Multi-objective aggregate production planning model with fuzzy parameters and its solving methods

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Abstract: Aggregate production planning (APP) is considered as mid-term decision planning. The purpose of multi-period APP is to set up overall production levels for each product category to meet fluctuating or uncertain demand in the near future and to set up decisions and policies on the subject of hiring, lay-offs, overtime, backorder, subcontracting, facilities and inventory. In this study, we develop a new multi-objective linear programming model for general APP for multi-period and multi-product problems. We assume that, there is uncertainty in critical input data (i.e., market demands and unit costs). This model is suitable for 24-hour production systems. To show practicality of our model, we will implement this model in a case study. Finally, we propose an interactive solution procedure for achieving the best compromise solution.

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1. Introduction

Aggregate production planning (APP) determines the best way to meet forecast demand in the intermediate future, often from 3 to 18 months ahead, by adjusting regular and overtime production rates, inventory level, labor levels, subcontracting and backorder rates, and other controllable variables. APP aims to identify production, inventory and work force levels to meet fluctuating demand requirements over an intermediate-range planning horizon (Chase and Aquilano 1992). Other forms of family disaggregation plans that are derived from APP are master production schedule, material requirement planning and capacity plan. In this research, we will develop a multiple objective APP model for multi-product and multi-period. Also, in this model, we will assume that costs and demand and resources are imprecise which have triangular possibility distributions. In this section, we review some pervious researches. Baykasoglu (2001) extended Masud's (1980) model by adding subcontracting and setup decisions or setup cost of product, setup time of product, and subcontracting cost of product. He developed a multi-objective linear goal programming model that with the following model objectives: 1) maximization of profit, 2) minimization of workforce changes, 3) minimization of inventory investment, and 4) minimization of backorder. To solve the model, he applied Tabu search, due to the high complexity of the model and the large number of model constraints. Techawiboonwong and Yenradee (2003) developed a linear model for multi-product APP that enabled workers to be transferred among production lines. Their model had only one objective function or minimizing total cost. In reality, when a worker performs any task for a long time, they get used to the task. Then, if that worker is transferred to operate a different task, their skill with the new task would likely be lower than that with the old task. This also impacts productivity. A feasible way to evaluate the extent to which productivity falls is to evaluate training cost and cost due to the loss of production. Therefore, they add to their model cost of transferring workers as a cost parameter. They compared two situations: 1) the worker cannot transfer among production lines, and 2) the worker can transfer between lines. The results of APP model showed that the total cost increases about 5% when the workers did not transfer among production lines; however, the goals and model inputs in these APP models were assumed to be crisp. In the real world, APP problems with deterministic parameters are unsuitable for yielding an effective solution. In the real word, APP problems, input data or parameters such as demand, resources, and costs, are generally imprecise due to incomplete or unobtainable information (Baykasoglu, and Gocken, 2010; Wang, and Liang, 2004). These imprecise parameters can be defined as random numbers with probability distribution, fuzzy numbers or interval numbers (Baykasoglu, and Gocken, 2010). Recently, several researches have been conducted on fuzzy APP. Mula et al. (2006) reviewed more than 87 articles focused on production planning under uncertainty. The results of their study revealed that, most of the analytical models were crisp or mentioned only one type of uncertainty and assumed the simple structure of the production process. Therefore, they suggested that more researches need to be done in the future about production planning under uncertainty. Aliev, et al. (2007) proposed an APP model in a supply chain. In this model, market demand, production capacity and storage capacity in production environment were uncertain. They solved their model by using the genetic algorithm to achieve near- optimal solutions. They compared their fuzzy model with crisp and disintegrated approaches. Comparison of the results of fuzzy and crisp integrated production-distribution aggregate planning showed that the profit achieved by the fuzzy model was 5-10% higher than the crisp model, especially, when the actual demand declined from the forecasted value or the capacity plummeted over the planning horizon. Torabi et al. (2009) developed a fuzzy hierarchical production planning comprised of two decision-making levels. Their proposed model attempted to maximize total profit or maximize the difference of the revenue and operation costs. In their model, all cost parameters were imprecise. Their results showed that hierarchical production planning models with fuzzy data are more practical than hierarchical production planning models with crisp data. Other APP models are Fung et al. (2003), Wang et al. (2003), Sillekens et al. (2011). However, the majority of APP models have cost-related objectives, whereas non-cost objectives are often sought by managers. This article is organized as follows: Section 2 introduces the problem and lays the frameworks for problem formulation of multi-objective APP model. Section 3 proposes a procedure for solving fuzzy APP model. Implementation of APP model (case study) is given in Section 4. Finally, Section 5 gives some conclusions and suggestions for further studies.

2. Fuzzy aggregate production planning model

In this section, we develop a new mathematical model for APP problem. The characteristics and assumptions of the model can be described as follows.it is assumed that company produces N types of products to fulfil market demand over planning horizon T. A manufacturer wants to find, for each product and each planning period, the 'best' production level and inventory level. It also wants to find the workforce level for each period. The forecasted maximum demands for each product do not necessarily remain constant from period to period. In addition, a portion of the forecasted demand (which can also vary from period to period) must be satisfied in that particular period. The rest of the forecasted demands can either be backordered or not satisfied at all; however, all backorders must be fulfilled within the next period. Assigning a set of crisp values for model parameters is inappropriate for dealing with ambiguous APP decision problems; thus, we assume that environmental coefficients and related parameters are uncertain (Gen et al., 1992; Wang and Liang, 2004; Alive et al., 2007; Torabi et al., 2009; Baykasouglu et al., 2010; Tang et al., 2003). Therefore, forecast demand-related operating costs, and machine and labor capacities are fuzzy over the planning horizon.

In this research, we assume that our imprecise input data are triangular positive fuzzy numbers R= (R_1, R_2, R_3) as shown in Figure 1. The membership function is as follows:

$$\mu(x)=\begin{cases}[(x-R_2)/(R_2-R_1)]+1 & (R_1\le x\le R_2),\\ [(x-R_2)/(R_2-R_3)]+1 & (R_2\le x\le R_3)\\ 0 & (x\le R_1 \text{ OR } x\ge R_3)\end{cases}$$

where x is decision variable (1)

Other assumptions are:

1) Average hiring and firing cost are considered;

2) Raw materials are always available without shortage; **Appendix 1:** Transformation method (Okada et.al 1991, Gen et.al 1992)

Suppose we have fuzzy multi-objective model as follows:

$$\text{Max } z_k=\Sigma\ c_{kj}\ x_j,\ k=1,2,..,q_1$$

$$\text{Min } z_k=\Sigma\ c_{kj}\ x_j,\ k=q_1+1,..,q=q_1+q_2$$

Subject to:

$$\Sigma\ a_{ij}\ x_j\le b_i\quad i=1,2,\ldots,m_1$$

$$\Sigma\ a_{ij}\ x_j\ \ge b_i\quad i=m_1+1,\ldots,m_2$$

$$\Sigma\ a_{ij}\ x_j=b_i\quad i=m_2+1,\ldots,m=m_1+m_2+m_3$$

$$x_j\ge 0\quad j=1,\ldots,n$$

where

$c_{kj}=(c_{kj1},c_{kj2},c_{kj3})$ is a fuzzy coefficient of the k-th objective function and j-th decision variable

$a_{ij}=(a_{ij1},a_{ij2},a_{ij3})$ is a fuzzy technical coefficient of the i-th constraint and j-th decision variable

$b_i=(b_{i1},b_{i2},b_{i3})$ is a fuzzy available resource of i-th constraint

x_j is decision variables. All fuzzy parameters in this multi-objective model are triangular fuzzy numbers.

Crisp multi-objective linear model can be computed by following formulation:

$$\text{Max } z_k=\Sigma\ [(1-\alpha)c_{kj3}+\alpha\ c_{kj2}]\ x_j,\ k=1,2,..,q_1$$

$$\text{Min } z_k=\Sigma\ [(1-\alpha)c_{kj1}+\alpha\ c_{kj2}]x_j,\ k=q_1+1,..,q=q_1+q_2$$

Constraint:

$$\Sigma\ [(1-\alpha)a_{ij1}+\alpha\ a_{ij2}]\ x_j\le(1-\alpha)b_{i3}+\alpha\ b_{i2},\quad i=1,2,\ldots,m_1$$

$\Sigma\ [(1-\alpha)a_{ij3}+\alpha\ a_{ij2}\ ]x_j\ \geq (1-\alpha)b_{i1}+\alpha\ b_{i2},\ i= m_1+1,\ldots,\ m_2$

$\Sigma\ [(1-\alpha)a_{ij1}+\alpha\ a_{ij2}\ ]x_j \leq(1-\alpha)b_{i3}+\alpha b_{i2},\quad i= m_2+1,\ldots,m$

$\Sigma\ [(1-\alpha)a_{ij3}+\ \alpha\ a_{ij2}\ ]\ x_j \geq(1-\alpha)b_{i1}+\alpha b_{i2},\ \ i= m_2+1,\ldots,m$

$x_j\geq0 \quad j= 1,\ldots,n$

where α is a cutoff value between zero to one, $\alpha=[0,0.1,0.2,0.3,\ldots,1]$.

Appendix 2: A sample of input data ($\alpha= 0$) for Shahab Shishe company example (max-min method)

Max λ

λ-((1/189023)*(395587-(0.016p_{11} + 0.016p_{12} + 0.016p_{13} + 0.016p_{14} + 0.007p_{21} +0.007p_{22} +0.007p_{23} + 0.007p_{24} + 440w_1 +440w_2+ 440w_3+440w_4+ 0.003I_{11}+ 0.003I_{12}+ 0.003I_{13} + 0.003I_{14}+0.0008I_{21}+ 0.0008I_{22}+ 0.0008I_{23}+ 0.0008I_{24})))<=0;

λ-((1/22)*(22- (Wh_1+ Wh_2+ Wh_3+ Wh_4+ WL_1+ WL_2+ WL_3+ WL_4)))<=0;

λ-((1/.028)*(1-[(B_{11}+ B_{12}+ B_{13}+ B_{14}+ B_{21}+ B_{22}+ B_{23}+ B_{24})/17490000])-.971)<=0;

λ>=0;

λ<=1;

$W_1<80$; $W_2<80$; $W_3<80$; $W_4<80$;

$W_1>58$; $W_2>58$; $W_3>58$; $W_4>58$;

$W_1 - Wh_1 + WL_1 = 68$;

$W_2 - W_1 - Wh_2 + WL_2 = 0$;

$W_3 - W_2 - Wh_3 + WL_3 = 0$;

$W_4 - W_3 - Wh_4 + WL_4 = 0$;

$P_{11} > -500000$;

$P_{12} + I_{11} - B_{11} > 1970000$;

$P_{13} + I_{12} - B_{12} > 1600000$;

$P_{14} + I_{13} - B_{13} > 1500000$;

$P_{21} > -4540000$;

$P_{22} + I_{21} - B_{21} > 2650000$;

$P_{23} + I_{22} - B_{22} > 2550000$;

$P_{24} + I_{23} - B_{23} > 1880000$;

$0.018p_{11}+0.013p_{21}- 744w_1 < 0$; $0.018p_{12}+0.013p_{22}- 744w_2 < 0$;

$0.018p_{13}+0.013p_{23}- 744w_3 < 0$;

$0.018p_{14}+0.013p_{24}- 744w_4 < 0$;

$0.0002p_{11}+0.00015p_{21}<744$;

$0.0002p_{12}+0.00015p_{22}<744$;

$0.0002p_{13}+0.00015p_{23}<744$;

$0.0002p_{14}+0.00015p_{24}<744$;

$P_{11}- I_{11}+B_{11} < -320000$;

$P_{11} - I_{11} + B_{11} > -360000$;

$I_{11} - B_{11} - I_{12} + B_{12} + p_{12} < 2130000$;

$I_{11} - B_{11} - I_{12} + B_{12} + P_{12} > 2070000$;

$I_{12} - B_{12} - I_{13} + B_{13} + P_{13} < 1760000$;

$I_{12} - B_{12} - I_{13} + B_{13} + p_{13} > 1700000$;

$I_{13} - B_{13} - I_{14} + B_{14} + p_{14} < 1660000$;

$I_{13} - B_{13} - I_{14} + B_{14} + p_{14} > 1580000$;

$P_{21} - I_{21} + B_{21} < -4480000$;

$P_{21} - I_{21} + B_{21} > -4500000$;

$I_{21} - B_{21} - I_{22} + B_{22} + P_{22} < 2740000$;

$I_{21} - B_{21} - I_{22} + B_{22} + P_{22} > 2700000$;

$I_{22} - B_{22} - I_{23} + B_{23} + P_{23} < 2660000$;

$I_{22} - B_{22} - I_{23} + B_{23} + P_{23} > 2620000$;

$I_{23} - B_{23} - I_{24} + B_{24} + P_{24} < 1940000$;

$I_{23} - B_{23} - I_{24} + B_{24} + P_{24} > 1920000$;

End.

3) Production system cannot operate with less than a certain number of workers (minimum workforce level);

4) Overtime and subcontracting are not allowed.

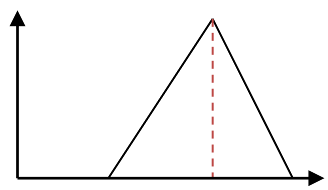

Figure 1. Triangular fuzzy number R

After reviewing the literature and considering practical solutions, the proposed fuzzy APP model selected total cost, changes in work force level, and customer service as objective functions (for total costs, see Gen et al., 1992; Wang and Liang, 2004; Alive et al., 2007; Baykasouglu et al., 2010; Tang et al., 2003; for changes in workforce level, see Gen et al., 1992; Baykasouglu et al., 2010; Masud, 1981; and for customer service, see Chen and Liao, 2003). The three objective multi-period multi-product APP model with fuzzy parameters can be formulated as follows:

Indices:

n product type

t planning period

Decision variables:

p_{nt} units of production for product n in period t (units)

w_t work force level in period t (man-day)

In_t inventory level for product n in period t (units)

B_{nt} backorder level for product n in period t (units)

wht worker hired in period t (man-day)

wlt worker lay-off in period t (man-day)

Parameters and constants:

N total number of products

T total number of planning periods in the planning horizon

cp_{nt} production cost for product n in period t (\$/unit)

c_t labor cost in period t (\$/man-period)

ci_{nt} inventory carrying cost for product n in period t (\$/unit-period)

ch_t cost to hire one worker in period t (\$/man-day)

cl_t cost to lay off one worker in period t (\$/man-day)

cs_{nt} cost of stock out product n in period t (\$/unit-period)

Hn hours of labor needed for each unit for product n

U_{nt} hours of machine usage per unit of nth product in period t (machine-hour/unite)

R_t maximum working hours in period t (a month), equal to regular working hours per worker per day* working shift in per day* working days for period t

In_0 initial inventory level for product n (units)

B_{n0} initial backorder level for product n (units)

w_0 initial work force level (man-day)

D_{nt} forecasted demand for product n in period t (units)

$D_{min,n}$ minimum demand for product n in period t (units)

w_{max} maximum workforce level available in period t (man-day)

w_{min} minimum workforce level available in period t (man-day)

$M_{t\ max}$ maximum machine capacity available in period t (machine-hour)

cp_{nt}, c_t, D_{nt}, $D_{min,n}$, w_{max}, U_{nt} and $M_{t\ max}$ are triangular fuzzy numbers.

Objective functions:

(1)Minimize total cost:

$$\min Z_1 = \sum_{n=1}^{N} \sum_{t=1}^{T} \tilde{\ } p_{nt} + c_t\, w_t + I_{nt}\ ci_{nt} \quad (2)$$

(2) Minimize the changes in work force level:

$$\min \quad Z_2 = \sum_{t=1}^{T} (\, wh_t + wl_t\,) \quad (3)$$

(3) Maximize customer service:

$$\max\ Z_3 = (\,1 - \sum_{n=1}^{N} \sum_{t=1}^{T} B_{nt} \ /\ \sum_{n=1}^{N} \sum_{t=1}^{T} D_{nt}\,) \quad (4)$$

The constraints:

$$w_{min} \le w_t \le w_{max} \quad (5)$$

$$w_t = w_{t-1} + wh_t - wl_t \quad (6)$$

$$\sum_{n=1}^{N} H_n\, p_{nt} \le R_t\ w_t \quad (7)$$

$$I_{nt} - B_{nt} = I_{n,t-1} - B_{n,t-1} + p_{nt} - D_{nt} \quad (8)$$

$$p_{nt} + I_{n,t-1} - B_{n,t-1} \ge D_{min,n} \quad (9)$$

$$\sum_{n=1}^{N} U_{nt}\, p_{nt} \le M_{t\ max} \quad (10)$$

$$p_{nt}, w_t, I_{nt}, B_{nt}, wh_t, wl_t \ge 0;\ n=1,\dots,N;\ t=1,\dots,T \quad (11)$$

3. Solving procedure

For solving our fuzzy APP model, the following steps are required:

Step 1: Formulate a fuzzy model

Step 2: Transform fuzzy model into crisp model by using the transformation method proposed by Okada et al. (1991), (see Appendix 1).

Step 3: Solve the crisp multi-objective linear model after setting α (α-cut) parametrically through multi-objective genetic algorithm.

Step 4: Determine the weight of each objective function by using fuzzy AHP method proposed by Chang (1996).

Step 5: Finding the best compromise solution by using TOPSIS method proposed by Hwang and Masud (1980).

3-1. Fuzzy AHP

In this study, we used fuzzy AHP method proposed by Chang (1996) for determining the weight of each objective or minimizing total cost, minimizing the change in workforce level and maximizing customer service. For these purposes, managers of the company evaluated three objectives. As a result of their evaluation, the metrics of pairwise comparison are given in Table 1. Then, we calculated fuzzy synthetic extent by Equation 12.

$$S_i = \sum m_{gi}^{j} .\ [\, \sum \sum\ m_{gi}^{j}\ ]^{-1} \quad (12)$$

where all m_{gi}^{j} (j=1,…,m) are triangular fuzzy numbers. m_{gi}^{1}, m_{gi}^{2},… m_{gi}^{m} are values of extent analysis of ith objective for m goals.

3-2. Genetic algorithm

The genetic algorithm is a powerful method for combinatorial optimization problems. Our implementation of genetic algorithm is presented as follows.

3-2-1. Selection

Selection provides the opportunity to deliver the gene of a good solution to the next generation. In this study employs the pareto tournament method proposed by Horn, Nafpliotis and Goldberg in 1994. In this method, two candidates for selection are picked at random from the population. A comparison set of individuals is also picked randomly from population. If one candidate is dominated by comparison set but the other is not dominated, the non-dominated candidate is selected for reproduction. If both candidates are either non-dominated or dominated, a sharing method according to the niche count is used to choose the winner. Candidate with the smaller niche count is selected as the winner. In this research, tournament size is equal 2.

Table 1. Pairwise comparison of three objectives

	Z_1	Z_2	Z_3
Z_1	(1, 1, 1)	(3/2, 2, 5/2)	(5/2, 3, 7/2)
Z_2	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/5, 1, 3/2)
Z_3	(2/7, 1/3, 2/5)	(2/3, 1, 5/2)	(1, 1, 1)

Where Z_1= minimize total cost, Z_2=minimize changes in workforce level and Z_3= maximization customer service.
Degree of possibility of $M_1 \geq M_2$ is calculated by Equation 13.

$V(M_1 \geq M_2) = 1$ if $m_1 \geq m_2$,

$V(M_2 \geq M_1) = l_1\text{-}u_2/(m_2\text{-}u_2) - (m_1\text{-}l_1)$, otherwise (13)

When $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are triangular fuzzy numbers.

The results of computation are: s_1= (0.35, 0.55, 0.8), s_2= (0.12, 0.22, 0.36), and s_3= (0.14, 0.21, 0.44); and V $(s_1 \geq s_2)$=1, v $(s_1 \geq s_3)$=1, v $(s_3 \geq s_2)$=1, v $(s_2 \geq s_1)$=0.14, v $(s_3 \geq s_1)$= 0.2, v $(s_2 \geq s_3)$=0.94. Finally the weights of each objective after normalization are W= (0.75, 0.1, 0.15).

3-2-2. Reproduction

The best chromosomes that have a lower fitness function are selected for reproduction.

3-2-3. Crossover

Scattered crossover is used in this study. Scattered crossover creates binary vectors randomly. It then selects the genes where the binary vector is one from the first parent, and genes where the vector is a zero from the second parent and then combines them to form the child. For example, parent1= [2 3 4 5] and parent2= [1 2 3 4] and the random crossover vector is [11 0 0], and the child will be [2 3 3 4].

3-2-4. Mutation

In our implemented GA, we used adaptive feasible mutation. Adaptive feasible mutation randomly generates directions that are adaptive with respect to the least successful or the unsuccessful generation.

3-2-5 Fitness function

The objective functions are chosen as the fitness function that it defined in Section 2.

3-2-6. Termination condition

The search process stops if the number of generations exceeds the maximum number of generations, or if some specified number of generations is reached without improving upon the of best-known solution.

Solutions or individuals are real decision variables such as production, inventory and backorders.

3-3. TOPSIS

Among the many famous multiple-criteria decision-making methods for ranking and selecting numerous possible alternatives through the measuring Euclidean distances, TOPSIS (technique for order performance by similarity to ideal solution) is a practical and useful technique first introduced by Hwang and Yoon (1981).(Goyal et al., 2012). TOPSIS is based on the concept that a chosen alternative should not only be nearest to the positive ideal solution (PIS) but also should be furthest from the negative ideal solution (NIS) (Goyal et al., 2012, Sajedi et.al, 2013).To determine a compromise solution, we will use TOPSIS method as follows:

$$U_j^+ = \left(\sum_{K=1}^{q} W_k (Z_{kj} - Z_k^B)^2 / Z_k^{*2} \right)^{1/2}, \quad j = 1,\ldots, v \qquad (14)$$

$$U_j^- = \left(\sum_{K=1}^{q} W_k (Z_{kj} - Z_k^W)^2 / Z_k^{*2} \right)^{1/2}, \quad j = 1,\ldots, v \qquad (15)$$

$$Z_k^* = \left(\sum_{j=1}^{v} Z_{kj}^2 \right)^{1/2} \quad k= 1, 2,\ldots, v \qquad (16)$$

Z_k^B : the best value of k-th objective function

Z_k^W : the worse value of k-th objective function

W_k weight of k-th objective function which is obtained in Section 4-1.

We obtain the nearest nondominated solution to best value using the following equation.

$e_j^* = U_j^- / U_j^+ + U_j^-$, $0 < e_j^* < 1$,

$j = 1, 2,\ldots, v$ (17)

The greater value of e_j^* will be selected as a compromise solution.

3-4. Multi-objective technique

There are several ways to solve multi-objective linear programming models in the literature; among them, the fuzzy programming approaches are more common. Zimmermann (1978) proposed the first fuzzy approach for solving multi-objective linear programming problems, called max–min approach. Max-min is single-phase method which tends to maximize overall satisfaction degree of objective (surrogate objective of λ). In this method, the following formulation is used for solving multi-objective linear problems:

Max λ

Subject to: $\lambda \leq (z_k(x) - z_k^{NIS}) / (z_k^{PIS} - z_k^{NIS})$, k=1, …, L

$\lambda \leq (w_s^{NIS} - w_s(x)) / (w_s^{NIS} - w_s^{PIS})$, s=1, …, r,

$\lambda \in [0,1]$, $x \in X$, where k is maximum objectives and s is minimum objectives.

The linear membership function of these objective functions can now be computed as follows:

$$\mu_{z1}=\begin{cases} 1 & \text{if } z_1< z_1^{PIS} \\ (z_1^{NIS}-z_1)/(z_1^{NIS}-z_1^{PIS}) & \text{if } z_1^{PIS}\le z_1\le z_1^{NIS} \\ 0 & \text{if } z_1> z_1^{NIS} \end{cases}$$

$$\mu_{z2}=\begin{cases} 1 & \text{if } z_2< z_2^{PIS} \\ (z_2^{NIS}-z_2)/(z_2^{NIS}-z_2^{PIS}) & \text{if } z_2^{PIS}\le z_2\le z_2^{NIS} \\ 0 & \text{if } z_2> z_2^{NIS} \end{cases}$$

$$\mu_{z3}=\begin{cases} 1 & \text{if } z_3> z_3^{PIS} \\ (z_3-z_3^{NIS})/(z_3^{PIS}-z_3^{NIS}) & \text{if } z_3^{PIS}\le z_3\le z_3^{NIS} \\ 0 & \text{if } z_3> z_3^{NIS} \end{cases}$$

In this research, besides solving our model by multi objective genetic algorithm, we will use max-min formulation to convert multi-objective problem into single-objective problem and then solve this single-objective problem through the simplex method (exact method). To compare the objective values obtained by LINGO with the results of the multi-objective genetic algorithm, a quality measure, the percent deviation of solution, is defined according to the following equation (Ramezanian et al. 2012):

% deviation = (objective function value of GA - objective function value LINGO/ objective function value LINGO)*100 (19)

4. Implementation of model (Case study)

4.1. Case description

Shahab Shishe Company is used as a case study to demonstrate the practicality of the proposed methodology. Shahab Shishe Company produces two types of products, tubes and bulbs, which are used for producing lamps. The APP decision problem for Shahab Shishe Company is described as follows.

There is a four-period planning horizon of four months based on the Iranian calendar. The model includes two types of product or tubes and bulbs. The forecast demand, minimum demand, production cost, payroll cost, maximum machine capacity and maximum workforce level are fuzzy numbers with triangular possibility distributions from period to period. Table 2-3 summarizes the forecast demand, minimum demand and cost data used by Shahab Shishe Company. Other relevant data are as follows. The hours of labor needed for each unit production are 0.018 for tubes and 0.013 for bulbs. Hours of machine usage for each of four planning periods are (0.0002, 0.00023, 0.00026) and (0.00015, 0.00018, 0.0002) for tubes and bulbs respectively. Maximum machine capacities are (700, 720, 744) for tubes and bulbs in each of the four planning periods. We have 31 working days per period (month), three working shifts of eight hours each, and 24 working hours per day. The initial workforce is 68 workers and the maximum workers allowed are (60, 70, 80) for periods 1 to 4.

Table 2. Forecast demand and minimum demand data

Item	**Period**			
	1	2	3	4
$\mathbf{D_{1t}}$	(2040000, 2060000, 2080000)	(2070000, 2100000, 2130000)	(1700000, 1730000, 1760000)	(1580000, 1620000, 1660000)
$\mathbf{D_{2t}}$	(2500000, 2510000, 2520000)	(2700000, 2720000, 2740000)	(2620000, 2640000, 2660000)	(1920000, 1930000, 1940000)
$\mathbf{D_{min1}}$	(1900000, 1960000, 2060000)	(1970000, 1990000, 2010000)	(1600000, 1650000, 1700000)	(1500000, 1550000, 1580000)
$\mathbf{D_{min2}}$	(2460000, 2480000, 2500000)	(2650000, 2670000, 2700000)	(2550000, 2580000, 2620000)	(1880000, 1900000, 1920000)

Table 3. Cost data

Product	cp_{nt} (\$/unit)	cs_{nt} (\$/unit)	ci_{nt} (\$/unit)	c_t (\$/worker)	ch_t (\$/worker)	cl_t (\$/worker)
Tubes	(0.016, 0.02, 0.024)	0.015	0.003	(440, 465, 480)	30	5
Bulbs	(0.007, 0.009, 0.011)	0.004	0.0008			

Also, the minimum workforce level is 58 workers. Furthermore, the company has 2400000 beginning inventory for tubes and 7000000 for bulbs.

4.2. Results of problem

We transformed fuzzy model into crisp model by using the transformation method proposed by Okada et al. (1991) by 11 different α from zero to one. The genetic algorithm was coded in MATLAB R2009 (a), and all tests were conducted on a laptop computer with a core i5-2410m processor 2.3 GHz with 4 GB of RAM. The results of solving crisp models by using multi objective genetic algorithm and run times (second) are given in Table 4. For this example the population size is equal to 180 and the number of generations is equal to 360. The computational times recorded in table 4 can be considered reasonable times for a problem of this size.

Table 4. Values of objective functions at different levels of α

α	Z1	Z2	Z3	Run time(s)
0	228172	4	0.981	2
0.1	231673	4	0.981	3
0.2	235232	4	0.982	2
0.3	238710	5	0.982	2
0.4	240878	5	0.982	3
0.5	245792	5	0.982	3
0.6	249984	5	0.984	2
0.7	253076	5	0.984	4
0.8	256213	5	0.983	3
0.9	258046	4	0.986	3
1	276802	4	0.962	3

By using TOPSIS, we can obtain compromise solutions ($\alpha = 0$) among dominated solutions. The solutions in $\alpha = 0$ are as follows:

P_{1t} = [416666, 1193334, 1700000, 1600000] or production units for tubes

P_{2t} = [0, 0, 810000, 1950000] or production units for bulbs

W_t = [68, 68, 69, 72] or workforce level in the four periods

I_{1t} = [776666, 0, 0, 0] or inventory level for tubes in four months

I_{2t} = [4480000, 1740000, 0, 0] or inventory level for bulbs in four months

B_{1t} = [0, 100000, 99999, 81104] or backorder level for tubes per period

B_{2t} = [0, 0, 70000, 40000] or backorder level for bulbs per period

Wh_t =[0, 0, 1, 3] or workers hired per period

Wl_t =[0, 0, 0, 0] or workers laid off per period.

$Z_1 = 228172$, $Z_2 = 4$, $Z_3 = 0.981$

Now, we compute exact solutions by max-min method and compare the results. Table 5 shows the total degree of satisfaction of the decision maker (λ) for this example, achieved through max-min method.

Table 5. Total degree of satisfaction at different levels of α

α	λ
0	0.79
0.1	0.73
0.2	0.6
0.3	0.54
0.4	0.58
0.5	0.61
0.6	0.54
0.7	0.59
0.8	0.6
0.9	0.56
1	0.52

As demonstrated above, maximum degree of satisfaction is 0.79(for α=0), which confirms our proposed solution strategy results. Optimum solutions for this problem in α=0 are as follows.

P_{1t} = [0, 1710000, 1700000, 1500000] or production units for tubes.

P_{2t} = [0, 0, 978234, 1721766] or production units for bulbs.

W_t = [67, 67, 67, 67] or workforce level in the four periods.

I_{1t} = [360000, 0, 0, 0] or inventory level for tubes in four months.

I_{2t} = [4500000, 1800000, 158234, 0] or inventory level for bulbs in four months.

B_{1t} = [0, 0, 0, 80000] or backorder level for tubes per period.

B_{2t} = [0, 0, 0, 40000] or backorder level for bulbs per period.

Wh_t =[0, 0, 0, 0] or workers hired per period.

Wl_t =[1, 0, 0, 0] or workers laid off per period.

$Z_1 = 221626$, $Z_2 = 1$, $Z_3 = 0.993$

Satisfaction degree of each objective function is: μ_{z1}=0.92, μ_{z2}=0.95 and μ_{z3}=0.79.

%Deviation between optimum result (max-min) and GA result z_1=2.9%. It can be concluded for a problem of this size, that max-min approach is most suitable.

5. Conclusion

In this paper, we formulated aggregate production planning with fuzzy parameters. To transformation a fuzzy APP model into a crisp model, we used the transformation method suggested by Okada et al. (1991). We solved crisp APP models (11 crisp models with 11 different α values ranging from zero to one) by using genetic algorithm. Then, we combined fuzzy AHP and TOPSIS to achieve the best compromise solution. This model is more suitable for 24-hour production systems. We used real-world problems to illustrate the practicality of our model.

For further studies, we can extend this model in a supply chain and develop a fuzzy multi-objective production/distribution planning model with some modification of the proposed model.

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Appendix 1: Transformation method (Okada et.al 1991, Gen et.al 1992)

Suppose we have fuzzy multi-objective model as follows: in all summation symbols (sigma) lower index is j=1 and upper index is n.

Max $z_k = \Sigma\ c_{kj}\ x_j$, k=1,2,.., q_1

Min $z_k = \sum_{J=1}^{n} c_{kj}\ x_j$, k=$q_1$+1,.., q=$q_1$+$q_2$

Subject to:

$\Sigma\ a_{ij}\ x_j \leq b_i$ i= 1,2,…,m_1

$\Sigma\ a_{ij}\ x_j \geq b_i$ i= m_1+1,…, m_2

$\Sigma\ a_{ij}\ x_j = b_i$ i= m_2+1,…,m=m_1+m_2+m_3

$x_j \geq 0$ j= 1,…,n

where

$c_{kj}=(c_{kj1},c_{kj2},c_{kj3})$ is a fuzzy coefficient of the k-th objective function and j-th decision variable

$a_{ij}=(a_{ij1},a_{ij2},a_{ij3})$ is a fuzzy technical coefficient of the i-th constraint and j-th decision variable

$b_i=(b_{i1},b_{i2},b_{i3})$ is a fuzzy available resource of i-th constraint

x_j is decision variables. All fuzzy parameters in this multi-objective model are triangular fuzzy numbers.

Crisp multi-objective linear model can be computed by following formulation:

Max $z_k= \Sigma\ [(1-\alpha)c_{kj3}+\alpha\ c_{kj2}]\ x_j$, k=1,2,.., q_1

Min $z_k= \Sigma\ [(1-\alpha)c_{kj1}+\alpha\ c_{kj2}]x_j$, k=$q_1$+1,.., q=$q_1$+$q_2$

Constraint:

$\Sigma\ [(1-\alpha)a_{ij1}+\alpha\ a_{ij2}]\ x_j \leq (1-\alpha)b_{i3}+\alpha\ b_{i2}$, i= 1,2,…,$m_1$

$\Sigma\ [(1-\alpha)a_{ij3}+\alpha\ a_{ij2}]x_j \geq (1-\alpha)b_{i1}+\alpha\ b_{i2}$, i= m_1+1,…, m_2

$\Sigma\ [(1-\alpha)a_{ij1}+\alpha\ a_{ij2}]x_j \leq (1-\alpha)b_{i3}+\alpha b_{i2}$, i= m_2+1,…,m

$\Sigma\ [(1-\alpha)a_{ij3}+\alpha\ a_{ij2}]\ x_j \geq (1-\alpha)b_{i1}+\alpha b_{i2}$, i= m_2+1,…,m

$x_j \geq 0$ j= 1,…,n

where α is a cutoff value between zero to one, α=[0,0.1,0.2,0.3,…,1].

Appendix 2: A sample of input data (α= 0) for Shahab Shishe company example (max-min method)

Max λ

λ-((1/189023)*(395587-(0.016p_{11} + 0.016p_{12} + 0.016p_{13} + 0.016p_{14} + 0.007p_{21} +0.007p_{22} +0.007p_{23} + 0.007p_{24} + 440w_1 +440w_2+ 440w_3+440w_4+ 0.003I_{11}+ 0.003I_{12}+ 0.003I_{13} + 0.003I_{14}+0.0008I_{21}+ 0.0008I_{22}+ 0.0008I_{23}+ 0.0008I_{24})))<=0;

λ-((1/22)*(22- (Wh_1+ Wh_2+ Wh_3+ Wh_4+ WL_1+ WL_2+ WL_3+ WL_4)))<=0;

λ-((1/.028)*(1-[(B_{11}+ B_{12}+ B_{13}+ B_{14}+ B_{21}+ B_{22}+ B_{23}+ B_{24})/17490000])-.971)<=0;

λ>=0;

λ<=1;

W_1<80; W_2<80; W_3<80; W_4<80;

W_1>58; W_2>58; W_3>58; W_4>58;

$W_1 - Wh_1 + WL_1 = 68$;

$W_2 - W_1 - Wh_2 + WL_2 = 0$;

$W_3 - W_2 - Wh_3 + WL_3 = 0$;

$W_4 - W_3 - Wh_4 + WL_4 = 0$;

P_{11}> -500000;

$P_{12} + I_{11} - B_{11} > 1970000$;

$P_{13}+ I_{12}-B_{12} > 1600000$;

$P_{14}+ I_{13}-B_{13} > 1500000$;

$P_{21} > -4540000$;

$P_{22}+I_{21}-B_{21}> 2650000$;

$P_{23}+I_{22}-B_{22}>2550000$;

$P_{24}+I_{23}-B_{23}> 1880000$;

0.018p_{11}+0.013p_{21}- 744w_1 < 0; 0.018p_{12}+0.013p_{22}- 744w_2 < 0;

0.018p_{13}+0.013p_{23}- 744w_3 < 0;

0.018p_{14}+0.013p_{24}- 744w_4 < 0;

0.0002p_{11}+0.00015p_{21}<744;

0.0002p_{12}+0.00015p_{22}<744;

0.0002p_{13}+0.00015p_{23}<744;

0.0002p_{14}+0.00015p_{24}<744;

$P_{11}- I_{11}+B_{11} < -320000$;

$P_{11} - I_{11} + B_{11} > -360000$;

$I_{11} - B_{11} - I_{12} + B_{12} + p_{12} < 2130000$;

$I_{11} - B_{11} - I_{12} + B_{12} + P_{12} > 2070000$;

$I_{12} - B_{12} - I_{13} + B_{13} + P_{13} < 1760000$;

$I_{12} - B_{12} - I_{13} + B_{13} + p_{13} > 1700000$;

$I_{13} - B_{13} - I_{14} + B_{14} + p_{14} < 1660000$;

$I_{13} - B_{13} - I_{14} + B_{14} + p_{14} > 1580000$;

$P_{21} - I_{21} + B_{21} < -4480000$;

$P_{21} - I_{21} + B_{21} > -4500000$;

$I_{21} - B_{21} - I_{22} + B_{22} + P_{22} < 2740000$;

$I_{21} - B_{21} - I_{22} + B_{22} + P_{22} > 2700000$;

$I_{22} - B_{22} - I_{23} + B_{23} + P_{23} < 2660000$;

$I_{22} - B_{22} - I_{23} + B_{23} + P_{23} > 2620000$;

$I_{23} - B_{23} - I_{24} + B_{24} + P_{24} < 1940000$;

$I_{23} - B_{23} - I_{24} + B_{24} + P_{24} > 1920000$;

End.

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