# On Adomian's Decomposition Method for solving nonlocal perturbed stochastic fractional integro-differential equations

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**Abstract:** Adomian decomposition method (ADM) is applied to approximately solve stochastic fractional integro-differential equations involving nonlocal initial condition. The convergence of the ADM for the considered problem is proved. The mean square error between approximate solution and accurate solution is also given.
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## 1. Introduction

Recently a great deal of interest has been focused on the application of ADM for the solution of many different problems. For example in [1-7] boundary value problems, algebraic equations, nonlinear differential equations, partial differential equations, stochastic nonlinear oscillator and nonlinear Sturm-Liouville problems are considered. The ADM, which accurately computes the series solution, is of great interest to applied sciences. This method provides the solution in a rapidly convergent series with components that are elegantly computed. The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients as the method does not need linearization, weak nonlinearity assumptions or perturbation theory. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution. In this paper, we investigate the applicability of ADM to the following nonlocal perturbed random fractional integro-differential equations.

$$\frac{\partial^\alpha x(t;\omega)}{\partial t^\alpha} = h\big(t, x(t;\omega)\big) + \int_0^t k(t,\tau;\omega)\, f\big(\tau, x(\tau;\omega)\big)d\tau ,$$

with the nonlocal condition

$$x(0;\omega) + \sum_{i=1}^{p} c_i x(t_i,\omega) = x_0(\omega)$$

where

$0 < \alpha \le 1,\ t \in R_+ = [0,\infty),\ 0 < t_1 < \ldots < t_p < \infty,$ the fractional derivative is provided by the Caputo derivative and

(i) $\omega \in \Omega$, the supporting set of a probability measure space $(\Omega, \mathcal{A}, P)$ ;

(ii) $x(t;\omega)$ is the unknown stochastic process for $t \in R_+$;

(iii) $h(t,x)$ is called the stochastic perturbing term and it is a scalar function of $t \in R_+$and scalar $x \in R$ ;

(iv) $k(t,\tau;\omega)$ is a stochastic kernel defined for $t$ and $\tau$ satisfying $0 \le \tau \le t < \infty$; and

(v) $f(t,x)$ is a scalar function of $t \in R_+$, scalar $x \in R$ and will be specified later.

The purpose of this paper is to apply the ADM to obtain an approximate random solution $x(t;\omega)$ to the nonlocal Cauchy problem $(1.1), (1.2)$. We shall utilize the new formula for Adomian's polynomials developed by El-Kalla in [6] to prove the convergence of ADM and to estimate for the error between a truncated n+1-terms and accurate solution. The studied problem may be considered a generalization to the work of G. Adomain in [8]. The nonlocal Cauchy problem $(1.1), (1.2)$ has applications in many fields such as viscoelasticity, fluid mechanics and electromagnetic theory, see for example [9-13].

## 2. Preliminaries

Let $(\Omega, \mathcal{A}, P)$ denote a probability measure space, that is $\Omega$ is a nonempty set known as the sample space, $\mathcal{A}$ is a sigma-algebra of subsets of $\Omega$, and $P$ is a complete probability measure on $\mathcal{A}$. We let $x(t;\omega), t \in R_+ , \omega \in \Omega$ , denote a stochastic process whose index set is $R_+$. Let $L_2(\Omega, \mathcal{A}, P)$ be the space of all random variables $x(t;\omega), t \in R_+$, which have a second moment (or square- summable) with respect to $P$-measure for each $t \in R_+$. That is:

$$E\{|x(t;\omega)|^2\} = \int_\Omega |x(t;\omega)|^2\, dp(\omega) < \infty .$$

The norm of $x(t;\omega)$ in $L_2(\Omega, \mathcal{A}, P)$ is defined for each $t \in R_+$ by:

$$\|x(t;\omega)\| = \|x(t;\omega)\|_{L_2(\Omega,\mathcal{A},P)}$$
$$= [E\{|x(t;\omega)|^2\}]^{1/2} = \left\{\int_\Omega |x(t;\omega)|^2 dP(\omega)\right\}^{\frac{1}{2}}$$

With respect to the functions in equation $(1.1)$, we make the following assumptions:

The random solution $x(t;\omega)$ will be considered as a function of $t \in R_+$ with values in the space $L_2(\Omega,\mathcal{A},P)$. The functions $h(t,x), f(t,x)$ under convenient conditions will be functions of $t \in R_+$ with values in $L_2(\Omega,\mathcal{A},P)$.

Let $L_\infty(\Omega,\mathcal{A},P)$ be the space of all measurable and $P$ -essentially bounded random variables of $\omega \in \Omega$. With respect to the stochastic kernel, we will assume that, for each $t$ and $\tau$ satisfying $0 \le \tau \le t < \infty$, $k(t,\tau;\omega)$ is essentially bounded with respect to $P$, So that the product of $k(t,\tau;\omega)$ and $f(\tau,x(\tau;\omega))$ will always be in $L_2(\Omega,\mathcal{A},P)$ for each fixed $t$ and $\tau$. The norm of $k(t,\tau;\omega)$ in $L_\infty(\Omega,\mathcal{A},P)$ will be denoted and defined by

$$|||k(t,\tau;\omega)||| = P - ess\ sup_{\omega\in\Omega}|k(t,\tau;\omega)|.$$

Also the mapping $(t,\tau) \to k(t,\tau;\omega)$ from the set $\Delta = \{(t,\tau): 0 \le \tau \le t < \infty\}$ into $L_\infty(\Omega,\mathcal{A},P)$ is continuous.

And further, whenever $\tau_n \to \tau$ as $n \to \infty$, $P - ess\ sup_{\omega\in\Omega}|k(s,\tau_n;\omega) - k(s,\tau;\omega)| \to 0$ as $n \to \infty$. It will be assumed also that for each fixed $t$ and $\tau$, $P - ess\ sup_{\omega\in\Omega}|\,k(s,\tau;\omega)| \le M_{(t,\tau)}$ uniformly for $\tau \le s \le t$, where $M_{(t,\tau)} > 0$ is some constant depending only on $t$ and $\tau$, $0 \le \tau \le t < \infty$.

**Definition 2.1**

We define the space $C_c = C_c(R_+, L_2(\Omega,\mathcal{A},P))$ to be the space of all continuous functions from $R_+$ into $L_2(\Omega,\mathcal{A},P)$ and define a topology on $C_c$ by means of the following family of seminorms

$$\|x(t;\omega)\|_n = sup_{0\le t\le n}\|x(t;\omega)\|,\ n = 1,2,\ldots\ldots$$

It is known that such a topology is metrizable and that the metric space $C_c$ is complete.

Finally, let $B, D \subset C_c(R_+, L_2(\Omega,\mathcal{A},P))$ be Banach spaces and let $T$ be a linear operator from $C_c(R_+, L_2(\Omega,\mathcal{A},P))$ into itself.

Now we give the following definitions with respect to $B$, $D$, and $T$.

**Definition 2.2**

The pair of Banach spaces $(B, D)$ is said to be admissible with respect to the operator $T$ if and only if $T(B) \subset D$.

**Definition 2.3**

The Banach space $B$ is said to be stronger than the space $C_c$ if every convergent sequence in $B$ , with respect to its norm, will also converge in $C_c$.(but the converse is not true in general).

**Definition 2.4**

We call $x(t;\omega)$ a random solution of equation (1.1) if $x(t;\omega) \in C_c$ for each $t \in R_+$, satisfies the equation (1.1) for every $t > 0$ and satisfies the nonlocal initial condition, almost surely.

Now by using the definitions of the fractional derivatives and integrals, it is suitable to rewrite the considered problem in the form:

$$x(t;\omega) = x(0;\omega) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} h(\tau, x(\tau;\omega))d\tau + \frac{1}{\Gamma(\alpha)}\int_0^t\int_0^s (t-s)^{\alpha-1}k(s,\tau;\omega)f(\tau,x(\tau;\omega))d\tau ds$$

Changing the order of integration

$$x(t;\omega) = x(0;\omega) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} h(\tau, x(\tau;\omega))d\tau + \frac{1}{\Gamma(\alpha)}\int_0^t \boldsymbol{K}(t,\tau;\omega)f(\tau,x(\tau;\omega))d\tau \quad (2.1)$$

where

$$\boldsymbol{K}(t,\tau;\omega) = \int_\tau^t (t-s)^{\alpha-1}k(s,\tau;\omega)\,ds \quad (2.2)$$

Now define the integral operators $T_1$ and $T_2$ on $C_c(R_+, L_2(\Omega,\mathcal{A},P))$ as follows:

$$(T_1x)(t;\omega) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}x(\tau;\omega)\,d\tau \quad (2.3)$$

$$(T_2x)(t;\omega) = \frac{1}{\Gamma(\alpha)}\int_0^t \boldsymbol{K}(t,\tau;\omega)\,x(\tau;\omega)d\tau \quad (2.4)$$

**Lemma 2.1**

Assume that $\sum_{i=1}^{p} c_i \neq -1$. then the nonlocal Cauchy problem $(1.1)$, $(1.2)$ is equivalent to the following integral equation

$$x(t;\omega) = \Lambda x_0(\omega)$$
$$-\Lambda\left(\sum_{i=1}^{p} c_i\left[(T_{1i}hx)(t_i,\omega) + (T_{2i}fx)(t_i,\omega)\right]\right)$$
$$+(T_1hx)(t;\omega) + (T_2fx)(t;\omega) \qquad (2.5)$$

where: $\Lambda = \left[1+\sum_{i=1}^{p} c_i\right]^{-1}$, $T_1$ and $T_2$ are defined by $(2.3)$,$(2.4)$ respectively and

$$(T_{1i}x)(t_i;\omega) = \frac{1}{\Gamma(\alpha)}\int_0^{t_i}(t_i-\tau)^{\alpha-1}x(\tau;\omega)\,d\tau$$

$$(T_{2i}x)(t_i;\omega) = \frac{1}{\Gamma(\alpha)}\int_0^{t_i}\boldsymbol{K}(t_i,\tau;\omega)x(\tau;\omega)\,d\tau$$

$i = 1,2,3 \ldots\ldots\ldots.p$

see El-borai et al. in [14] for the proof.

**Theorem 2.1**

Suppose the random equation $(1.1)$ satisfies the following conditions:

(i) $B$ and $D$ are Banach spaces stronger than $C_c$ and the pair $(B,D)$ is admissible with respect to each of the operators, $T_1$ and $T_2$ defined by $(2.3)$, $(2.4)$ respectively;

(ii) $x(t;\omega) \to h\big(t,x(t;\omega)\big)$ is an operator on the set $\boldsymbol{S} = \{x(t;\omega) \in D: \|x(t;\omega)\|_D \le \rho\}$, with values in $B$ satisfying:

$$\|h(t,x(t;\omega)) - h(t,y(t;\omega))\|_B \le \lambda_1\|x(t;\omega) - y(t;\omega)\|_D$$

for $x(t;\omega), y(t;\omega) \in \boldsymbol{S}$, $\rho > 0$ and $\lambda_1 > 0$ are constants;

(iii) $x(t;\omega) \to f\big(t,x(t;\omega)\big)$ is an operator on $\boldsymbol{S}$ with values in $B$ satisfying:

$$\|f(t,x(t;\omega)) - f(t,y(t;\omega))\|_B \le \lambda_2\|x(t;\omega) - y(t;\omega)\|_D$$

for $x(t;\omega), y(t;\omega) \in \boldsymbol{S}$ and $\lambda_2 > 0$ constant,

(iv) $x_0(\omega) \in D$.

Then there exists a unique random solution $x(t;\omega) \in \boldsymbol{S}$ of equation$(1.1)$, provided that

$$\left[(K_1\lambda_1 + K_2\lambda_2)\left(1+|\Lambda|\sum_{i=1}^{p}|c_i|\right)\right] < 1$$

and

$$|\Lambda|\,\|x_0(\omega)\|_D +$$
$$(K_1\|h(t,0)\|_B + K_2\|f(t,0)\|_B)\left(1+|\Lambda|\sum_{i=1}^{p}|c_i|\right)$$
$$\le \rho\left(1-(K_1\lambda_1+K_2\lambda_2)\left(1+|\Lambda|\sum_{i=1}^{p}|c_i|\right)\right)$$

where $K_1$ and $K_2$ are the norms of $T_1$ and $T_2$, respectively

see El-borai et al. in [14] for the proof.

## 3. Main results

The Adomian decomposition method consists of decomposing the unknown solution of $(2.5)$ into a sum of an infinite number of components defined by the decomposition series

$$x(t,\omega) = \sum_{k=0}^{\infty} x_k(t,\omega) \qquad (3.1)$$

where the components $x_k(t,\omega), k \ge 0$ are to be determined in a recursive manner. The ADM concerns itself with finding the components $x_0(t,\omega), x_1(t,\omega), x_2(t,\omega) \ldots\ldots\ldots\ldots$ individually.

The nonlinear operator $N$ that appears in the considered equation is decomposing by a special representation of infinite series of the so-called Adomian Polynomials, that is

$$N(t,x) = \sum_{k=0}^{\infty}\boldsymbol{A}_k(t,x_0,x_1,\ldots,x_k)$$

where $\boldsymbol{A}_k, k \ge 0$ are called the Adomian polynomials and are given by the following Adomian's technique

$$\boldsymbol{A}_k = \frac{1}{k!}\frac{d^k}{d\lambda^k}\left[N\left(t,\sum_{j=0}^{k}\lambda^j x_j\right)\right]_{\lambda=0} \quad ,k \ge 0$$

where the parameter $\lambda$ is simply a device for tagging appropriate terms for grouping together. The Adomian polynomials are not unique and can be evaluated for all forms of nonlinearity. (see for example [15-17]).

Now by substituting the series $(3.1)$ and the Adomian polynomials into the left side and right side of $(2.5)$ respectively gives

$$\sum_{k=0}^{\infty} x_k(t,\omega) = -\Lambda\sum_{i=1}^{p} c_i\left[\left(T_{1i}\sum_{k=0}^{\infty}A_k\right)(t_i,\omega) + \left(T_{2i}\sum_{k=0}^{\infty}B_k\right)(t_i,\omega)\right]$$

$$+\left(T_1\sum_{k=0}^{\infty}A_k\right)(t;\omega)+\left(T_2\sum_{k=0}^{\infty}B_k\right)(t;\omega)$$

where $A_k, B_k, k \geq 0$ are the Adomian polynomials of the nonlinear terms $h(t,x)$ and $f(t,x)$ respectively, they can be evaluated for all forms of nonlinearity and are given by

$$A_k=\frac{1}{k!}\frac{d^k}{d\lambda^k}\left[h\left(t,\sum_{j=0}^{k}\lambda^j x_j\right)\right]_{\lambda=0}\quad ,k\geq 0$$

and

$$B_k=\frac{1}{k!}\frac{d^k}{d\lambda^k}\left[f\left(t,\sum_{j=0}^{k}\lambda^j x_j\right)\right]_{\lambda=0}\quad ,k\geq 0$$

Now we identify the zeroth component by all the terms that are not included under the integral sign in $(2.5)$. Therefore, we obtain the following recurrence relation:

$$x_0(t;\omega)=\Lambda x_0(\omega)$$

$$x_{k+1}(t;\omega)=-\Lambda\sum_{i=1}^{p}c_i(T_{1i}\,A_k)(t_i,\omega)$$

$$-\Lambda\sum_{i=1}^{p}c_i(T_{2i}\,B_k)(t_i,\omega)+(T_1A_k)(t;\omega)$$

$$+(T_2B_k)(t;\omega)\quad ,k\geq 0\qquad (3.2)$$

where $x_0(t;\omega)$ is referred to as the zeroth component. The m+1–components truncated series solution is thus obtained as

$$S_m=\sum_{k=0}^{m}x_k(t,\omega)$$

with

$$\lim_{m\to\infty}S_m=x(t;\omega)\qquad (3.3)$$

Now, the convergence of the Adomain decomposition scheme for the considered problem will be established by the following theorem.

**Theorem 3.1**

The series solution $(3.3)$ of the nonlocal Cauchy problem $(1.1), (1.2)$ using the ADM converges provided that $0\leq\beta<1$ and $\|x_1(t,\omega)\|_D\leq d_1$, where $d_1$ is a positive constant and

$$\beta=(\lambda_1K_1+\lambda_2K_2)\left(1+|\Lambda|\sum_{i=1}^{p}|c_i|\right)$$

**Proof:**

Since the approximated series solution is $x(t;\omega)=\sum_{k=0}^{\infty}x_k(t,\omega)$, so we can define the following sequence of partial sums

$$\{S_m\}=\left\{\sum_{k=0}^{m}x_k(t,\omega)\right\}$$

since

$$h(t,x(t;\omega))=\sum_{k=0}^{\infty}A_k(t;\omega),$$

$$f(t,x(t;\omega))=\sum_{k=0}^{\infty}B_k(t;\omega),$$

Then we have

$$h(t,S_n)=\sum_{k=0}^{n}A_k(t;\omega),$$

and

$$f(t,S_m)=\sum_{k=0}^{m}B_k(t;\omega)$$

Now let $S_m$ and $S_n$ be two arbitrary partial sums with $m>n$, yields

$$S_m=\sum_{k=0}^{m}x_k(t,\omega)=\Lambda x_0(\omega)$$

$$-\Lambda\sum_{i=1}^{p}c_i\left(T_{1i}\sum_{k=0}^{m-1}A_k\right)(t_i,\omega)$$

$$-\Lambda\sum_{i=1}^{p}c_i\left(T_{2i}\sum_{k=0}^{m-1}B_k\right)(t_i,\omega)$$

$$+\left(T_1\sum_{k=0}^{m-1}A_k\right)(t;\omega)$$

$$+\left(T_2\sum_{k=0}^{m-1}B_k\right)(t;\omega)$$

$$S_n=\sum_{k=0}^{n}x_k(t,\omega)=\Lambda x_0(\omega)$$

$$-\Lambda \sum_{i=1}^{p} c_i \left( T_{1i} \sum_{k=0}^{n-1} A_k \right)(t_i,\omega)$$

$$-\Lambda \sum_{i=1}^{p} c_i \left( T_{2i} \sum_{k=0}^{n-1} B_k \right)(t_i,\omega)$$

$$+\left( T_1 \sum_{k=0}^{n-1} A_k \right)(t;\omega) + \left( T_2 \sum_{k=0}^{n-1} B_k \right)(t;\omega)$$

We shall prove that the sequence $\{S_n\}$ is a Cauchy sequence in the Banach space $D$ .

$$S_m - S_n$$
$$= -\Lambda \sum_{i=1}^{p} c_i \left( T_{1i} \left( \sum_{k=0}^{m-1} A_k - \sum_{k=0}^{n-1} A_k \right) \right)(t_i,\omega)$$
$$-\Lambda \sum_{i=1}^{p} c_i \left( T_{2i} \left( \sum_{k=0}^{m-1} B_k - \sum_{k=0}^{n-1} B_k \right) \right)(t_i,\omega)$$
$$+\left( T_1 \left( \sum_{k=0}^{m-1} A_k - \sum_{k=0}^{n-1} A_k \right) \right)(t;\omega)$$
$$+\left( T_2 \left( \sum_{k=0}^{m-1} B_k - \sum_{k=0}^{n-1} B_k \right) \right)(t;\omega)$$

Taking the norm in $D$ for both sides, we have

$$\|S_m - S_n\|_D \le |\Lambda| \sum_{i=1}^{p} |c_i|\, K_1 \left\| \sum_{k=n}^{m-1} A_k(t_i,\omega) \right\|_B$$

$$+|\Lambda| \sum_{i=1}^{p} |c_i|\, K_2 \left\| \sum_{k=n}^{m-1} B_k(t_i,\omega) \right\|_B$$

$$+K_1 \left\| \sum_{k=n}^{m-1} A_k(t,\omega) \right\|_B + K_2 \left\| \sum_{k=n}^{m-1} B_k(t,\omega) \right\|_B$$

$$= K_1 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \left\| \sum_{k=n}^{m-1} A_k(t;\omega) \right\|_B$$

$$+K_2 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \left\| \sum_{k=n}^{m-1} B_k(t;\omega) \right\|_B$$
$$= K_1 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \|h(t,S_{m-1}) - h(t,S_{n-1})\|_B$$

$$+K_2 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \|f(t,S_{m-1}) - f(t,S_{n-1})\|_B$$

$$= K_1 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \lambda_1 \|S_{m-1} - S_{n-1}\|_D$$
$$+K_2 \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \lambda_2 \|S_{m-1} - S_{n-1}\|_D$$
$$= (\lambda_1 K_1 + \lambda_2 K_2) \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right) \|S_{m-1} - S_{n-1}\|_D$$

Let

$$\beta = (\lambda_1 K_1 + \lambda_2 K_2) \left( 1 + |\Lambda| \sum_{i=1}^{p} |c_i| \right)$$

Then

$$\|S_m - S_n\|_D \le \beta \|S_{m-1} - S_{n-1}\|_D$$

Let $m = n + 1$, then

$$\|S_{n+1} - S_n\|_D \le \beta \|S_n - S_{n-1}\|_D$$

$$\le \beta^2 \|S_{n-1} - S_{n-2}\|_D \dots\dots \le \beta^n \|S_1 - S_0\|_D$$

From triangle inequality we have

$$\|S_m - S_n\|_D \le \|S_m - S_{m-1}\|_D + \|S_{m-1} - S_{m-2}\|_D$$

$$+ \dots + \|S_{n+2} - S_{n+1}\|_D + \|S_{n+1} - S_n\|_D$$

$$\le (\beta^{m-1} + \beta^{m-2} + \dots + \beta^{n+1} + \beta^n) \|S_1 - S_0\|_D$$

By the same way we have

$$\|S_{m^*} - S_n\|_D \le \beta^n (1 + \beta + \beta^2 + \cdots) \|S_1 - S_0\|_D$$

Since $0 \le \beta < 1$, then

$$\|S_{m^*} - S_n\|_D \le \frac{\beta^n}{1-\beta} \|S_1 - S_0\|_D$$
$$= \frac{\beta^n}{1-\beta} \|x_1(t,\omega)\|_D \to 0 \ as\ n \to \infty$$

Since $\|x_1(t,\omega)\|_D < d_1$, and hence $\{S_n\}$ is a Cauchy sequence in the Banach space $D$, therefore, $\{S_n\}$ converges to the solution $x(t,\omega) \in D$, that is the series $\sum_{k=0}^{\infty} x_k(t;\omega)$ converges. This completes the proof.

**Lemma 3.1**

The mean square error of the solution $(3.3)$ is estimated to be

$$E\left(x\,(t,\omega)-\sum_{j=0}^{n}x_j(t;\omega)\right)^2\leq\frac{\beta^{2n}}{(1-\beta)^2}\|x_1(t,\omega)\|_D^2$$

**Proof:**

From theorem 3.1 we proved that

$$\|S_{m^*}-S_n\|_D\leq\frac{\beta^n}{1-\beta}\|x_1(t,\omega)\|_D$$

If $m^*\rightarrow\infty$, then $S_{m^*}\rightarrow x\,(t,\omega)$, then we have

$$\|x\,(t,\omega)-S_n\|_D\leq\frac{\beta^n}{1-\beta}\|x_1(t,\omega)\|_D$$

which means that

$$E\left(x\,(t,\omega)-\sum_{j=0}^{n}x_j(t;\omega)\right)^2\leq\frac{\beta^{2n}}{(1-\beta)^2}\|x_1(t,\omega)\|_D^2$$

Hence the required result follows.

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