Hybridizing Rough Sets and Double Sets (An approach for increasing decision accuracy)

[1]O. A. E.Tantawy, [2]A. M. Kozae, [1]H I. Mustafa and [3]Shehab.A. Kandil

[1]Mathematics Department, Faculty of Science, Zagazig University, Egypt; [2]Mathematics Department, Faculty of Science, Tanta University, Egypt; [3]Canadian International College (CIC), Egypt
Shehab_ali@cic-cairo.com

Abstract: Hybrid methods are important recent approaches for studying uncertainty of concepts and decisions in information systems. For example, combining rough sets and fuzzy sets, rough sets and genetic algorithm, rough sets and topology among other approaches. In this work, we introduce the notion of relative double sets and give examples and investigate some of its properties and characterizations. The suggested type of double sets is constructed using information system and is connected with uncertain concepts in information systems. The class of relative double sets related to a decision set in decision systems is used in finding more accurate approximations for uncertain concepts in general and specially for decision sets. Consequently, decision makers and takers can have new choices for more accurate decisions.

[O. Tantawy, A. M. Kozae, H I. Mustafa and Shehab.A. Kandil. **Hybridizing Rough Sets and Double Sets (An approach for increasing decision accuracy).** *Life Sci J* 2013;10(4):2915-2923]. (ISSN:1097-8135). http://www.lifesciencesite.com. 388

Key words: Double sets, rough sets, relative double sets, information systems.

1.Introduction

Decision making and taking under uncertain information is a problem of key importance when dealing with knowledge from real situations. Obtaining the precise numbers required by many uncertainty handling formalisms can be a problem when building real systems. Many theories for reasoning under uncertainty exist , the oldest formalism for reasoning under uncertainty is probability theory, which, according to Shafer [14] was founded by Pascal and Fermat in an exchange of letters in 1654. Over the subsequent 340 years the theory has been well defined and its capabilities extensively explored, so that the rules for propagating values are established without question, and may be found in any textbook on probability (for instance [8]). The theory of rough sets allows us to handle uncertainty without the need for precise numbers, and so has some advantages in such situations.

In past several years of 21st, rough set theory (see [9, 10, 11]) has developed significantly due to its wide applications. Various generalized rough set models have been established and their properties or structures have been investigated intensively (see [5, 6, 7, 12, 13]). One of the interesting research topics in RST is to modify this theory via topology (see[1, 2]).

Another new of research related to RST is the hybridization of this theory with fuzzy set theory (see[15]) and other theories of uncertainty to the best of our knowledge, hybridization of RST and double sets did not take the suitable interest of researches the purpose of this paper to introduce an approach for hybridizing RST and double set theory to increase the accuracy of approximation for uncertain concepts in general and specially decision concepts.

This paper is organized as follows: article 1 is concerned with basic concepts of double sets . 2 is devoted to give a detail account on principals of rough set concepts. In 3 . we introduce the concept of relative double sets and give examples for this concept. The notion of relative double sets in information systems is investigated and its importance for increasing accuracy is discussed.

I. Preliminaries

This introductory article is considered as a background for the martial included in this paper.

1. Double Sets:

In this section, we state the basic definition of double set and we sate the properties of double sets, the quasi – coincident relation in the sense of double set theory **.**

Definition [3].

Let X be a non empty set.

a) A double set α is an ordered pair (A_1 , A_2) $\in P(X)\times P(X)$ such that $A_1 \subseteq A_2$.

b) The family of all double sets on X , will be denoted by D(X).

i.e $D(X)=\{(A_1,A_2):(A_1,A_2)\in P(X)\times P(X), A_1\subseteq A_2\}$.

c) The double set χ =(X , X) is called the universal double set,

d) The double set Φ = (φ , φ) is called the empty double set.

We introduce the following example:

Example: Let $X = \{a,b,c\}$

$$Then\ D(X) = \left\{ \begin{array}{l} (\phi,\phi),(\phi,\{a\}),(\{a\},\{a\}),(\phi,\{b\}),(\{b\},\{b\}),(\phi,\{c\}),(\{c\},\{c\}), \\ (\phi,\{a,b\}),(\{a\},\{a,b\}),(\{b\},\{a,b\}),(\{a,b\},\{a,b\}),(\phi,\{a,c\}), \\ (\{a\},\{a,c\}),(\{a,c\},\{a,c\}),(\{c\},\{a,c\}),(\phi,\{b,c\}),(\{b\},\{b,c\}), \\ (\{c\},\{b,c\}),(\{b,c\},\{b,c\}),(\phi,X),(\{a\},X),(\{b\},X),(\{c\},X), \\ (\{a,b\},X),(\{a,c\},X),(\{b,c\},X),(X,X) \end{array} \right\}$$

Theorem:

If X has n elements i.e $|X| = n$ then $|D(X)| = 3^n$

Proof: $D_B(X) = \{(A,B) : A \subseteq B\}$

Put $D_B = \{(A,B) : A \subseteq B\}$ for every $B \subseteq X$.

Then $\{D_B : B \subseteq X\}$ is a partition for D(X).

Now $D(X) = \bigcup_{B \subseteq X} D_B$ and $|D(X)| = \sum_{B \subseteq X} |D_B|$

Noting that $D_\phi = \{(\phi,\phi),\} \Rightarrow |D_\phi| = 1$

$$D_{\{x_i\}} = \{(\phi,\{x_i\}),(\{x_i\},\{x_i\})\} \Rightarrow |D_{\{x_i\}}| = \binom{n}{1}2 \qquad i = 1,2,3,\ldots,n$$

$$D_{\{x_i,x_j\}} = \{(\phi,\{x_i,x_j\}),(\{x_i\},\{x_i,x_j\}),(\{x_j\},\{x_i,x_j\}),(\{x_i,x_j\},\{x_i,x_j\})\}$$

$$\Rightarrow |\eta_{\{x_i,x_j\}}| = \binom{n}{2}2^2 \Rightarrow$$

$$|D(X)| = 1 + \binom{n}{1}2 + \binom{n}{2}2^2 + \binom{n}{3}2^3 + \ldots + \binom{n}{n}2^n = (1+2)^n = 3^n$$

Definition[3]. Let $\alpha = (A_1, A_2), \beta = (B_1, B_2) \in$ D (X). Then:

a) $\alpha \subseteq \beta$ if and only if $A_i \subseteq B_i$, i =1,2

b) $\alpha = \beta$ if and only if $A_i = B_i$, i =1, 2

c) $\alpha \cup \beta = (A_1 \cup B_1, A_2 \cup B_2)$

d) $\alpha \cap \beta = (A_1 \cap B_1, A_2 \cap B_2)$

e) $\alpha \setminus \beta = (A_1 \setminus B_2, A_2 \setminus B_1)$

f) $\alpha^c = (A_2^c, A_1^c)$ where α^c is the complement of α.

Definition [3]. Let $x \in X$. Then the double sets $x_{\frac{1}{2}} = (\varphi, \{x\})$ and $x_1 = (\{x\}, \{x\})$ are said to be double points in X.

The set of all double points of X will be denoted by X_P

. i.e. $X_P = \{x_t : x \in X, t = \frac{1}{2}, 1\}$.

Proposition [3] If $\alpha = (A_1, A_2) \in$ D (X), then:

a- $x_{\frac{1}{2}} \in \alpha \Leftrightarrow x \in A_2$

b- $x_1 \in \alpha \Leftrightarrow x \in A_1$.

c- $x_t \notin \alpha \Leftrightarrow x \notin A_2$.

d- $x_1 \in \alpha \Rightarrow x_{\frac{1}{2}} \in \alpha$

Definition[3]. Two double sets α and β are said to be quasi- coincident, denoted by $\alpha\, Q\, \beta$, if $A_1 \cap B_2 \neq \varphi$ or $A_2 \cap B_1 \neq \varphi$.

α is not quasi-coincident with β, denoted this by $\alpha\, \overline{Q}\, \beta$, if $A_1 \cap B_2 = \varphi$ and $A_2 \cap B_1 = \varphi$.

Theorem [3]

Let $\alpha, \beta, \gamma, x_t, y_r \in$ D(X). Then:

a) $\alpha\, Q\, \beta \Rightarrow \alpha \cap \beta \neq \underline{\varphi}$.

b) $\alpha\, Q\, \beta$ iff $x_t\, Q\, \beta$, for some $x_t \in \alpha$.

c) $\alpha \overline{Q} \beta \Leftrightarrow \alpha \subseteq \beta^c$.

d) $x \neq y \Rightarrow x_t \overline{Q} y_r$ for every r, t ∈ { $\frac{1}{2}$, 1}.

e) $x_t \overline{Q} \alpha \Leftrightarrow x_t \in \alpha^c$.

f) $x_t \overline{Q} y_r \Leftrightarrow x \neq y$ or $x = y$, t = r= $\frac{1}{2}$.

g) $x_t Q y_r \Leftrightarrow x = y$ and t + r >1.

h) $\alpha \overline{Q} \alpha^c$.

i) $\alpha \subseteq \beta$ iff $x_t Q \alpha$ implies $x_t Q \beta$.

j) $\alpha = \bigcup \{x_t : x_t \overline{Q} \alpha^c\}$.

k) $\alpha \overline{Q} \beta$, $\gamma \subseteq \beta \Rightarrow \alpha \overline{Q} \gamma$.

l) $x_t \overline{Q} (\alpha \cup \beta) \Leftrightarrow x_t \overline{Q} \alpha \wedge x_t \overline{Q} \beta$.

Definition[3] Let η_1, $\eta_2 \subseteq$ P (X). The double product of η_1 and η_2 is denoted by $\eta_1 \hat{\times} \eta_2$ and is defined by

$$\eta_1 \hat{\times} \eta_2 = \{(A_1, A_2) \in \eta_1 \times \eta_2 : A_1 \subseteq A_2\}.$$

Definition [3] Let $\eta \subseteq D(X)$, then all first component of η is denoted by $\pi_1(\eta)$ and all second component of η is denoted by $\pi_2(\eta)$ .i.e $\pi_1(\eta) = \{A_1 \subseteq X : (A_1, A_2) \in \eta\}$ and $\pi_2(\eta) = \{A_2 \subseteq X : (A_1, A_2) \in \eta\}$

Definition [4] Let X be a non- empty set. Then a family $\eta \subseteq D(X)$ is called a double topology on X if it satisfies, the following axioms:

a) $\underline{\varphi} = (\varphi, \varphi)$, $\underline{X} = (X, X) \in \eta$.

b) If $\underline{A}, \underline{B} \in \eta$, then $\underline{A} \cap \underline{B} \in \eta$

c) If $\{\underline{A_s} : s \in S\} \subseteq \eta$,then $\bigcup_{s \in S} \underline{A_s} \in \eta$.

The pair (X, η) is called a double topological space (DT-space, for short). Each member of η is called an open double set in X. The complement of an open double set is called a closed double set. A double topological space is called a double stratified space if η contains the double set (ϕ, X). For any $\underline{A} \in D(X)$, the double closure of $\underline{A}$ is denoted by $DCl\,\underline{A}$ or $\overline{\underline{A}}$ and defined by $\overline{\underline{A}} = \cap\{\underline{B} : \underline{B} \in \eta^c \text{ and } \underline{A} \subseteq \underline{B}\}$.

Also, the double interior of $\underline{A}$ is denoted by $DInt\,\underline{A}$ or $\underline{A}^0$ and defined by $\underline{A}^o = \cup\{\underline{B} : \underline{B} \in \eta \text{ and } \underline{B} \subseteq \underline{A}\}$.

2.Basics of Rough Sets[9]

Consider the following information system

U \ A	a_1	a_2	a_3	$D = \{d\}$
x_1	3	2	2	**1**
x_2	2	2	2	**0**
x_3	2	2	2	**1**
x_4	1	1	2	**0**
x_5	2	1	1	**1**
x_6	3	1	1	**0**

1- Using the above information system , the relation $IND(C) = \{(x_i, x_j) \in U^2 \mid \forall a \in C, a(x_i) = a(x_j)\}$ is defined on U

$IND(C) = \{ (x_1, x_1), (x_2, x_2), (x_2, x_3), (x_3, x_2), (x_3, x_3), (x_4, x_4), (x_5, x_5), (x_6, x_6)\}$.

The partition (elementary set) induced by $IND(C)$.

$$[x_i]_C = \{x_j \in U \mid (x_i, x_j) \in IND(C)\}$$

The set of all partitions (elementary set) induced by $IND(C)$ is:

$$U / C = \{\{x_1\}, \{x_2, x_3\}, \{x_4\}, \{x_5\}, \{x_6\}\}$$

2- **Finding the set of all partitions (concepts) induced by**

Indiscernibility relation $IND(D)$

$IND(D) = \{(x_i, x_j) \in U^2 \mid \forall d \in D, d(x_i) = d(x_j)\}$.

$IND(D) = \{ (x_1, x_1), (x_1, x_3), (x_1, x_5), (x_2, x_2), (x_2, x_4), (x_2, x_6),$

$(x_3,x_1),(x_3,x_3),(x_3,x_5),$
$(x_4,x_2),(x_4,x_4),(x_4,x_6),$
$(x_5,x_5),(x_5,x_3),(x_5,x_5),$
$(x_6,x_2),(x_6,x_4),(x_6,x_6)\}$.

The partition (concepts) induced by $IND(D)$

$$[x_i]_D = \{x_j \in U \mid (x_i,x_j) \in IND(D)\}$$

The set of all partitions (concepts) induced by $IND(D)$

The set of all partitions (concepts) induced by $IND(D)$ is:

$$U/D = \{\{x_1,x_3,x_5\},\{x_2,x_4,x_6\}\}$$

Lower and upper approximations

Let X denote the subset of elements of the universe U $(i.e.\ X \subseteq C)$. X can be approximated using only the information contained within condition attribute $A\ (A \subseteq C)$ by constructing the lower and the upper approximation of the set X.

Consider the approximation of the set X in the following figure:

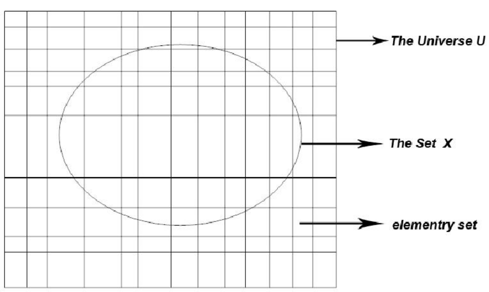

Figure (2.1):

Each square in Figure (2.1) represent an elementary set (equivalence class) induced by the indiscernibility relation $IND(A), A \subseteq C$ [..].

The lower approximation of the set X using the information contained within condition attributes $A\ (A \subseteq C)$ is the union of all elementary sets X_i contained in X [..], as shown in Figure (2.2), more formally $\underline{A}X = \bigcup_{X_i \subseteq X} X_i$

The lower approximation of the set X **.**

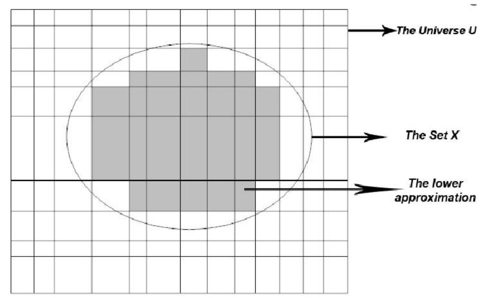

Figure (2.2):

The upper approximation of the set X using the information contained with in condition attributes $A\ (A \subseteq C)$ is the union of all elementary sets which have a non-empty intersection with X [..], as shown in Figure (2.3), more formally $\overline{A}X = \bigcup_{X_i \cap X \neq \varphi} X_i$

The upper approximation of the set X

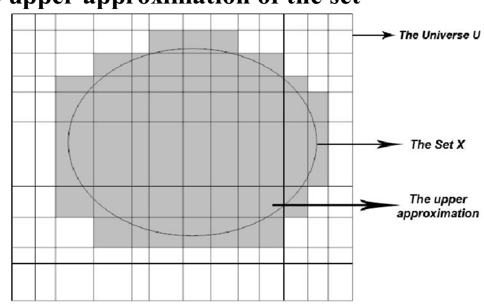

Figure (2.3):

Positive, Boundary and Negative Region

The positive region of the set X, is the set of these objects, which can, with certainty, be classified in the set X (or belonging to the set X), using the information contained within condition attributes A [9], as shown in Figure (2.4), more formally $POS_A(X) = \underline{A}X$.

Positive region of X

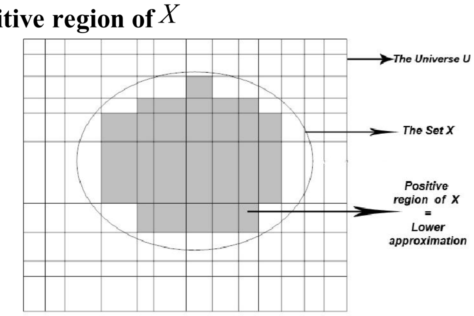

Figure (2.4):

The boundary region is the difference between these two approximations, which contains all the objects that can't be classified with certainty as belonging or not to the set X, using the information contained within condition attributes A, as shown in Figure (2.5), more formally $BN_A(X) = \overline{A}X - \underline{A}X$

The boundary region of X.

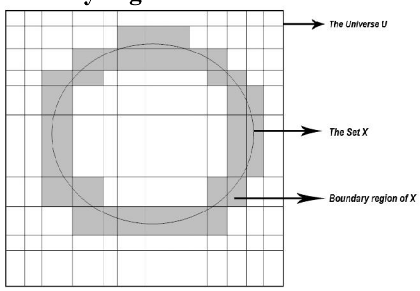

Figure (2.5):

The negative region of X is the set of objects, which without ambiguity, that cannot be classified in the set X (or as not belonging to the set X) using the information contained within condition attributes A, as shown in Figure (2.6), more formally $NEG_A(X) = U - \overline{A}X$.

Negative region of X.

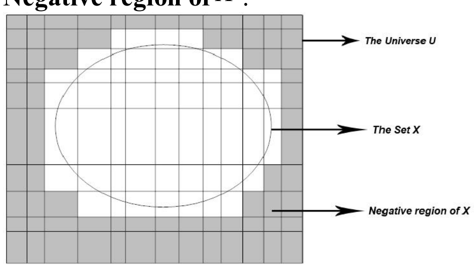

Figure (2.6):

Remark

The set X is said to be rough set if the boundary region is non-empty [.9.].

Accuracy of approximation

An accuracy measure of the set X in $A \subseteq C$ is defined as: $\mu_A(X) = \frac{|\underline{A}X|}{|\overline{A}X|}$, $0 \le \mu_A(X) \le 1$

where $|\cdot|$ denotes the cardinality of a set (the number of objects contained in the lower (upper) approximation of the set X).

Remark

If X is definable in U (i.e., $\overline{A}X = \underline{A}X$), then $\mu_A(X) = 1$, if X is undefinable in U (i.e., $\overline{A}X \ne \underline{A}X$), then $\mu_A(X) < 1$ [..].

Example:

The sets of all partitions (elementary sets) induced by $IND(C)$ are:

$X_1 = [x_1]_C = \{x_1\}$.

$X_2 = [x_2]_C = [x_3]_C = \{x_2, x_3\}$.

$X_3 = [x_4]_C = \{x_4\}$.

$X_4 = [x_5]_C = \{x_5\}$.

$X_5 = [x_6]_C = \{x_6\}$.

The sets of all partitions (concepts) induced by $IND(D)$ are:

$Y_1 = [x_1]_D = [x_3]_D = [x_5]_D = \{x_1, x_3, x_5\}$.

$Y_2 = [x_2]_D = [x_4]_D = [x_6]_D = \{x_2, x_4, x_6\}$.

1. The lower and upper approximations of Y_i **using the information contained in condition attribute** C **is:**

The lower approximation is: $\underline{C}Y_i = \bigcup_{X_i \subseteq Y_i} X_i$

The upper approximation is: $\overline{C}Y_i = \bigcup_{X_i \cap Y_i \ne \varphi} X_i$

For Y_1,

The lower approximation $\underline{C}Y_1 = \bigcup_{X_i \subseteq Y_1} X_i$

$X_1 = \{x_1\} \subset Y_1$.

$X_2 = \{x_2, x_3\} \not\subset Y_1$.

$X_3 = \{x_4\} \not\subset Y_1$.

$X_4 = \{x_5\} \subset Y_1$.

$X_5 = \{x_6\} \not\subset Y_1$.

The lower approximation is the union of all elementary sets contained in X, then $\underline{C}Y_1 = X_1 \cup X_4 = \{x_1, x_5\}$.

It means that the lower approximation can be with certainty classified as members of X on the basis of knowledge in condition attribute C.

The upper approximation $\overline{C}Y_1 = \bigcup_{X_i \cap Y_i \ne \varphi} X_1$

$X_1 = \{x_1\} \cap Y_1 = \{x_1\} \ne \phi$.

$X_2 = \{x_2, x_3\} \cap Y_1 = \{x_3\} \ne \phi$.

$X_3 = \{x_4\} \cap Y_1 = \varphi$.

$X_4 = \{x_5\} \cap Y_1 = \{x_5\} \ne \phi$.

$X_5 = \{x_6\} \cap Y_1 = \phi$

The upper approximation is the union of all elementary sets which have a non-empty intersection with X, then $\overline{C}Y_1 = X_1 \cup X_2 \cup X_4 = \{x_1, x_2, x_3, x_5\}$.

It means that the upper approximation can be only classified as possible members of X on the basis of knowledge in condition attribute C.

For Y_2

The lower approximation is $\underline{C}Y_2 = X_3 \cup X_5 = \{x_4, x_6\}$.

The upper approximation is $\overline{C}Y_2 = X_2 \cup X_3 \cup X_5 = \{x_2, x_3, x_4, x_6\}$.

3.Relative double set:

In this section, we introduce the notion of relative double sets and give examples and investigate some of its properties and characterizations.

Definition:

Let $U \neq \phi, Y \subseteq U$. Then pair (A, B) $\in D(U)$ is a double set of U relative to Y if $A \subseteq Y \subseteq B$.

The family of all double sets of U relative to Y is denoted by $D_Y(U)$ **and is given by** $D_Y(U) = \{(A, B) \in D(U) : A \subseteq Y \subseteq B\}$

Example: Let U= {a,b,c,d,e}, Y={a,b,d}. Then $D_Y(U) = \{(A, B) \in D(U) : A \subseteq Y \subseteq B\}$

This can be listed in the following table

$Y \cup P(Y')$ / P(Y)	$B_1 = Y$	$B_2 = Y \cup \{c\}$	$B_3 = Y \cup \{e\}$	$B_4 = Y \cup \{c, e\}$
$A_1 = \phi$	(A_1, B_1)	(A_1, B_2)	(A_1, B_3)	(A_1, B_4)
$A_2 = \{a\}$	(A_2, B_1)	(A_2, B_2)	(A_2, B_3)	(A_2, B_4)
$A_3 = \{b\}$	(A_3, B_1)	(A_3, B_2)	(A_3, B_3)	(A_3, B_4)
$A_4 = \{d\}$	(A_4, B_1)	(A_4, B_2)	(A_4, B_3)	(A_4, B_4)
$A_5 = \{a, b\}$	(A_5, B_1)	(A_5, B_2)	(A_5, B_3)	(A_5, B_4)
$A_6 = \{a, d\}$	(A_6, B_1)	(A_6, B_2)	(A_6, B_3)	(A_6, B_4)
$A_7 = \{b, d\}$	(A_7, B_1)	(A_7, B_2)	(A_7, B_3)	(A_7, B_4)
$A_8 = Y$	(A_8, B_1)	(A_8, B_2)	(A_8, B_3)	(A_8, B_4)

Definition :

Let $\eta, \delta \subseteq P(X)$. Then the direct union of η, δ is denoted by $\eta \coprod \delta$ and its defined by $\eta \coprod \delta = \{A \cup B : A \in \eta, \beta \in \delta\}$.

Lemma 1: The family $D_Y(U)$ has the following properties:-

1) $D_Y(U) \subseteq D(U)$
2) $\pi_1(D_Y(U)) = \{A_1 : (A_1, A_2) \in D_Y(U)\} = P(Y)$
3) $\pi_1(D_Y(U))$ is the discrete topology on Y
4) $\tau_1(Y) = \pi_1(D_Y(U)) \cup \{U\}$ is a topology on U
5) $\tau'_1(Y) = (\{Y'\} \coprod P(Y)) \cup \{\phi\}$
6) $\pi_2(D_Y(U)) = \{A_2 : (A_1, A_2) \in D_Y(U)\}$
7) $\tau_2(Y) = \pi_2(D_Y(U)) \cup \{\phi\}$ is a topology on U which is called the included set topology.

$$\tau'_2(Y) = \left(\{Y\} \coprod P(Y')\right) \cup \{U\}$$

8)
9)
$D_Y(U) = \pi_1(D_Y(U)) \times \pi_2(D_Y(U))$
$= \{(A_1, A_2) : A_1 \in \pi_1(D_Y(U)), A_2 \in \pi_2(D_Y(U))\}$

10) $\pi_1(D_Y(U)) \times \pi_2(D_Y(U))$ is a double topology on U.

11) $|U| = n,\ |Y| = m \Rightarrow |D_Y(U)| = 2^n$.

12) If $Y = U$, then $D_U(U) = \{(A_1, U) : A_1 \subseteq U\}$

13) If $Y = \phi$, then $D_\phi(U) = \{(\phi, A_2) : A_2 \subseteq U\}$

Relative double sets in information systems

If (U,R) is an approximation space , then $(\underline{Y}, \overline{Y}) \in D_Y(U)$ where $\underline{Y}$ is the lower approximation of Y and $\overline{Y}$ is the upper approximation of Y.

Remark

If X is definable in U (i.e., $\overline{A}X = \underline{A}X$), then $\mu_A(X) = 1$, if X is undefinable in U (i.e., $\overline{A}X \neq \underline{A}X$), then $\mu_A(X) < 1$.

Example:

Find here all double sets relative to Y1 and Y2

Example:

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and let $Y_1 = \{x_1, x_3, x_5\}$. Then,

1. $\pi_1(D_{Y_1}(U)) = P(Y_1) = \begin{Bmatrix} A_1 = \phi, A_2 = \{x_1\}, A_3 = \{x_3\}, A_4 = \{x_5\} \\ , A_5 = \{x_1, x_3\}, A_6 = \{x_1, x_5\}, A_7 = \{x_3, x_5\}, A_8 = \{x_1, x_3, x_5\} \end{Bmatrix}$
2. $\pi_1(D_{Y_1}(U))$ is the discrete topology on Y_1
3. $\tau_1(Y_1) = \pi_1(D_{Y_1}(U)) \cup \{U\}$ is a topology on U
4. $\tau_1'(Y_1) = \{Y_1' \cup A_1 : A_1 \subseteq Y_1\} \cup \{\phi\} = (\{Y_1'\} \coprod P(Y_1)) \cup \{\phi\}$
5. $\pi_2(D_{Y_1}(U)) = \begin{Bmatrix} B_1 = Y_1, B_2 = Y_1 \cup \{x_2\}, B_3 = Y_1 \cup \{x_4\}, B_4 = Y_1 \cup \{x_6\}, \\ B_5 = Y_1 \cup \{x_2, x_4\}, B_6 = Y_1 \cup \{x_4, x_6\}, B_7 = Y_1 \cup \{x_2, x_6\}, B_8 = U \end{Bmatrix}$ $= \{Y_1\} \coprod P(Y_1)$
6. $\pi_2(D_{Y_1}(U)) = \{A_2 : (A_1, A_2) \in D_{Y_1}(U)\}$
7. $\tau_2(Y_1) = \pi_2(D_{Y_1}(U)) \cup \{\phi\}$ is a topology on U
8. $\tau'_2(Y_1) = \left(\{Y_1\} \coprod P(Y_1) \right) \cup \{U\}$
9. $\pi_1(D_{Y_1}(U)) \times \pi_2(D_{Y_1}(U))$ $= \{(A_i, B_j) : 1 \le i \le 8, 1 \le j \le 8\}$

$|D_{Y_1}(U| = 2^6 = 64$

10. $\pi_1(D_{Y_1}(U) \times \pi_2(D_{Y_1}(U) \cup \{\phi, U\}$ is a double topology on U.
11. $(\underline{Y}_1 = \{x_1, x_5\}, \overline{Y}_1 = \{x_1, x_2, x_3, x_5\}) = (A_6, B_2) \in D_{Y_1}(U)$.
12. $DInt(\underline{Y}_1, \overline{Y}_1) = (\underline{Y}_1, \overline{Y}_1) = (A_6, B_2)$
13. $DCl(\underline{Y}_1, \overline{Y}_1) = (Y_1, \{x_1, x_2, x_3, x_5\}) = (Y_1, B_2)$
14. $DBN(\underline{Y}_1, \overline{Y}_1) = (Y_1, \{x_1, x_2, x_3, x_5\}) \setminus (A_6, B_2)$ $= (Y_1 \setminus B_2, B_2 \setminus A_6) = (\phi, \{x_2, x_3\})$

Example

If (U,R) is the information table given in section 2

$Y_1 = \{x_1, x_3, x_5\},\ \underline{Y}_1 = \{x_1, x_5\},\ \overline{Y}_1 = \{x_1, x_2, x_3, x_5\}$

we construct two algorithms for increasing the accuracy of the decisions in this information system in the following steps

The first algorithm

1-Find the decision subset U/D

2-Find the family of all relative double sets with respect to the decision subset $D_d(U)$

3-Choose the largest first component $\pi_1(D_d(U)) = \{A_1 \subseteq U : (A_1, A_2\} \in D_d(U)\}$

4-Choose the smallest second component $\pi_2(D_d(U)) = \{A_2 \subseteq U : (A_1, A_2\} \in D_d(U)\}$

5- Use these two subsets as lower and upper approximations.

The second algorithm

1-Find the decision subset U/D

2-Find the family of all relative double sets with respect to the decision subset $D_d(U)$.

3- Choose the largest first component $\pi_1(D_d(U)) = \{A_1 \subseteq U : (A_1, A_2\} \in D_d(U)\}$ with largest

$$\alpha_1 = \frac{card\ A_1}{card\ \underline{Y}_1}$$

4- Choose the smallest second component $\pi_2(D_d(U)) = \{A_2 \subseteq U : (A_1, A_2\} \in D_d(U)\}$ with largest

$$\alpha_2 = \frac{card\ A_2 \cap \overline{Y_1}}{card\ \overline{Y_1}}$$

5- Use these two subsets as lower and upper approximations.

$A_1 \subset Y_1$	$A_2 \supset Y_1$	$A_1 \subset \underline{Y}_1$	α_1	$A_2 \cap \overline{Y_1}$	α_2
$\{x_1\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_1\}$	$\frac{1}{2}$	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$
$\{x_3\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_3\}$	$\frac{1}{2}$	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$
$\{x_5\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_5\}$	$\frac{1}{2}$	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$
$\{x_1,x_3\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_1,x_3\}$	$\frac{1}{2}$	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$
$\{x_1,x_5\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_1,x_5\}$	1	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$
$\{x_3,x_5\}$	$\{x_1,x_2,x_3,x_5\}$ $\{x_1,x_3,x_4,x_5\}$ $\{x_1,x_3,x_5,x_6\}$	$\{x_3,x_5\}$	1	$\overline{Y_1}$ $\{x_1,x_3,x_5\}$ $\{x_1,x_3,x_5\}$	1 $\frac{3}{4}$ $\frac{3}{4}$

Conclusion

The concept of relative double set initiated in this work can be used in modifying the hybridization of rough set and double set in both of theory and applications. In the theoretical context it can be used in defining lower (upper) relative double sets using lower and upper approximations, and in the construction of decision rules in information systems. On the other hand it opens the way for wide range of uncertain concepts approximation by finding a class of approximations for each concept and choosing the best approximation whose lower approximation is the union of all relative double set first component, and whose upper approximation is the intersection of all relative double sets second components.

References

1. H. M. Abu-Donia, New Rough Set Approximation Spaces, Abstract and Applied Analysis, Volume 2013, Article ID 189208, 7 pages.
2. D. P. Acharjya, B. K. Tripathy, Topological Characterization, Measures of Uncertainty and Rough Equality of Sets on Two Universal Sets, I.J. Intelligent Systems and Applications 2(2013) 16-24.
3. D. Coker, A note on intuitionistic sets and intuitionistic points, Turkish J.Math. 20 (1996) 343-351.
4. D. Çoker, An introduction to intuitionistic Topological Spaces, BUSEFAL 81 (2000) 51–56.
5. Estaji, M.R. Hooshmandasl, B. Davvaz, Rough set theory applied to lattice theory, Information Sciences 200 (2012) 108–122.
6. M. Kondo, On the structure of generalized rough sets, Information Sciences 176 (2006) 589–600.
7. X. Kang, D. Li, S. Wang, K. Qu, Rough set model based on formal concept analysis, Information Sciences 222 (2013) 611–625.
8. V. Lindley, Introduction to probability theory and statistics from a bayesian
9. point of view, Cambridge University Press, Cambridge, 1965.
10. Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning about Data, Kluwer, Dordrecht, 1991.
11. Z. Pawlak, Rough sets and intelligent data analysis, Information Sciences 147 (2002) 1–12.
12. Z. Pawlak, A. Skowron, Rudiments of rough sets, Information Sciences 177 (2007) 3–27.
13. Z. Pawlak, A. Skowron, Rough sets: some extensions, Information Sciences 177 (2007) 28–40.
14. Z. Pawlak, A. Skowron, Rough sets and Boolean reasoning, Information Sciences 177 (2007) 41–73.
15. G. Shafer, The early development of mathematical probability, Working Paper, School of Business, The University of Kansas.
16. E, Q. Chen, S. Zhao, D. Yeung, X. Wang , Hybridization of Fuzzy and Rough Sets: Present and Future, Fuzzy Sets and Their Extensions: Representation, Aggregation and Models Studies in Fuzziness and Soft Computing Volume 220(2008) 45-64.

12/11/2013