Fitting GARCH Models to Crude Oil Spot Price Data

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Abstract: Crude oil price forecasting has generated interest across the globe for over forty decades. This interest in crude oil spot price is mainly due to the volatility of the product which results in fat tails in the distribution of the series. The price of this energy commodity has always been highly volatile. Since crude oil price variability affects other sectors and stock market, the prediction of future crude oil prices has become crucial. The aim of this paper is to apply the GARCH model in crude oil time series modeling in order to illustrate the advantages of these nonlinear models and we fit three GARCH models namely; GARCH–N, GARCH–t and GARCH–G to forecasting crude oil spot prices. The study adopted two crude oil prices from West Texas Intermediate and Brent to evaluate the performance of the models developed. The results revealed that GARCH–N model is the best for forecasting for Brent and that GARCH–G model is the best for the forecasting of WTI crude oil spot prices judging by their Mean Squared Error (MSE) and the Mean Absolute Error (MAE).

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1. Introduction

Crude oil is a naturally occurring and flammable liquid found in rock formations in the earth. It consists of a complex mixture of hydrocarbons of various molecular weights in addition to other organic compounds. The main characteristics of crude oil are generally classified according to its sulphur and density content which the petroleum industry measured by its American Petroleum Institute (API) gravity.

However, the geographical location of crude oil production is altogether another matter. In the crude oil market, the two current references or pricing markers are West Texas Intermediate (WTI) and Europe Brent. The former is the base grade traded, as 'light sweet crude', on the New York Mercantile Exchange (NYMEX) for delivery at Cushing, Oklahoma. While the latter trades on London's International Petroleum Exchange (IPE) for delivery at SullomVoe and is also one of the grades acceptable for delivery of the NYMEX contract (Lin and Tamvakis, 2001).

Crude oil has taken on an increasingly important role in the world's economy as nearly two-third of the world's energy demands is met from crude oil (Alvarez-Ramirez et al., 2003). It is stated that crude oil is also the world's largest and actively traded commodity, accounting for over 10% of the total world's trade (Verleger, 1993).As special commodities, crude oil is a commodity traded across boundaries amongst interested parties in the oil business in many climes. As is common with most traded commodity, market forces of demand and supply equally impact strongly in determining the price of crude oil (Hagen, 1994; Stevens, 1995), however these are not the only factors that affect crude oil prices as there are others such as: weather, economic, political and psychological factors also coupled with the time of shipment from one country to the other. All this accounts for the instability in crude oil market which results in non-linearity, variations and high irregularity of the series (Watkins and Plourde, 1994)

In the year 1982, Engle (1982) proposed the Autoregressive Conditional Heteroskedasticity (ARCH) process. However, studies have revealed that the need for selection of high ARCH order to deal with the dynamics of the conditional variance. The high ARCH order implies that several parameters would have to be estimated and the computations are tedious.

Some years after Engel proposed the ARCH process, Bollerslev (1986), proposed the Generalized ARCH (GARCH) model as a way out of the high ARCH orders in volatility series. This model is uses an infinite ARCH specification and allows for the reduction of the number of parameters to be estimated amongst the class of the infinite number to just a few.

In Bollerslev's GARCH model, the conditional variance is a linear function that comprises of past squared innovations and previously computed conditional variances.

Marzo and Zagalia (2007) studied the forecasting properties of linear GARCH models for closing-day futures prices on crude oil sold on the

NYMEX. They compared volatility models based on the normal, Student's and Generalized Exponential distribution (GED). Their main focus was on out-of-sample predictability. From the tests for predictive ability, the results showed that the GARCH-GED model fares best for short horizons from one to three days ahead. Carbide and Moya's (2003) concluded that a normal distribution assumption is mainly driven by standard GARCH frameworks which are imposed on the risk future usually in future time.

Fan et al. (2008b) carried out an estimation using the GARCH type model based on Generalized Error Distribution (GED-GARCH) for VAR of returns on crude oil spot market. They stressed that the historical simulation with ARMA forecast method did not have an advantage over others to forecast the return in out-of-sample data. The results revealed that there is a significant two-way risk spillover effect between crude oil markets.

Hung et al. (2008) adopted the GARCH model with the heavy-tailed (HT) distribution to estimate the one-day-ahead VAR for WTI and Brent spots and further compares the accuracy and efficiency with the GARCH–N and GARCH-models. First for each series considered, the out-of-sample VAR forecast of GARCH-HT models outperformed alternative models in terms of failure rate in back tests at all confidence levels.

In Liu and Hung (2010), and Bali (2007) we see the presentation of skewed generalized error distribution (SGED) GARCH models while the work of Gokcan (2000) is a comparison between GARCH(1,1) and EGARCH models for the most efficient in stock market forecasting.

Chuang et al. (2007) used different assumptions of distribution to evaluated the forecast accuracy of GARCH (1,1) model by using data obtained from the stock market. The results obtained showed that a combined model involving GARCH (1,1) model combined with either logistic distribution, the scaled student's distribution or the Risk metrics models is better at forecasting stock and/or foreign exchange markets while Curto and Pinto (2009) used ARMA-GARCH (1,1) models combined with Normal, Student's t and stable Paretian distributional assumptions and concluded form the results that ARMA-GARCH (1,1) model combined with stable Paretian fits returns better than normal distribution and was equally found to be record slight improvement over Student's t distribution.

However, beside these studies into the volatility forecasting performance of GARCH models in the economic sector, we have not come across literature that fit GARCH models to highly volatility series like crude oil data, so in this paper attempt is made at achieving this using the Brent and west Texas intermediate spot oil prices.

The price of the energy commodity is highly volatile throughout time. Since crude oil price variability affects other sectors and stock market, the prediction of future crude oil prices becomes crucial. In this paper we fit three GARCH models namely; GARCH–N, GARCH–t and GARCH–G and forecasted crude oil spot prices.

The remaining part of this paper is structured as follows: Section 2 discusses the methodology and the distributions of error in GARCH models. Section 3 explicitly describes the data used for the study of three types of GARCH models to be used. The measurements used to evaluate forecast performance are discussed in Section 4. Section 5 discusses the report of the experimental results while the conclusion is given in section6.

2. GARCH

Engle (1982) introduced the Autoregressive Conditional Heteroscedastic (ARCH) model which was later generalized by Bollerslev (1986), named Generalized Autoregressive Conditional Heteroscedastic model (GARCH). The term "conditional" implies the level of association on the past sequence of observations and the "autoregressive" describes the feedback mechanism that incorporates past observations into the present (Laux et al., 2011).

A GARCH (p, q) model for a given time series X_t is defined by:

$$X_t = \mu + \varepsilon_t,$$

$$\varepsilon_t = \eta_t \sqrt{h_t},$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

While the GRACH (1, 1) the variance equation can be expressed as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Where, μ is a constant; η_t are independently and identically distributed (i.i.d) variable with $E(\eta_t) = 0$ and $Var(\eta_t) = 1$; η_t is independent of σ_t^2; $\omega > 0, \alpha_i \geq 0$ and $\beta_j \geq 0$ are non – negative constant with $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ to ensure the positive of conditional variance and stationary as well. when $q = 0$ the model becomes an Autoregressive Conditional heteroscedasticity (ARCH) model.

GARCH (p, q) models with normal error distribution hardly captures leptokurtic (high courtesies' and heavy-tailed) of the original time series but several non-normal error distributions have been proposed in the literature. Hansen (1994) employed the skewed t distribution to detect measure of skewness and excess kurtosis in the time series. Lee and Pai (2010) estimated volatility prediction of the GARCH models by using the student-t and SGED distributions.

1.1 The distribution of error ε_t

This study considered three types of error distributions and these are the normal distribution, the student-t and the generalized error distribution (GED). These three distributions are defined as follows:

1. **Normal Distribution** (Woldzimeirz, 2005)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

2. **Student – t Distribution** (Hogg and Craig, 1978)

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}},$$

Where, ν denotes the number of degrees of freedom and Γ denotes the Gamma function.

3. **Generalized Error Distribution (GED)** (Nadarajah, 2005)

$$F(x) = \frac{e^{1/2\left|\frac{x-\mu}{\sigma}\right|^{1/k}}}{2^{k+1}\sigma\Gamma(k+1)}$$

2. Data

In this study, two main crude oil price series, West Texas Intermediate (WTI) crude oil spot price and Brent crude oil spot price were chosen as experimental samples. The main reason of selecting these two oil price indicators is that, these two crude oil prices are the most famous benchmark prices, which are widely used as the basis of many crude oil price formulae. The two crude oil price data used in this study are daily data, and are freely obtainable from the energy information administration (EIA) For WTI crude oil spot price; we needed the daily data from January 1, 1986 to September 30, 2006, excluding public holidays, with a total of 5237 observations.

For Brent crude oil spot price, the sampling data covers the period from May 20, 1987 to September 30, 2006 with a total of 4933 observations. The main reason of different starting points is that, the EIA website only provides the Brent data since May 20, 1987. Similarly, we took the data from May 20, 1987 to December 31, 2002 as in-sample (training periods) training set (3965 observations), and took the data from January 1, 2003 to September 30, 2006 as out-of-sample (testing period) testing set (968 observations), which was used to evaluate the performance of prediction, based on evaluation criteria. The oil price series for WTI and Brent are shown in figure 1andfigure2 respectively.

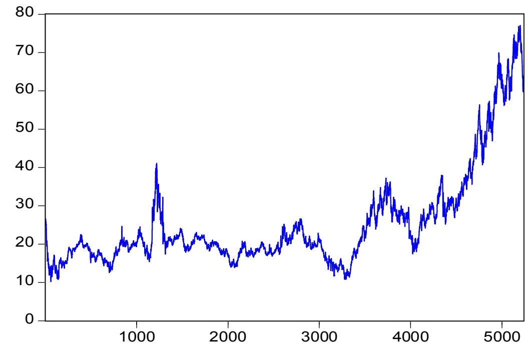

Figure 1 Time plot for WTI daily

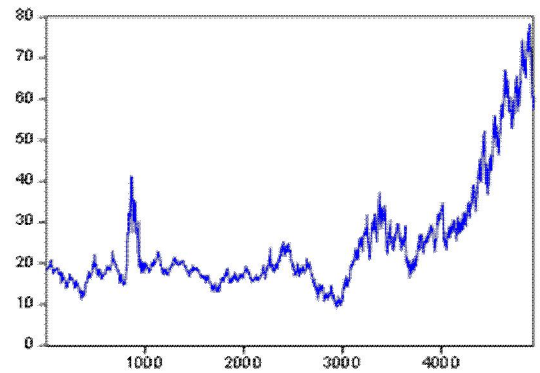

Figure 2 Time plot for Brent daily

3. Evaluation of volatility forecasts

Several forecast accuracy measures can be found in the literatures such as (Lui et al, (2009) and Awartani and Corradi's (2005)).This study adopted two very popular measures for evaluating the forecast accuracy of the volatility of the series and these are: mean absolute error (MAE) and mean square error (MSE). These measures are defined as follows:

Mean Absolute Error (MAE) is given by:

$$MAE = \frac{1}{N}\sum_{t=1}^{N}\left|(X_t - \bar{X})^2 - \hat{p}_t\right|$$

and Mean Squared Error (MSE) is given by:

$$MSE = \frac{1}{N}\sum_{t=1}^{N}\left[X_t - \bar{X})^2 - \hat{p}_t\right]^2$$

Where

X_t: is the return of the horizon before the current time t,

$\bar{X}$: the average return,

$\hat{p}_t$: is the forecast value of the conditional variance over n steps ahead horizon at the current time t.

4. Experiment Results

For the observed crude oil spot prices X_t , the daily log-returns are calculated as $\overline{X}_t = \log(X_t / X_{t-1})$. Table 1 provides a summary of descriptive statistics for the considered return series.

Table 1: Summery Statistics of WTI and Brent Crude oil

	WTI	Brent
Mean	-0.00017	0.000234
Median	-0.00058	0.000000
Maximum	0.406396	0.173333
Minimum	-0.19151	-0.361214
Std. Dev.	0.025661	0.023395
Skewness	1.021473	-0.865649
Kurtosis	20.61832	20.09403
Jarque-Bera	68630.60	60664.25
Probability	0.000000	0.000000
Sum	-0.90052	1.154288
Sum Sq. Dev.	3.447306	2.698854
Observations	N= 5236	N = 4932

We observed for WTI that the mean and median of daily log returns were negative indicating that overall crude oil spot prices decreased during the time period considered. The magnitude of the median return (-0.00017%) was very low in comparison to its standard deviation (0.026%). Further, the large kurtosis of 20.618 indicated the leptokurtic characteristics of daily returns.

Brent observed that the mean and median of daily log returns were positive indicating that overall crude oil spot prices increased during the time period studied. The magnitude of the average return (0.0234%) was very low in comparison to its standard deviation (2.3395%). Further, the large kurtosis of 20.094 indicated the leptokurtic characteristics of daily log returns.

Apparently, the series have a distribution with tails that are significantly faster than those of a normal distribution. This is an indication of non-normality, this result is supported by the Jarque and Bera (1980) test statistic, which rejected the null hypothesis of a normal distribution at 0.01and 0.05 levels of significance. Figure 3 and Figure 4provide a plot of the time series for the daily log-returns as well as a histogram of the return distribution for WTI crude oil prices and Brent crude oil prices. The figures indicated heteroscedasticity and volatility clustering for the return series that also exhibited a number of rather isolated extreme returns for both WTI and Brent crude oil prices. We further tested for stationarity of the log return series using the Augmented Dick Fuller (1979) (ADF) and Phillips Perron (1988) (PP) unit root tests.

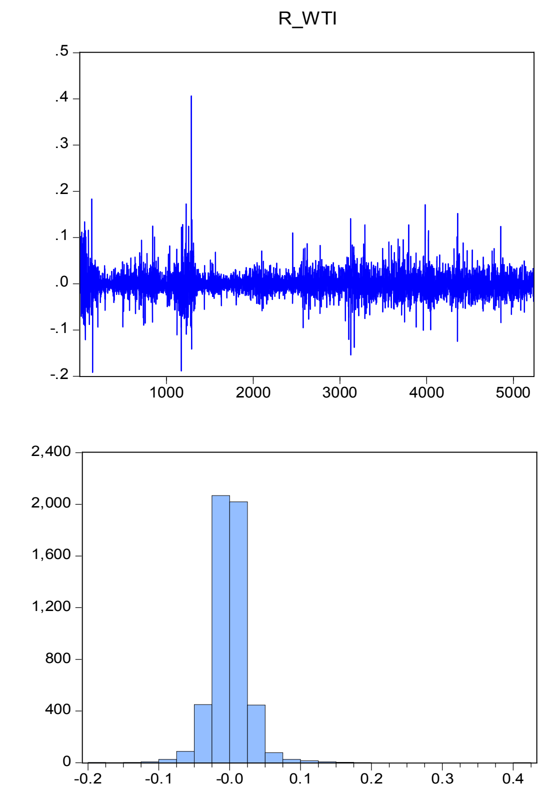

Figure 3: Time Series and distribution of log returns for WTI

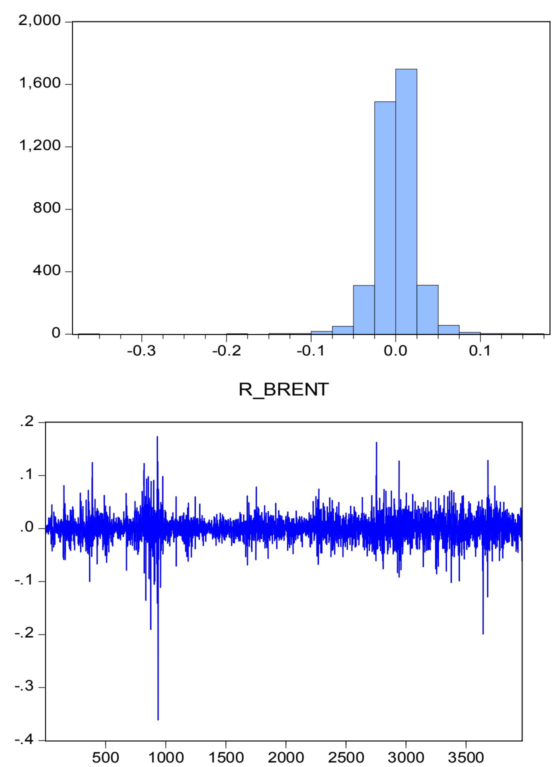

Figure 4: Time Series and distribution of log returns for Brent daily crude oil prices

The ADF test was set to a lag length 0 using the Schwarz Information Criterion (SIC) and the PP test was conducted using the Bartlett Kernel spectral estimation method. Results are reported in (Table 2 and Table 3). The results revealed that for both tests, the null hypothesis of a unit root was rejected. So, the log return series crude oil spot prices can be considered to be stationary.

Table 2: Result for Augmented Dickey – Fuller and Phillips – Perron unites root Tests for WTI crude oil spot log return series

	T-Statistic	Prop. *
Augmented Dickey-Fuller test statistic	-45.84707	0.0001
Phillips-Perron test statistic	-73.84772	0.0001

Table 3: Result for Augmented Dickey – Fuller and Phillips – Perron unites root Tests for Brent crude oil spot log return series

	T-Statistic	Prop. *
Augmented Dickey-Fuller test statistic	-59.87176	0.0001
Phillips-Perron test statistic	-59.81871	0.0001

Table 4: ARCH LM test

	WTI		
No.of lags	1	5	10
Q-statistic	36.918	221.91	315.88
P-value	0.000	0.000	0.000
	Brent		
No.of lags	1	5	10
Q-statistic	51.666	413.77	515.29
P-value	0.000	0.000	0.000

The test is conducted at different numbers of lags. Values in parenthesis indicate the p – value at all lags which strongly indicates the presence of ARCH effect in both series.

Next, we performed the ARCH LM test to determine if there was any ARCH effect in the residuals. Table 4 shows that the LM test for both data showed a significant presence of ARCH effect with low p–value of 0.0000.GARCH models were examined at different lags based on the p-values. So, we rejected the null hypothesis of no ARCH effect and detected a strong presence of ARCH effect as expected for most financial time series.

As the return exhibited an ARCH effect, we used GARCH–type models. Franses and Van Dijk (1996) and Gokcan (2000) have also shown that models with a small lag like GARCH (1, 1) are sufficient to cope with the changing variance. According to Brooks (2002), the lag order (1, 1) model is sufficient to capture all of the volatility clustering that is present from the data

Explain all parameters in GARCH (1,1)-N, GARCH (1,1)-t and GARCH (1,1)-G for WTI and Brent respectively are significant at the 5 % level. For each index data set, LM test supported the absence of ARCH effect in the residuals. A high value of α_1 means that volatility is spiky and quick, reacts to market movements (Dowd, 2002).

Tables 5 – 10 show all coefficients tests at the 5 % level of significance that were found to be highly significant.

Table 5: Parameters Estimates and diagnostic of GARCH (1, 1)-N Models for WTI

Parameter	Normal	P-value
μ	-8.54E-05	0.7218
ω	6.59E-06	0.0000
α_1	0.103229	0.0000
β_1	0.894631	0.0000
LM test	0.160742	0.6885

Table 6: Parameters Estimates and diagnostic of GARCH (1, 1)-t Models for WTI

Parameter	Student-t	P-value
μ	-0.00050	0.0428
ω	6.95E-06	0.0000
α_1	0.069140	0.0000
β_1	0.921041	0.0000
v	5.422923	0.0000
LM test	0.388099	0.5333

Table 7: Parameters Estimates and diagnostic of GARCH (1, 1)-GED Models for WTI

Parameter	GED	P-value
μ	-0.00046	0.0521
ω	6.72E-06	0.0000
α_1	0.079143	0.0000
β_1	0.912355	0.0000
v	1.277640	0.0000
LM test	0.070948	0.7900

Table 8: Parameters Estimates and diagnostic of GARCH (1, 1)-N Models for Brent

Parameter	Normal	P-value
μ	0.000330	0.1884
ω	6.10E-06	0.0000
α_1	0.087434	0.0000
β_1	0.905735	0.0000
LM test	0.034965	0.8517

Table 9: Parameters Estimates and diagnostic of GARCH (1, 1)-t Models for Brent

Parameter	Student-t	P-value
μ	0.000465	0.0557
ω	5.25E-06	0.0000
α_1	0.070849	0.0000
β_1	0.922314	0.0000
ν	5.820882	0.0000
LM test	0.342608	0.5583

Table 10: Parameters Estimates and diagnostic of GARCH (1, 1)-GED Models for Brent

Parameter	GED	P-value
μ	0.000414	0.0795
ω	5.55E-06	0.0000
α_1	0.078184	0.0000
β_1	0.914116	0.0000
ν	1.338290	0.0000
LM test	0.036058	0.8494

The results for the fitted GARCH model for crude oil spot prices are presented in Table 11, it is observed that for Brent the minimum value of MAE and MSE were obtained in – sample modeling and forecasting the model GARCH-N. We also observed that for WTI the minimum value of MAE is in the in- sample modeling for the GARCH-t model, but the minimum value of MSE is in the in –sample modeling for the GARCH-G model. But, we can see for WTI the minimum value of MSE and MAE is in the out of sample of the modeling for GARCH-G model. Based on AIC criterion, GARCH (1, 1) with an error distribution STD is the best fitted GARCH model for WTI and Brent in sample modeling also, GARCH (1, 1) with STD for WTI and Brent out sample forecasting.

In Table 12, the results of the volatility GARCH model for crude oil spot prices are presented, it is observed that for WTI and Brent the minimum values of MAE and MSE were obtained in – sample modeling and forecasting is the model GARCH – N. Based on AIC criterion, GARCH (1, 1) with Normal distribution is the best fitted GARCH model for WTI in the in-sample modeling, on the other hand in the out-of-sample forecasting GARCH (1, 1) with Generalized error distribution GED is the best for WTI.But the best fitted GARCH model for Brent in- sample modeling and out-of-sample forecasting is GARCH with Normal distribution.

Table 11: The Best Fitted GARCH Using the Data WTI and Brent Crude Oil Spot

In – sample Modeling			
Models	**MSE**	**MAE**	**AIC**
WTI			
GARCH-N	0.353	0.377	-4.820116
GARCH-t	0.321	**0.349**	**-4.901769**
GARCH-G	**0.304**	0.380	-4.891365
Brent			
GARCH-N	**0.752**	**0.702**	-4.945741
GARCH-t	1.212	0.951	**-5.008167**
GARCH-G	1.017	0.853	-5.000991
Out – of – sample forecast			
Models	**MSE**	**MAE**	**AIC**
WTI			
GARCH-N	1.027	0.739	-4.675200
GARCH-t	3.226	1.384	**-4.726873**
GARCH-G	**0.906**	**0.696**	-4.712744
Brent			
GARCH-N	**3.590**	**0.695**	-4.849699
GARCH-t	6.155	1.384	**-4.876067**
GARCH-G	5.060	0.696	-4.869370

Notes: The bold value in each row is the minimum value

Table 12: The Best Volatility GARCH Using the WTI and Brent Crude Oil Spot

In – sample Modeling			
Models	**MSE**	**MAE**	**AIC**
WTI			
GARCH-N	**0.3896065**	**0.6237626**	**-13.36822**
GARCH-t	0.770397	0.876954	-12.59418
GARCH-G	0.585208	0.764381	-12.86207
Brent			
GARCH-N	**0.00001120**	**0.0012914**	**-13.34326**
GARCH-t	0.00001437	0.001666	-13.15075
GARCH-G	0.00001306	0.001515	-13.15821
Out – of – sample forecast			
Models	**MSE**	**MAE**	**AIC**
WTI			
GARCH-N	**0.3910498**	**0.625338**	-13.83803
GARCH-t	0.774241	0.879908	-12.60133
GARCH-G	0.5878333	0.766701	**-14.32352**
Brent			
GARCH-N	**0.00000289**	**0.001175**	**-13.50564**
GARCH-t	0.00000490	0.001568	-13.27169
GARCH-G	0.00000405	0.001412	-13.41909

5. Conclusion

This paper investigates the volatility forecasting capability of GARCH (p,q) models with three different types of error distributions and applies them to two main crude oil prices-WTI crude oil spot price and Brent crude oil spot price and the prices were transformed to log returns. Descriptive statistics showed that WTI and Brent returned the presence of

Skewness in the series. A test of stationarity was carried out on the time series data using unit root and the null hypothesis was rejected furthermore by using ARCH-LM test, we found the presence of very high ARCH effect in the residuals of the series. Having differentiated the series to take care of the non-stationary nature and the ARCH effect we obtained the estimates of the GARCH models using the popular Quasi-Maximum likelihood estimation. Different lags were examined GARCH (1,1) model for both series WTI and Brent were found to be the most successful model, in line with previous literature. Rechecking was done by using the ARCH-LM test which then showed no presence of ARCH effect. The results for the crude oil the minimum values of MAE and MSE for the Brent were obtained in-sample modeling and forecasting the model GARCH-N, but for the WTI in the in-sample model the minimum values of MAE is GARCH-t and the minimum values of MSE is GARCH-G, on the other hand the minimum values of MAE and MSE in the out-of-sample forecast is GARCH-G. Based on the AIC criterion GARCH with an error distribution STD is the best fitted GARCH model for both WTI and Brent in the in-sample modeling and out-of-sample forecasting. For the volatility GARCH the minimum values of MAE and MSE were obtained in the in-sample modeling and out-of-sample forecasting for WTI is GARCH-N, in addition for the Brent the minimum values of MAE and MSE is GARCH-N in the in-sample modeling and out-of-sample forecasting. Based on the AIC criterion GARCH with Normal distribution is the best volatility GARCH model for WTI in the in-sample modeling, but in the out-of-sample forecasting GARCH with Generalized error distribution GED is the best for WTI. And for the Brent in the in-sample modeling and out-of-sample forecasting is the GARCH with Normal distribution. Since, the leverage effect to the characteristics of time series on asset prices that "bad news" tend to increase volatility more than "good news", it is expected that the occurrence of the crisis will significantly increase this impact.

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