs not be the universal IFS.

The intersection of an IFS and its complement needs not to be the empty IFS.

The statement;  $(x^{(\alpha,\beta)} \notin \underline{A} \Leftrightarrow x^{(\alpha,\beta)} \in \underline{A}^c)$  may not be true in general.

(i) In fuzzy setting, we observe that if  $\{A_i : i \in J\}$  be a family of fuzzy sets, then for any fuzzy point  $x^{(\alpha)}$ ,  $x^{(\alpha)} \in \bigcup_{i \in J} A_i$  iff  $x^{(\alpha)} \in A_i$  for some  $i \in J$  and  $J$  is finite. But this is not true in the intuitionistic setting.

This is shown in the following examples:

**Example 2.9:**[9] Let  $X = \{a, b, c\}$  and  $\underline{A} = \langle (a^{0.4}, b^{0.3}, c^{0.5}), (a^{0.5}, b^{0.3}, c^{0.7}) \rangle$ . Then:

(i)  $\underline{A}^c = \langle (a^{0.5}, b^{0.7}, c^{0.3}), (a^{0.6}, b^{0.7}, c^{0.5}) \rangle$ .

(ii)  $\underline{A} \cup \underline{A}^c = \langle (a^{0.5}, b^{0.7}, c^{0.5}), (a^{0.6}, b^{0.7}, c^{0.7}) \rangle \neq \underline{1}$ .

(iii)  $\underline{A} \cap \underline{A}^c = \langle (a^{0.4}, b^{0.3}, c^{0.3}), (a^{0.5}, b^{0.3}, c^{0.5}) \rangle \neq \underline{0}$ .

(iv) If  $x = a$ , then  $a^{(0.4,0.7)} \notin \underline{A}$  and  $a^{(0.4,0.7)} \notin \underline{A}^c$ .

**Example 2.10:**[9] Let  $X = \{a, b\}$  and consider the following IFSs;

$\underline{A} = \langle (a^{0.3}, b^{0.4}), (a^{0.5}, b^{0.7}) \rangle$  and  $\underline{B} = \langle (a^{0.4}, b^{0.2}), (a^{0.4}, b^{0.3}) \rangle$ . Then we have:

$\underline{A} \cup \underline{B} = \langle (a^{0.4}, b^{0.4}), (a^{0.5}, b^{0.7}) \rangle$ ,  $x^{(0.4,0.7)} \in \underline{A} \cup \underline{B}$  while  $x^{(0.4,0.7)} \notin \underline{A}$  and  $x^{(0.4,0.7)} \notin \underline{B}$ .

**Definition 2.11:**[9] Let  $\underline{A} = \langle A^1, A^2 \rangle$  and  $\underline{B} = \langle B^1, B^2 \rangle$  be two IFSs on a non-empty set  $X$  and  $x^{(\alpha,\beta)}$  be an IFp in  $X$ . Then:

(i)  $\underline{A}$  and  $\underline{B}$  are said to be quasi-coincident, and denoted by  $\underline{A} q \underline{B}$  if  $A^1 q B^2$  or  $A^2 q B^1$ .

If  $\underline{A}$  is not quasi-coincident with  $\underline{B}$ , then we write  $\underline{A} \not q \underline{B}$  i.e.  $\underline{A} \not q \underline{B}$  if  $A^1 \not q B^2$  and  $A^2 \not q B^1$ .

(ii)  $x^{(\alpha,\beta)} q \underline{A}$  if  $\alpha > A_2^c(x)$  or  $\beta > A_1^c(x)$  i.e.  $x^{(\alpha,\beta)} q \underline{A}$  if  $x^{(\alpha)} q A^2$  or  $x^{(\beta)} q A^1$ .

The properties of the quasi-coincident relation were given in the following theorem:

**Theorem 2.12:**[9] Let  $f : X \rightarrow Y$  be a function,  $\underline{A}, \underline{B}, \underline{C} \in \Pi^X$ ,  $\underline{D}, \underline{E} \in \Pi^Y$  and  $\{\underline{A}_i : i \in J\} \subseteq \Pi^X$  and  $x^{(\alpha,\beta)}, y^{(\gamma,\delta)} \in X^{IP}$ . Then:

(1)  $\underline{A} q \underline{B} \Leftrightarrow \underline{A} \subseteq \underline{B}^c$ .

(2)  $\underline{A} \cap \underline{B} = \underline{0} \Rightarrow \underline{A} q \underline{B}$ .

(3)  $x^{(\alpha,\beta)} q \underline{A} \Leftrightarrow x^{(\alpha,\beta)} \in \underline{A}^c$ .

(4)  $\underline{A} q \underline{A}^c$ .

(5)  $\underline{A} q \underline{B}, \underline{C} \subseteq \underline{B} \Rightarrow \underline{A} q \underline{C}$ .

(6)  $\underline{A} \subseteq \underline{B} \Leftrightarrow (x^{(\alpha,\beta)} q \underline{A} \Rightarrow x^{(\alpha,\beta)} q \underline{B}) \forall x^{(\alpha,\beta)}$  in  $X$ .

(7)  $\underline{A} q \underline{B} \Leftrightarrow x^{(\alpha,\beta)} q \underline{B}$ , for some  $x^{(\alpha,\beta)} \in \underline{A}$ .

(8)  $x^{(\alpha,\beta)} q (\bigcup_{i \in J} \underline{A}_i) \Leftrightarrow \exists i \in J$  such that  $x^{(\alpha,\beta)} q \underline{A}_i$ .

(9) If  $x^{(\alpha,\beta)} q (\bigcap_{i \in J} \underline{A}_i)$ , then  $x^{(\alpha,\beta)} q \underline{A}_i$  for all  $i \in J$ .

(10)  $x \neq y \Rightarrow x^{(\alpha,\beta)} \not\prec y^{(\gamma,\delta)}$  for all  $\alpha, \beta, \gamma, \delta \in I$ .

(11)  $x^{(\alpha,\beta)} \prec y^{(\gamma,\delta)} \Leftrightarrow x \neq y$  or  $x = y$  and  $(\alpha + \delta \leq 1$  and  $\beta + \gamma \leq 1)$ .

(12)  $\underline{A} \prec \underline{B} \Rightarrow f(\underline{A}) \prec f(\underline{B})$  if  $f$  is bijection.

(13)  $\underline{D} \prec \underline{E} \Rightarrow f^{-1}(\underline{D}) \prec f^{-1}(\underline{E})$ .

**Definition 2.13:**[10] A semi-topogenous intuitionistic fuzzy order on X is a binary relation  $\prec$  on  $\Pi^X$  satisfying the following axioms :

(IFO1)  $\underline{0} \prec \underline{0}$  and  $\underline{1} \prec \underline{1}$  ;

(IFO2)  $\underline{A} \prec \underline{B}$  implies  $\underline{A} \subseteq \underline{B}$  ;

(IFO3)  $\underline{A} \subseteq \underline{A}'$ ,  $\underline{B}' \subseteq \underline{B}$  implies  $\underline{A} \prec \underline{B}$ .

**Definition**

**2.14:**[10] A semi-topogenous intuitionistic fuzzy order  $\prec$  on a set X is called :

(i) topogenous intuitionistic fuzzy order on X if it satisfies the condition ;

$$\underline{A}_i \prec \underline{B}_i \ (i = 1, 2, 3, \dots, n) \text{ imply } \bigcup_{i=1}^n \underline{A}_i \prec \bigcup_{i=1}^n \underline{B}_i \text{ and } \bigcap_{i=1}^n \underline{A}_i \prec \bigcap_{i=1}^n \underline{B}_i.$$

(ii) perfect semi-topogenous intuitionistic fuzzy order on X if it satisfies the condition :

$$\underline{A}_i \prec \underline{B}_i \ (i \in J) \text{ implies } \bigcup_{i \in J} \underline{A}_i \prec \bigcup_{i \in J} \underline{B}_i.$$

(iii) biperfect topogenous intuitionistic fuzzy order on X if it satisfies the condition :

$$\underline{A}_i \prec \underline{B}_i \ (i \in J) \text{ implies } \bigcup_{i \in J} \underline{A}_i \prec \bigcup_{i \in J} \underline{B}_i \text{ and } \bigcap_{i \in J} \underline{A}_i \prec \bigcap_{i \in J} \underline{B}_i.$$

Where J is any index set.

**Definition 2.15 :**[10] Let  $\prec_1$  and  $\prec_2$  be two semi-topogenous (resp. topogenous, perfect semi-topogenous, biperfect topogenous) intuitionistic fuzzy orders, then we say that  $\prec_2$  is finer than  $\prec_1$  (or  $\prec_1$  is coarser than  $\prec_2$ ), denoted by  $\prec_1 \subseteq \prec_2$  if  $\underline{A} \prec_1 \underline{B}$  implies  $\underline{A} \prec_2 \underline{B}$ .

**Definition 2.16:**[10] The complement of a semi-topogenous (resp. topogenous, perfect semi-

topogenous, biperfect topogenous) intuitionistic fuzzy order  $\prec$  on X, denoted by  $\prec^c$  is defined by ;

$\underline{A} \prec^c \underline{B}$  iff  $\underline{B}^c \prec \underline{A}^c$ . Also  $\prec$  is called symmetrical if  $\underline{A} \prec \underline{B}$  implies  $\underline{A} \prec^c \underline{B}$ .

**Definition 2.17:**[10] A syntopogenous intuitionistic fuzzy structure on a set X is a non-empty family  $\underline{S}$  of topogenous intuitionistic fuzzy orders on X having the following two properties :

(IFS1) If  $\prec, \prec' \in \underline{S}$ , then there exist an  $\prec'' \in \underline{S}$  finer than  $\prec$  and  $\prec'$ .

(IFS2) If  $\prec \in \underline{S}$ , then there exists an  $\prec' \in \underline{S}$  such that  $\underline{A} \prec \underline{B}$  implies the existence of  $\underline{C} \in \Pi^X$  such that  $\underline{A} \prec', \underline{C} \prec', \underline{B}$ , (i. e.  $\prec \subseteq \prec',^2$ ).

The pair  $[X, \underline{S}]$  is called syntopogenous intuitionistic fuzzy space. In case  $\underline{S}$  consists of a single topogenous intuitionistic fuzzy order it is called a simple syntopogenous structure (topogenous intuitionistic fuzzy structure). If all topogenous intuitionistic fuzzy orders of an syntopogenous intuitionistic fuzzy structure  $\underline{S}$  on X are perfect, it is called perfect syntopogenous intuitionistic fuzzy structure or (intuitionistic fuzzy syntopology), and the space  $[X, \underline{S}]$  is called syntopological intuitionistic fuzzy space. If all topogenous intuitionistic fuzzy orders of a syntopogenous intuitionistic fuzzy structure  $\underline{S}$  on X are biperfect, it is called biperfect syntopogenous intuitionistic fuzzy structure or (biperfect syntopology).

**Definition 2.18:**[ ] A syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  is said to be:

**(T<sup>0</sup>)- space** If for any intuitionistic fuzzy points (IFPs)  $x^{(\alpha,\beta)}, y^{(\gamma,\delta)}$  with  $x^{(\alpha,\beta)} \leq y^{(\gamma,\delta)}$  there is  $\prec \in \underline{S}$  such that  $x^{(\alpha,\beta)} \prec y^{(\gamma,\delta)}$  or  $y^{(\gamma,\delta)} \prec x^{(\alpha,\beta)}$ .

**(T<sup>1</sup>)- space** If for any intuitionistic fuzzy points (IFPs)  $x^{(\alpha,\beta)}, y^{(\gamma,\delta)}$  with  $x^{(\alpha,\beta)} \leq y^{(\gamma,\delta)}$  there is  $\prec \in \underline{S}$  such that  $x^{(\alpha,\beta)} \prec y^{(\gamma,\delta)}$ .

$(T^2)$ - space If for any intuitionistic fuzzy points (IFPs)  $x^{(\alpha,\beta)}$ ,  $y^{(\gamma,\delta)}$  with  $x^{(\alpha,\beta)} \leq y^{(\gamma,\delta)}$  there is  $\prec \prec \in \underline{S}$  and  $\underline{A} \in \Pi^X$  such that  $x^{(\alpha,\beta)} \prec \prec \underline{A} \prec \prec y^{(\gamma,\delta)}$ .

$(T^2)$ - space If for any intuitionistic fuzzy points (IFPs)  $x^{(\alpha,\beta)}$ ,  $y^{(\gamma,\delta)}$  with  $x^{(\alpha,\beta)} \leq y^{(\gamma,\delta)}$  there are  $\prec \prec \in \underline{S}$  and  $\underline{A}, \underline{B} \in \Pi^X$  such that  $\underline{A} \cap \underline{B} = \underline{0}$ ,  $x^{(\alpha,\beta)} \prec \prec \underline{A}$  and  $y^{(\gamma,\delta)} \prec \prec \underline{B}$ .

**3. Intuitionistic fuzzy filter**

**Definition 3.1:** Let  $\underline{\mathfrak{F}}$  be a non-empty family of intuitionistic fuzzy sets on X (IFSSs), then  $\underline{\mathfrak{F}}$  is said to be an intuitionistic fuzzy filter on X if it is satisfying the following:

1.  $\underline{0} \notin \underline{\mathfrak{F}}$ ;
2. If  $\underline{A}, \underline{B} \in \underline{\mathfrak{F}}$ , then  $\underline{A} \cap \underline{B} \in \underline{\mathfrak{F}}$ ;
3. If  $\underline{A} \in \underline{\mathfrak{F}}$  and  $\underline{A} \subseteq \underline{B}$ , then  $\underline{B} \in \underline{\mathfrak{F}}$ .

**Definition 3.2:** Let  $\underline{\mathfrak{B}}$  be a non-empty family of intuitionistic fuzzy setson X (IFSSs), then  $\underline{\mathfrak{B}}$  is called an intuitionistic fuzzyfilter base if it is satisfying:

1.  $\underline{0} \notin \underline{\mathfrak{B}}$ ;
2. If  $\underline{A}, \underline{B} \in \underline{\mathfrak{B}}$ , then  $\underline{A} \cap \underline{B} \in \underline{\mathfrak{B}}$ ;

A family  $\underline{\mathcal{L}}$  is called a sub base of an intuitionistic fuzzyfilter base if it is nonempty and the of any finite numbers of elements of  $\underline{\mathcal{L}}$  is not  $\underline{0}$ .

**Lemma 3.3:(i)** If  $\underline{\mathcal{L}}$  is a sub base of an intuitionistic fuzzyfilter, then the family  $\underline{\mathfrak{B}}(\underline{\mathcal{L}})$  consisting of all finite intersections of the elements of  $\underline{\mathcal{L}}$  is an intuitionistic fuzzyfilter base.

(ii) If  $\underline{\mathfrak{B}}$  is an intuitionistic fuzzyfilter base, then the family  $\underline{\mathfrak{F}}(\underline{\mathfrak{B}})$  consisting of all IFSSs  $\underline{A}$  such that  $\underline{A} \supseteq \underline{B}$  for some  $\underline{B} \in \underline{\mathfrak{B}}$  is an intuitionistic fuzzyfilter.

(iii)  $\underline{\mathfrak{B}}(\underline{\mathcal{L}})$  and  $\underline{\mathfrak{F}}(\underline{\mathfrak{B}})$  are uniquely determined  $\underline{\mathcal{L}}$  and  $\underline{\mathfrak{B}}$  respectively.

**Lemma 3.4:**(1) Let  $\{\underline{\mathfrak{F}}_\alpha : \alpha \in \Gamma\}$  be a family of intuitionistic fuzzyfilters on a set X. Then:

- i.  $\bigcap_{\alpha \in \Gamma} \underline{\mathfrak{F}}_\alpha = \{\bigcap_{\alpha \in \Gamma} \underline{F}_\alpha : \underline{F}_\alpha \in \underline{\mathfrak{F}}_\alpha\}$  is also an intuitionistic fuzzyfilter on X.

- ii.  $\bigcup_{\alpha \in \Gamma} \underline{\mathfrak{F}}_\alpha = \{\bigcup_{\alpha \in \Gamma} \underline{F}_\alpha : \underline{F}_\alpha \in \underline{\mathfrak{F}}_\alpha\}$  is also an intuitionistic fuzzyfilter on X.

- (2)  $\underline{\mathfrak{B}}_1$  and  $\underline{\mathfrak{B}}_2$  be two intuitionistic fuzzyfilter bases. Then  $\underline{\mathfrak{F}}(\underline{\mathfrak{B}}_1) \subseteq \underline{\mathfrak{F}}(\underline{\mathfrak{B}}_2)$  iff for any  $\underline{B} \in \underline{\mathfrak{B}}_1$  there exists  $\underline{A} \in \underline{\mathfrak{B}}_2$  such that  $\underline{A} \subseteq \underline{B}$ .

**Theorem 3.5:** Let  $\underline{\mathfrak{F}}$  be an intuitionistic fuzzyfilter on X, and  $Y \subseteq X$ . Then  $\underline{\mathfrak{F}} \setminus Y = \{\underline{F} \setminus Y : \underline{F} \in \underline{\mathfrak{F}}\}$  is an intuitionistic fuzzyfilter on Y if  $\underline{F} \setminus Y \neq \underline{0}$  for every  $\underline{F} \in \underline{\mathfrak{F}}$ .

**Proof:**

Since  $\underline{F} \setminus Y \neq \underline{0}$  for every  $\underline{F} \in \underline{\mathfrak{F}}$ , then  $\underline{0} \notin \underline{\mathfrak{F}} \setminus Y$ . Let  $\underline{A} \setminus Y, \underline{B} \setminus Y \in \underline{\mathfrak{F}} \setminus Y$ , then clearly

$(\underline{A} \setminus Y) \cap (\underline{B} \setminus Y) = (\underline{A} \cap \underline{B}) \setminus Y$ . Since  $\underline{\mathfrak{F}}$  is an intuitionistic fuzzyfilter, then  $\underline{A} \cap \underline{B} \in \underline{\mathfrak{F}}$  and hence  $(\underline{A} \setminus Y) \cap (\underline{B} \setminus Y) \in \underline{\mathfrak{F}} \setminus Y$ . Let  $\underline{F} \in \underline{\mathfrak{F}}$  and  $\underline{B} \in Y$  such that  $\underline{F} \setminus Y \subseteq \underline{B}$ . Choose an IFSS  $\underline{C} \in \Pi^X$  such that  $\underline{C} \setminus Y = \underline{B}$ . Since  $\underline{\mathfrak{F}}$  is an intuitionistic fuzzyfilter on X, then  $\underline{C} \in \underline{\mathfrak{F}}$ , and hence  $\underline{C} \setminus Y = \underline{B} \in \underline{\mathfrak{F}} \setminus Y$ .

**Theorem 3.6:** Let  $f: X \rightarrow Y$  be a function and  $\underline{\mathfrak{F}}$  be an intuitionistic fuzzyfilter on X. Then  $f(\underline{\mathfrak{F}}) = \{f(\underline{F}) : \underline{F} \in \underline{\mathfrak{F}}\}$  is an intuitionistic fuzzy basefilter on Y.

**Proof:**

- (i) Suppose that  $\underline{0} \in f(\underline{\mathfrak{F}})$ , then there exist  $\underline{F} \in \underline{\mathfrak{F}}$  such that  $f(\underline{F}) = \underline{0}$  which imply  $\underline{F} = \underline{0}$  which contradict with  $\underline{0} \notin \underline{\mathfrak{F}}$ . Hence  $\underline{0} \notin f(\underline{\mathfrak{F}})$ .
- (ii) Let  $f(\underline{A}), f(\underline{B}) \in f(\underline{\mathfrak{F}})$  such that  $\underline{A}, \underline{B} \in \underline{\mathfrak{F}}$ , then  $f(\underline{A} \cap \underline{B}) \subseteq f(\underline{A}) \cap f(\underline{B}) \in f(\underline{\mathfrak{F}})$ . Hence  $f(\underline{\mathfrak{F}})$  is an intuitionistic fuzzy basefilter on Y.

**Theorem 3.7:** Let  $f: X \rightarrow Y$  be a surjection function and  $\underline{E}$  be an intuitionistic fuzzyfilter on Y. Then  $f^{-1}(\underline{E}) = \{f^{-1}(\underline{G}) : \underline{G} \in \underline{E}\}$  is an intuitionistic fuzzy basefilter on X.

**Proof:**

Suppose that  $\underline{0} \in f^{-1}(\underline{E})$ , then by surjection there exist  $\underline{0} \in \underline{E}$  which contradict with  $\underline{0} \notin \underline{E}$ . Hence  $\underline{0} \notin f^{-1}(\underline{E})$ .

(i) Let  $f^{-1}(\underline{G}), f^{-1}(\underline{G}') \in f^{-1}(\underline{E})$  such that  $\underline{G}, \underline{G}' \in \underline{E}$ .

Since  $\underline{E}$  is an intuitionistic fuzzy filter then  $\underline{G} \cap \underline{G}' \in \underline{E}$ , so  $f^{-1}(\underline{G}) \cap f^{-1}(\underline{G}') = f^{-1}(\underline{G} \cap \underline{G}') \in f^{-1}(\underline{E})$ . Hence  $f^{-1}(\underline{E})$  is an intuitionistic fuzzy base filter on  $X$ .

**Definition 3.8:** Let  $(X, \underline{\mathfrak{F}})$  and  $(Y, \underline{E})$  be two intuitionistic fuzzy filters, then a function  $f: (X, \underline{\mathfrak{F}}) \rightarrow (Y, \underline{E})$  is called intuitionistic fuzzy filter continuous w.r.t  $(\underline{\mathfrak{F}}, \underline{E})$  if

$$f^{-1}(\underline{G}) \in \underline{\mathfrak{F}} \text{ for every } \underline{G} \in \underline{E}.$$

**Proposition 3.9:** (i) If  $f: (X, \underline{\mathfrak{F}}) \rightarrow (Y, \underline{E})$  and  $g: (Y, \underline{E}) \rightarrow (Z, \underline{\mathcal{H}})$  are two intuitionistic fuzzy filters continuous functions, then the composition  $g \circ f: (X, \underline{\mathfrak{F}}) \rightarrow (Z, \underline{\mathcal{H}})$  is also intuitionistic fuzzy filter continuous.

If  $f: (X, \underline{\mathfrak{F}}) \rightarrow (X, \underline{\mathfrak{F}})$  is an identity function, then  $f$  is intuitionistic fuzzy filter continuous.

If  $f: (X, \underline{\mathfrak{F}}) \rightarrow (Y, \underline{E})$  is intuitionistic fuzzy filter continuous function, and  $Z \subseteq X$  such that  $\underline{\mathfrak{F}} \setminus Z \neq \underline{0}$  for any  $\underline{F} \in \underline{\mathfrak{F}}$ . Then the restriction  $f \setminus Z: (Z, \underline{\mathfrak{F}} \setminus Z) \rightarrow (Y, \underline{E})$  is also intuitionistic fuzzy filter continuous.

**Proof:**

Straightforward.

#### 4. Convergence of intuitionistic fuzzy filters in syntopogenous intuitionistic fuzzy space

**Proposition 4.1:** Consider a syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  and  $\text{IFP}_{x^{(\alpha, \beta)}}$  of  $X$ .

Then the family  $\underline{B}_{S(x^{(\alpha, \beta)})} = \{ \underline{B} \in \Pi^X : x^{(\alpha, \beta)} \prec \prec \underline{B} \text{ for a suitable } \prec \prec \in \underline{S} \}$  forms an intuitionistic fuzzy filter on  $X$ .

**Proof:**

(i) If  $\underline{B} \in \underline{B}_{S(x^{(\alpha, \beta)})}$ , then there exist  $\prec \prec \in \underline{S}$  such that  $x^{(\alpha, \beta)} \prec \prec \underline{B}$  which implies that  $x^{(\alpha, \beta)} \in \underline{B}$  i.e.  $\underline{B} \neq \underline{0}$ .

(ii) Let  $\underline{B}, \underline{B}' \in \underline{B}_{S(x^{(\alpha, \beta)})}$ , then there exist  $\prec \prec, \prec \prec' \in \underline{S}$  such that  $x^{(\alpha, \beta)} \prec \prec \underline{B}$  and  $x^{(\alpha, \beta)} \prec \prec' \underline{B}'$ . Since  $\underline{S}$  is a syntopogenous intuitionistic fuzzy structure, then there exists  $\prec \prec'' \in \underline{S}$  finer than  $\prec \prec$  and  $\prec \prec'$ . Thus  $x^{(\alpha, \beta)} \prec \prec'' \underline{B}$  and  $x^{(\alpha, \beta)} \prec \prec'' \underline{B}'$ , which implies that  $x^{(\alpha, \beta)} \prec \prec'' \underline{B} \cap \underline{B}'$ . Hence  $\underline{B} \cap \underline{B}' \in \underline{B}_{S(x^{(\alpha, \beta)})}$ .

(iii) Let  $\underline{A} \subseteq \underline{B}$  and  $\underline{A} \in \underline{B}_{S(x^{(\alpha, \beta)})}$ , then there exists  $\prec \prec \in \underline{S}$  such that  $x^{(\alpha, \beta)} \prec \prec \underline{A} \subseteq \underline{B}$  which implies that  $x^{(\alpha, \beta)} \prec \prec \underline{B}$  i.e.  $\underline{B} \in \underline{B}_{S(x^{(\alpha, \beta)})}$ .

Hence  $\underline{B}_{S(x^{(\alpha, \beta)})}$  is an intuitionistic fuzzy filter in  $X$ .

**Definition 4.2:** Let  $[X, \underline{S}]$  be any syntopogenous intuitionistic fuzzy space, and  $x^{(\alpha, \beta)}$  be any  $\text{IFP}$  of  $X$ . Then the intuitionistic fuzzy filter given in the previous proposition is called the intuitionistic fuzzy filter of neighborhood of  $x^{(\alpha, \beta)}$  in  $\underline{S}$ .

**Definition 4.3:** Let  $\underline{S}$  be a syntopogenous intuitionistic fuzzy structure on  $X$ , and let  $\underline{\mathfrak{F}}$  be an intuitionistic fuzzy filter in  $X$ . We call  $\underline{\mathfrak{F}}$  converges to an  $x^{(\alpha, \beta)}$  in  $\underline{S}$ , denoted by  $\underline{\mathfrak{F}} \rightarrow x^{(\alpha, \beta)}$  iff  $\underline{B}_{S(x^{(\alpha, \beta)})} \subseteq \underline{\mathfrak{F}}$ .

Also we call  $x^{(\alpha, \beta)}$  the intuitionistic fuzzy limit point of the intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$ .

In more general manner we give the following definition.

**Definition 4.4:** An  $\text{IFP}_{x^{(\alpha, \beta)}}$  in a syntopogenous intuitionistic fuzzy structure  $\underline{S}$  is called an intuitionistic fuzzy cluster point of an intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  iff  $\underline{B} \cap \underline{F} \neq \underline{0}$  for every  $\underline{B} \in \underline{B}_{S(x^{(\alpha, \beta)})}$  and  $\underline{F} \in \underline{\mathfrak{F}}$ .

**Corollary 4.5:** From the above definition it follows that any intuitionistic fuzzy limit point of an intuitionistic fuzzy filter is an intuitionistic fuzzy cluster point of it.



**Theorem 4.6:**An IFpx  $(\alpha, \beta)$  in a syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  is an intuitionistic fuzzy cluster point of an intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  iff there exists an intuitionistic fuzzy filter  $\underline{\mathfrak{F}}'$  in  $\underline{S}$  which consists  $\underline{\mathfrak{F}}$  and converges to  $x^{(\alpha, \beta)}$  in  $\underline{S}$ .

**Proof:**

Let  $x^{(\alpha, \beta)}$  be an intuitionistic fuzzy cluster point of the intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  in a syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$ , then  $\underline{B} \cap \underline{F} \neq \underline{0}$  for every  $\underline{B} \in \underline{B}_{S(x(\alpha, \beta))}$  and  $\underline{F} \in \underline{\mathfrak{F}}$ , this follows that  $\underline{B}_{S(x(\alpha, \beta))} \cap \underline{\mathfrak{F}}$  is an intuitionistic fuzzy filter. Since  $\underline{B} \supseteq \underline{B} \cap \underline{F}$  and  $\underline{F} \supseteq \underline{B} \cap \underline{F}$  for every  $\underline{B} \in \underline{B}_{S(x(\alpha, \beta))}$  and  $\underline{F} \in \underline{\mathfrak{F}}$ . Also  $\underline{B} \cap \underline{F} \in \underline{B}_{S(x(\alpha, \beta))} \cap \underline{\mathfrak{F}}$ . Then  $\underline{B}, \underline{F} \in \underline{B}_{S(x(\alpha, \beta))} \cap \underline{\mathfrak{F}}$  for every  $\underline{B} \in \underline{B}_{S(x(\alpha, \beta))}$  and  $\underline{F} \in \underline{\mathfrak{F}}$ . It follows that  $\underline{\mathfrak{F}}, \underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{B}_{S(x(\alpha, \beta))} \cap \underline{\mathfrak{F}}$ . Hence  $\underline{B}_{S(x(\alpha, \beta))} \cap \underline{\mathfrak{F}}$  is an intuitionistic fuzzy filter includes  $\underline{\mathfrak{F}}$  and converges to  $x^{(\alpha, \beta)}$  in  $\underline{S}$ .

Conversely, Let  $\underline{\mathfrak{F}}'$  be an intuitionistic fuzzy filter includes  $\underline{\mathfrak{F}}$  and converges to  $x^{(\alpha, \beta)}$  in  $\underline{S}$ . Then  $\underline{\mathfrak{F}}'$  includes  $\underline{B}_{S(x(\alpha, \beta))}$ . Also for every  $\underline{F} \in \underline{\mathfrak{F}}'$  and  $\underline{B} \in \underline{B}_{S(x(\alpha, \beta))}$  imply  $\underline{B}, \underline{F} \in \underline{\mathfrak{F}}'$ , then  $\underline{B} \cap \underline{F} \neq \underline{0}$ . Hence  $x^{(\alpha, \beta)}$  is an intuitionistic fuzzy cluster point of  $\underline{\mathfrak{F}}$ .

**Corollary 4.7:** Let  $\underline{\mathfrak{F}}$  and  $\underline{\mathfrak{F}}'$  be two intuitionistic fuzzy filters in a syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  and  $x^{(\alpha, \beta)}$  be an IFpin  $\underline{S}$ . Then :

- (i)  $\underline{\mathfrak{F}} \supseteq \underline{\mathfrak{F}} \rightarrow x^{(\alpha, \beta)}(\underline{S}) \Rightarrow \underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S})$ .
- (ii)  $\underline{\mathfrak{F}} \rightarrow x^{(\alpha, \beta)}(\underline{S})$  and  $\underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S}) \Rightarrow \underline{\mathfrak{F}} \cup \underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S})$ .

**Proof:** (i) If  $\underline{\mathfrak{F}} \rightarrow x^{(\alpha, \beta)}(\underline{S})$ , then  $\underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{\mathfrak{F}} \subseteq \underline{\mathfrak{F}}'$ . Consequently  $\underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S})$ .

- (ii)  $\underline{\mathfrak{F}} \rightarrow x^{(\alpha, \beta)}(\underline{S}) \Rightarrow \underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{\mathfrak{F}}$ , and  $\underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S}) \Rightarrow \underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{\mathfrak{F}}'$  which imply  $\underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{\mathfrak{F}} \cup \underline{\mathfrak{F}}'$ . Hence  $\underline{\mathfrak{F}} \cup \underline{\mathfrak{F}}' \rightarrow x^{(\alpha, \beta)}(\underline{S})$ .

**Theorem 4.8:**A syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  is  $(T^1_2)$ -space iff there is no intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  which converges to two distinct IFPs of  $[X, \underline{S}]$ .

**Proof:**

Suppose that  $[X, \underline{S}]$  is  $(T^1_2)$ -space, and  $\underline{\mathfrak{F}}$  is an intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  which converges to two distinct IFPs  $x^{(\alpha, \beta)}, y^{(\gamma, \delta)}$ . Then  $\underline{B}_{S(x(\alpha, \beta))} \subseteq \underline{\mathfrak{F}}$  and  $\underline{B}_{S(y(\gamma, \delta))} \subseteq \underline{\mathfrak{F}}$  which implies  $\underline{B}_{S(x(\alpha, \beta))} \cap \underline{B}_{S(y(\gamma, \delta))} \neq \underline{0}$  for every  $\underline{B}_{S(x(\alpha, \beta))} \in \underline{B}_{S(x(\alpha, \beta))}, \underline{B}_{S(y(\gamma, \delta))} \in \underline{B}_{S(y(\gamma, \delta))}$ . Consequently for every  $\prec, \prec' \in \underline{S}$  and for every IFPs  $\underline{A}, \underline{B}$  with  $x^{(\alpha, \beta)} \prec \underline{A}, y^{(\gamma, \delta)} \prec' \underline{B}$  and  $\underline{A} \cap \underline{B} \neq \underline{0}$  which contradict that  $[X, \underline{S}]$  is  $(T^1_2)$ -space.

Conversely;

Suppose that there is no intuitionistic fuzzy filter  $\underline{\mathfrak{F}}$  which converges to two distinct IFPs of  $[X, \underline{S}]$ . Let  $x^{(\alpha, \beta)}$  and  $y^{(\gamma, \delta)}$  be two IFPs in  $X$ . For every  $\underline{B}_{S(x(\alpha, \beta))} \in \underline{B}_{S(x(\alpha, \beta))}, \underline{B}_{S(y(\gamma, \delta))} \in \underline{B}_{S(y(\gamma, \delta))}$  such that  $\underline{B}_{S(x(\alpha, \beta))} \cap \underline{B}_{S(y(\gamma, \delta))} \neq \underline{0}$  this implies the existence of intuitionistic fuzzy filter  $\underline{\mathfrak{F}} = \underline{B}_{S(x(\alpha, \beta))} \cap \underline{B}_{S(y(\gamma, \delta))}$  includes  $\underline{B}_{S(x(\alpha, \beta))}$  and  $\underline{B}_{S(y(\gamma, \delta))}$  and converges to  $x^{(\alpha, \beta)}$  and  $y^{(\gamma, \delta)}$  which contradicts the hypothesis. So  $\prec, \prec' \in \underline{S}$  such that  $x^{(\alpha, \beta)} \prec \underline{\square}_{(\square, \square)}, y^{(\gamma, \delta)} \prec' \underline{\square}_{(\square, \square)}$  and  $\underline{\square}_{(\square, \square)} \cap \underline{\square}_{(\square, \square)} = \underline{0}$ . Hence  $[X, \underline{S}]$  is  $(T^1_2)$ -space.

**Corollary 4.9:** If  $[X, \underline{S}]$  is syntopogenous intuitionistic fuzzy space, such that there is no

intuitionistic fuzzy filter  $\underline{\square}$  which have two distinct IFcluster points. Then  $[X, \underline{S}]$  is  $(T^2)$ - space.

**Proof:**

Since ever IF limit point for an intuitionistic fuzzy filter is an IFcluster point of it. Then theorem 4.8 yield the proof.

Now we will define a kind of intuitionistic fuzzy filters in the syntopogenous intuitionistic fuzzy spaces which characterized by uniqueness of limit in

the  $(T^2)$ syntopogenous intuitionistic fuzzy space.

**Definition 4.10:** We call an intuitionistic fuzzy filter(intuitionistic fuzzy filter base)  $\underline{\square}$  a quasi-coincident iff  $\underline{F} \not\subseteq \underline{F}'$  for every  $\underline{F}, \underline{F}' \in \underline{\square}$ .

**Definition 4.11:** For every intuitionistic fuzzy filter (intuitionistic fuzzy filter base) the quasi-coincident part of  $\underline{\square}$  is the maximal one of the set of quasi-coincident intuitionistic fuzzy filter (intuitionistic fuzzy filter base) which contained in  $\underline{\square}$ .

**Theorem 4.12:** In any syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$ , the intuitionistic fuzzy filter of neighborhoods  $\underline{\square}_{\square(\square, \square)}$  is quasi-coincident if the values  $\square \geq \square > 1/2$ .

**Proof:**

Straightforward.

**Corollary 4.13:** Every convergent filter in any syntopogenous intuitionistic fuzzy space  $[X, \underline{S}]$  contains a quasi-coincident part if its IF limit point  $x$

$(\alpha, \beta)$  satisfying  $\square \geq \square > 1/2$ .

**Proof:**

Straightforward.

## Reference

1. Atanassov K., "Intuitionistic Fuzzy Sets", VII ITKRs Session, Deposited in General Sci-Tech. Library of Sci. (Sofia, June 1983) 1684-1697.
2. Atanassov K., "Intuitionistic Fuzzy Sets", Fuzzy set and system, 20(1) (1986) 87-96.
3. Atanassov K., "More on intuitionistic fuzzy sets", Fuzzy sets and systems, 33(1) (1989) 37-46.
4. Atanassov K., "New operations defined over the intuitionistic fuzzy sets", Fuzzy sets and systems, 61(2) (1994) 137- 142.

5. Buhaesca T., "On the convexity of intuitionistic fuzzy sets", Itinerat seminar on Functional Equations (cluj- Napoca, 1988) 137-144.
6. Buhaesca T., "Some observation on intuitionistic fuzzy sets", Itinerat seminar on Functional Equations (cluj- Napoca, 1989).
7. Csaszar A., "Foundation Of General Topology", pergamon, Elmsford, N. Y., 1963.
8. Kataras A. K. and G. G. Petalas, "A unified Theory Of Fuzzy Topologies, Fuzzy Proximities, And Fuzzy Uniformities". Rev. Roumaine Math. Purse., Tome XXXVIII, No. 9, P. 845-856, Bucarest, 1983.
9. Kataras A. K. and G. G. Petalas, "On Fuzzy Syntopogenous Structures". Journal of mathematical analysis and applications, vol. 99, P. 219-236, March 1984.
10. Kandil A., O. Tantawy, M. M. Yakout and S. Wasbi, "On Intuitionistic Fuzzy Separation Axioms", Applied Mathematics and Information Sciences" an International Journal. 5(3)(2011) 274-292.
11. Kandil A., O. Tantawy and Z. Abueidab, "On intuitionistic fuzzy syntopogenous structures", J. Egypt Math. Soc. 16(2)(2008) 255-280.
12. Mondal T. K., S.K. Samanta, "Generalized intuitionistic fuzzy sets", J. Fuzzy Math. 10 (2002) 839-861.
13. Park J.H and J. K. Park, "Hausdorffness on generalized intuitionistic fuzzy filters", Information science (2009).
14. Tantawy O., "On Fuzzy Syntopogenous Structures", Ph.D thesis submitted to the faculty of science Zagazig university (1988).
15. Tantawy O., "Extension of fuzzy syntopogenous spaces", J. Fuzzy Math., 13(2005) 1- 13.
16. Tantawy O. F. M. Sleim and Z. Abueidab, "Separation axioms on syntopogenous intuitionistic fuzzy structures", Submitted to public.
17. Tantawy O. F. M. Sleim and Z. Abueidab, "Convergence of fuzzy filters in fuzzy syntopogenous structures", Submitted to public.
18. Zadeh L. A., "Fuzzy sets", Information control, 8(1965) 338-353.

11/12/2013