Evaluation of the Shear Strength of Dapped Ended Beam

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Abstract: Dapped end beams are precast members of concrete structures which are widely used in buildings and bridges. The re-entrant corner is the weakest portion of the beam, where stress concentration develops, such regions are known as disturbed regions. These regions cannot be analyzed with ordinary flexural analysis theory rather than another method named Strut and Tie Model (STM) is used. The same approach has been used in this research. In this research four reinforced concrete dapped end beams divided into two groups G-1 and G-2 having depths of 18" (457 mm) and 12" (305 mm) respectively were designed for the assumed external load. The beams were later tested under monotonic loads to study the shear strength of the dapped ends and compared with the assumed external designed loads. In case of G-1, failure loads were observed much higher than the design loads, which show that STM gives very conservative solution for the design of dapped ended beams of greater depth. Actual values of strut forces have been observed as quite closer to the values proposed by ACI 318-08 for both diagonal bottled shaped struts and horizontal prismatic struts. The experimental values of strength reduction factor for struts βs are also close to the values specified in ACI code. Whereas, in case of beams of G-2, failure loads were observed lower than the design loads, which shows that STM gives non realistic solution for the design of dapped end beams of lower depth. Actual values of strut forces and strength reduction factor βs are also very smaller as compared to the values proposed by ACI code.

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Keywords: Strut and Tie Model; non flexural; non prismatic; dapped ends

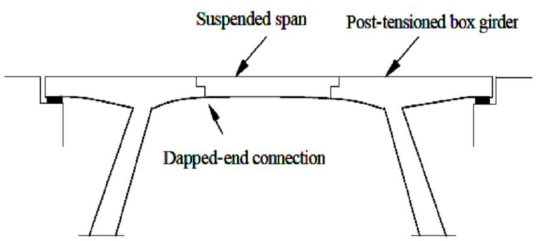
1. Introduction

Deep beams are those members whose clear spans do not exceed four times the overall member depth, or regions of beams with concentrated loads within twice the member depth from the support (ACI318-08). Since the early 1900s, sectional truss models have made great contributions towards the shear design of shallow beams, but they are not suitable for deep beams. The application of Strut and Tie Model (STM) for the analysis and design of non prismatic, non flexural members and typical disturbed regions in concrete structures has been increased in the last few decades (ASCE-ACI445) and recommended by numerous codes (CEB-FIP, 1990; AASHTO, 2004; CSA, 1994 ; EC2, 2004). Strut and Tie Model have been used to analyze and design RC concrete members like deep beams, pile caps, corbel, dapped ended beams and other non prismatic members with abrupt changes in geometries. Consequently, the stress flow around the discontinuity is severely disturbed (Lu et al., 2003; MacGregor and Wight, 2005). Dapped ended beams are most commonly used in the Pre-cast and pre-stressed concrete to mainly reduce the floor height. Some typical applications of dapped ended beams are given as follows (Huang et al., 2000).

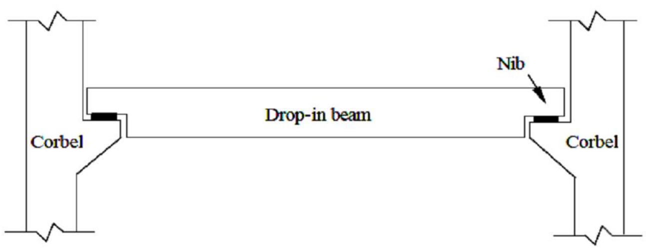
i. A cantilever and suspended span type of structure (Fig.1-a).
ii. As a drop-in beam between corbels (Fig.1b).
iii. As a hide-away type of beam-to-beam and beam-to-column connection (Fig.1c).

Historically dapped ended beams have been designed with various detailing and analytical approaches. Reynold (1969) developed suitable reinforcement details by evolving a design procedure for dapped-end members. He suggested some detailing guidelines for dapped end members. Sargious and Tadros (1970) presented a research work in which they used Finite Element Analysis (FEA) to determine the behavior and strength of dapped-end beams.

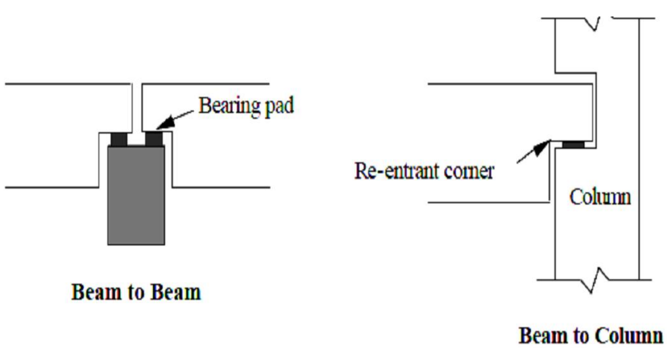
Werner and Dilger (1973) determined first cracking shear at reentrant corner using Finite Element Method (FEM). They determined concrete contribution to cracking shear and concluded that cracking shear at reentrant corner is in agreement with FEM using concrete tensile strength 6 $\sqrt{f_c'}$, 4 $\sqrt{f_c'}$ for practical design. total shear strength is the summation of shear strengths of concrete, shear reinforcement, and pre-stressing. They found that vertical and inclined shear reinforcement is equally efficient in resisting shear. Hamoud *et al* (1975) developed the mechanics of diagonal shear cracks in the dapped-end beam. They concluded that shear cracking load for beams with post-tensioned bars is equal to failure loads. Beams with low flexural reinforcement ratio and high pre-stress reinforcement failed in flexure, where as beams having low pre-stressing failed by concrete rupture. The ultimate shear strength increased with an increase in pre-stressing and Shear span to effective depth ratio (a/d). Mattock and Chan (1979) carried out an experimental investigation and applied corbel design concepts to dapped-ends. They proposed that the reduced depth of dapped-end may be designed as a corbel if shear span "a" is measured to the center of gravity of the hanger reinforcement. They proposed some basic guidelines for detailing of main reinforcement and shear reinforcement at the reentrant end of beam.



a. Cantilever suspended span bridge



b. Drop-in-Beam supported by Corbels



c. Hide Away type connections

Fig. 1: Some typical applications of dapped ended beams in pre-cast structures

There has been significant volume of literature on application of the STM to the non prismatic and discontinuous members (Schlaich et al; 1987 and Muttoni et al; 1997) but its application to dapped ended beam is limited.

Strut-and-tie models are a conservative and intuitive design methodology for solving the problems of dapped ended beams and other disturbed region in concrete. Chen *et al*, (2002) worked on the dapped ended beams with openings using various assumed STM and observed that all strut-and-tie test specimens carried loads greater than the factored design load specified. The use of orthogonal reinforcement patterns to simplify fabrication was both a valid and useful technique. The ACI 318-02 Code provisions of STM provided simple and conservative estimate for design of dapped ended beams.

Lu *et al* (2003) tested 12 high strength concrete dapped ended beams. He proposed a model for prediction of the shear strength of dapped ended beams by using the Constitutive law, equilibrium equations and compatibility conditions. The actual values were compared with the values predicted by the proposed model and the approach of PCI Design Handbook. He concluded that the PCI design method underestimated the shear strength of dapped ended beams and recommended to revise the methods by incorporating the proposed model.

Wang et al, (2005) performed tests on 24 of reinforced concrete (RC) dapped end beams and investigated the shear resistance behaviour. It was found that inclined stirrups and longitudinal bent reinforcement have more influence on the shear capacity than vertical stirrups. Some design suggestions for the ultimate shear strength of dapped end beams were offered.

Huang and Nanni (2006) reported the testing of three full-scale prestressed concrete double tee beams in shear at the re-entrant corner. Dapped-end of each beam was reinforced with internal steel reinforcement while other was strengthened with externally bonded carbon fiber reinforced polymer (CFRP) laminates. The tests showed that the reinforcement arrangements have significant influence on the member response.

Ley et al (2007) conducted tests on small scale dapped ended beams with opening with various Strut and Tie Models and different reinforcement patterns were proposed by the designers. The test results showed that all assumed models and reinforcement carried loads greater than the design factored loads. The STM provides flexibility in design. However STM could not predict the ultimate failure for the dapped ends. They recommended more full scale tests for design of dapped ended beam having opening with STM.

The literature review on the design of dapped ends of beams shows that little experimental data is available on the design of dapped ended beams particularly with STM. In this research, four dapped ended beams were designed for external assumed loads to be applied at the mid span with the Strut and Tie Model using the guidelines given by ASCE-ACI-445. The beams were later tested under monotonic loads at the mid span and actual shear strength and load carrying capacity of the beams were determined. The strut capacities corresponding to the failure loads were compared with the provisions of ACI 318-08.

Strut and Tie Model (STM) has been widely used for the analysis and design of disturbed region in concrete structures, however its application to dapped ended beams is very limited. The main significance of this research is to provide more experimental data on the application of STM to analysis and design of dapped ended beams. The objectives of the research can be summarized as:

i. To check the suitability of Strut and Tie Model (STM) for the design of reinforced concrete dapped end beams.
ii. To check the capacity of the compression struts corresponding to failure loads and compare the same with the provisions of ACI318-08
iii. The tests results shall supplement the limited secondary data available on the design of dapped ended beams using STM.

3. Experimental Investigation

To study the behavior of dapped ends of beams, four dapped ended beams two of sizes 9″×18″ (228 mm × 457 mm) and two of sizes 9”x12” (228 mm x 305 mm), having dapped ends of 9″×9″ (228 mm×228mmm) and length of 5″ (127 mm) were designed by using Strut and Tie Model (STM) for an external assumed loads. The forces in the assumed Struts and Tie were worked by resolving the assumed truss. The capacities of the diagonal and horizontal struts as well as nodal zones were checked against the limits given by ACI 318-08. The member forces in the struts satisfied the conditions prescribed by ACI 318-08. Main reinforcement was provided to resist the tensile forces in the horizontal direction and transverse reinforcement was provided to resist the tensile forces in vertical direction. Strain gauges were placed along the theoretical inclination of the strut at the dapped ends. The dapped ended beams designed by STM were tested under monotonic loads and the loads were gradually increased and the cracking pattern and corresponding loads were recorded. The beams were loaded till failure.

Materials:

For concrete, Ordinary Portland Cement (OPC), sand of fineness modulus 2.5 and coarse aggregates of ½ in (12.7mm) sizes or less were used. The cement, sand and coarse aggregates were mixed in nominal ratio of 1:1 ½:3 by volume and the water cement ratio were taken as 0.50. The average 28 days cylinder compressive strength was observed as 5140 psi (35.45 MPa). For main reinforcement deformed steel bars of specified yield strength of 60,000 psi (413.79 MPa) and transverse reinforcement of yield strength of 40,000 psi (275.86 MPa) were used.

Test Specimens:

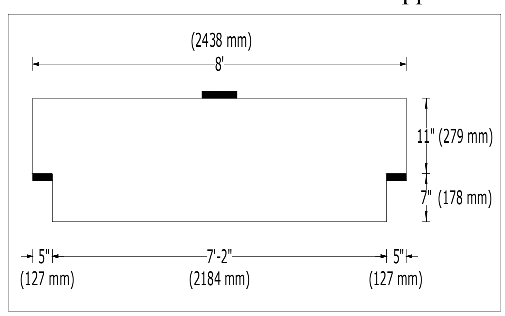
Four dapped ended beams, two of sizes 9″×18″ (228 mm×457 mm) and two of sizes 9”x12” (228 mm x 305 mm), having dapped ends of 9″×9″ (228 mm × 228 mm) and dapped length of 5″ (127 mm) were cast. The clear span of the beams, excluding the dapped ends was taken as 7′-2″ (2184.4 mm). The details of test specimen are given in Figs.2 and 3.

Design of dapped ended beams for external assumed load:

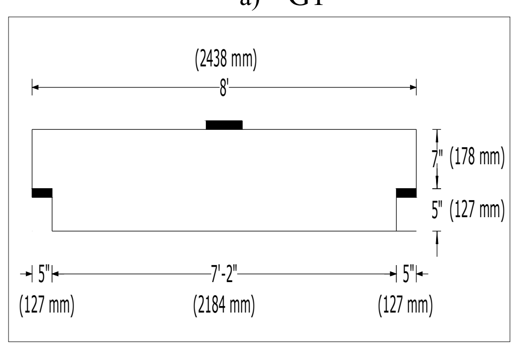
The beams were designed for the assumed external load of 55 kips (245 kN), using Strut and Tie Model (STM). The assumed STM is given in Fig.4. The detailed design procedure given by ASCE-ACI committee 445 was adopted. The dapped end was designed as disturbed region and the mid span was designed as simply supported beam under the assumed external load. The details of finally designed beam are shown in Figs.5 and 6.

Testing of beams:

The beams were tested under monotonic load applied at the centre of the beams, through a rigid plate. Deflection gauges were placed under the mid span and strain gauges were placed in the direction assumed struts at the intersection of dapped ends.



a) G1



b) G2

Figure 2: Categorization of Dapped-end Beam Models

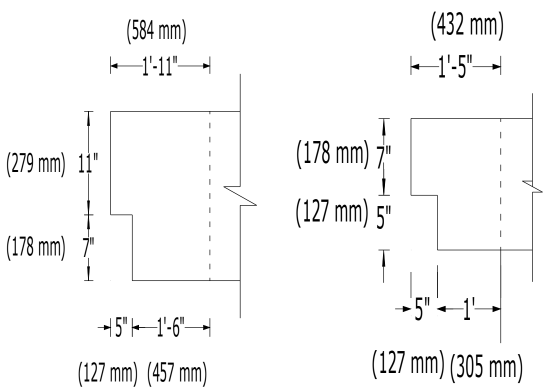


Figure 3: Defining D-region of Dapped-end Beam

The loads were applied through hydraulic system, transferring the load through calibrated proving ring, to evaluate the shear behavior of dapped-end beams.

When the load was applied at the mid span of dapped ended beam, the first diagonal crack appeared at the dapped end almost at half of the total failure loads, which was followed by diagonal crack of the same nature at the other end with slight increase in the load. With further increase in the external load, flexural crack appeared in the mid span region, which was followed by flexural shear crack in the critical region. When loads were further increased, the diagonal crack at the corner widened and caused failure of the beams at one of the end. A little difference in the pattern of the beam failures was noted in all the four beams.

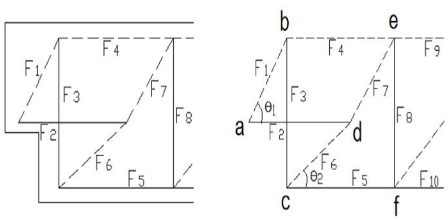


Figure 4: Truss Layout for Strut and Tie Model (STM)

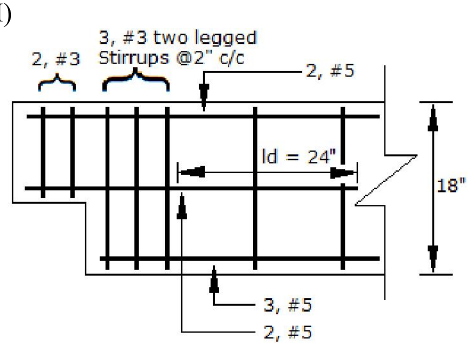


Figure 5 a: Reinforcement detail of Ties and Development Length

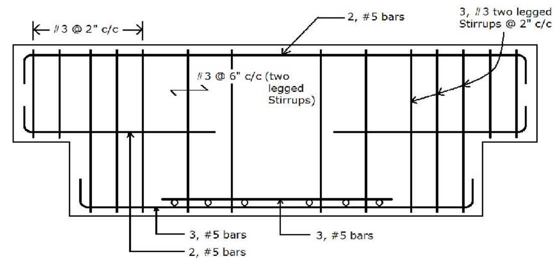


Figure 5b: Reinforcement Detail for Beam of Group G-1 (18" x 9")

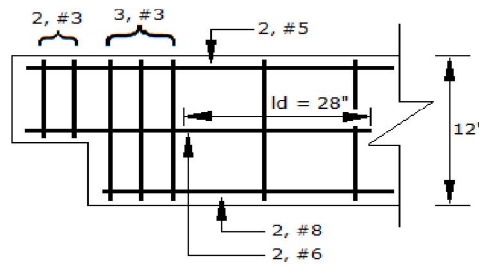


Figure 6a: Reinforcement detail of Ties and Development Length

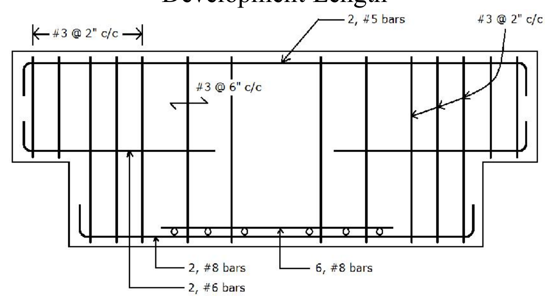


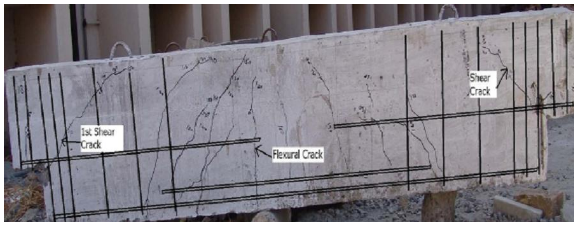
Figure 6.2: Reinforcement Detail for Beam of Group G-2 (12" x 9")

EXPERIMENTAL RESULTS AND DISCUSSION

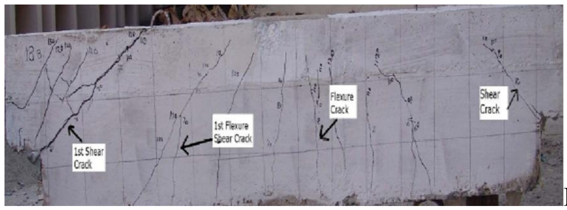
Failure loads and cracking pattern of the beams:

The typical behavior of the beams under increasing loads and details of the total loads carried with corresponding cracks are shown in Table 1. The cracking pattern of two dapped ended beams, one from G1 and G2 group each is given in Fig.7. The actual load carried by the beams of G-1 is at an average of 50% more than the assumed external load which shows that STM has provided a conservative solution for the tested beams of higher depth i.e 18"(457mm) but for G-2 beams of lower depth i.e 12"(305mm) the actual load carried was lesser than design loads.

The theoretical inclination of the strut for G-1 beams at the dapped end is 53.13º, whereas the actual inclination of the dapped ends measured from the cracks is 33.69º, which is lesser than the theoretical value but for G-2 beams the theoretical inclination of the strut is 41.98° and the actual inclination is 37.22° which is reasonably close to the theoretical value.



Beam of G-1



Beam of G-2

Figure 7: Dapped ended beams after cracking

Comparison of theoretical capacities of compression struts proposed by ACI and actual strength at the failure of the dapped ended beams:

According to ACI 318-08, the actual capacities of compressive struts are worked out by the equation:

$$F_{us} = 0.85 . \beta_s . f_c^{'} . w . b \qquad Eq. (1)$$

Where β_s is capacity reduction factor of struts proposed as $\beta_s = 1$ for prismatic strut and 0.75 for bottle shaped struts.

If the load carried by the compression struts is worked out from the failure loads and inclination of the struts, the actual capacity reduction factor can be determined from the test values as follows:

$$F_{actual} = F_{us} = 0.85 . \beta_s . f_c^{'} . w . b \qquad Eq. (2)$$

$$\beta_s = F_{actual} / 0.85 f_c^{'} . w . b = F_{actual} / F$$

$$F = 0.85 f_c^{'} . w . b$$

The values of β_s for diagonal and horizontal struts were worked out and the calculation to determine the actual capacity reduction factors for both diagonal and horizontal compressive struts is given in Table 2.

The comparison of the values of β_s with the proscribed value of ACI 318-08 of 0.75 shows that it was reasonably suitable to assume the horizontal struts a bottled shaped for the diagonal struts as the calculate means values of 0.76 and 0.72 for the struts *"ab"* and *"cd"* are quite closer to 0.75.

For the horizontal struts, prismatic shape of strut was assumed and the values of 1.0 as prescribed by ACI-318 was used but the actual mean value was observed as 0.72, which is non conservative, which shows that either the behavior of horizontal strut is bottle like or the ACI 318-08 has overestimated the capacity of horizontal strut. For safe design of dapped ends with STM, it is more advisable to assume the strut as bottle shaped.

Table 1: Comparison of design shear and failure shear

Groups	Beam	Design Shear V	Shear at 1st Flexure crack $V_{u (FL)}$		*Shear at 1st Diagonal crack At one end V_{cr}	Shear at 1st Diagonal crack At other end V_{cr}	Shear at test failure V_f		Observed Angles		Theoretical Angles		Failure to design shear ratio V_f / V	
			Entity	Mean			Entity	Mean	θ_1	θ_2	θ_1	θ_2	Entity	Mean
G-1 (9"x18")	B1	27.5	24.37	20.49	19.19	21.78	45.06	42.47	33.69	_	53.13	40.60	1.64	1.54
	B2	27.5	16.6		16.6	21.78	39.89		33.69	_	53.13	40.6	1.45	
G-2 (9"x12")	B3	27.5	-	–	11.25	11.25	19.03	17.74	37.22	_	41.98	41.98	0.69	0.65
	B4	27.5	-		6.69	11.25	16.44		37.22	_	41.98	41.98	0.60	

*Strut forces were calculated using these values

Table 2: Comparison of values of β_s between ACI code and observed values considering $F_{ab} = F_{us\,(ab)}$, $F_{cd} = F_{us\,(cd)}$ & $F_{be} = F_{us\,(be)}$

For strut ab $\beta_s = F_{ab}/\varphi\,(0.85)\,f_c'\,b\,w_s = 0.75\,F_{ab}/F_{us\,(ab)}$

For strut cd $\beta_s = F_{cd}/\varphi\,(0.85)\,f_c'\,b\,w_s = 0.75\,F_{cd}/F_{us\,(cd)}$

For strut be $\beta_s = F_{be}/\varphi\,(0.85)\,f_c'\,b\,w_s = 1.0\,F_{be}/F_{us\,(be)}$

Beams	At 1st diagonal crack			At test failure using theoretical angles			At test failure using observed angles		
	Strut ab	Strut cd	Strut be	Strut ab	Strut cd	Strut be	Strut ab	Strut cd	Strut be
B1	0.217	0.217	0.216	0.513	0.507	0.508	0.733	0.507	1.01
B2	0.188	0.188	0.187	0.453	0.453	0.449	0.647	0.453	0.90
B3	0.128	0.128	0.128	0.213	0.213	0.216	0.240	0.213	0.253
B4	0.111	0.111	0.111	0.187	0.187	0.187	0.207	0.187	0.220

Load- deformation behavior of dapped beams:

Strain gauges were placed at the dapped ends along the theoretical inclination of the struts. The typical load strain curve has been given in Fig.8 and Fig.9 for dapped ended beam B-1 of group G-1 and beam B-3 of group G-2 respectively. It is observed that, under gradual increase of load, initially a linear load-strain curve is obtained. For beam B-1 at load 30 kips strain is abruptly increased. This increase is associated with the formation of 1st diagonal shear crack. Similar nature is noted for beam B-3 of Group-2. At load of 20 kips strain abruptly increases with the formation of 1st diagonal crack.

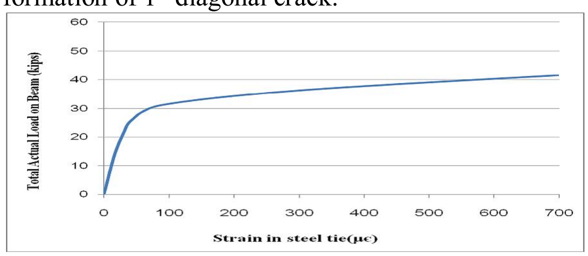


Figure 8: Load-Strain Relation for Longitudinal Reinforcement in Beam B-1

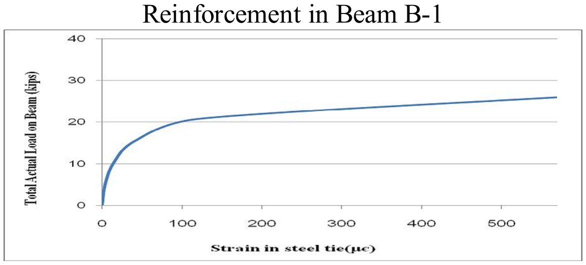


Fig. 9. Load-Strain Relation for Longitudinal Reinforcement in Beam B-3

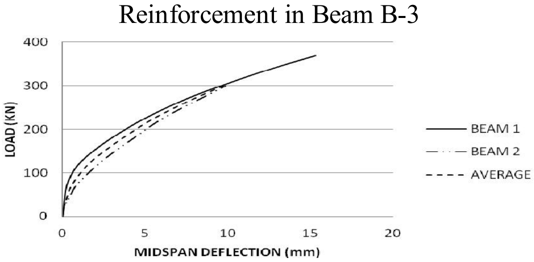


Fig. 10. Load-Deflection Curves for Group G-1

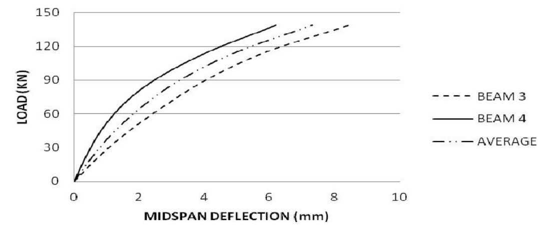


Figure 11: Load-Deflection Curves for Group G-2

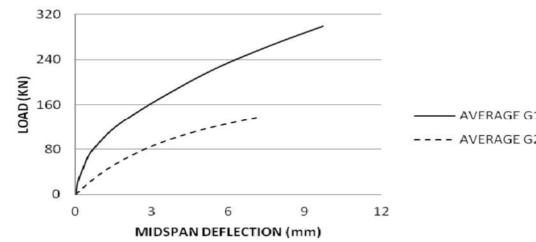


Fig. 12. Comparison of average deflection in the Beams of Group-1 and Group-2

Linear variable displacement transducers (LVDTs) were installed to measure the deflection at the mid span of beam specimen and the test progress was recorded for each beam. Load-deflection curves for the Beams Group-1 and Group-2 have been shown in Figure 10 and 11. Average deflection curves for both groups have been compared in Figure 12. This comparison exhibits that, Group-2 (h=12") sustained highest deflection up to the beam failure load. Beams of Group-1 (18") sustained comparatively lower deflection and carry highest ultimate load than beams of other group. The load deformation curve shows that the behavior of the dapped end is almost elastic up to a load of 30 kips (133.44 kN) and beyond this point, it behaves like inelastic material, where the deformation increases substantially with very little increase in the load. The beam has been designed for 27.5 kips (120 kN) to be carried by each end, which lies within the elastic region. The deformation of the dapped end in the inelastic region is not reliable as cracking of concrete has led to substantial deformation with very little increase in the loads. Hence the assumption of STM to take the concrete strength corresponding to elastic regions seems to be reasonable.

FURTHER RESEARCH

Due to limited experimental data on the design of dapped ends by STM, it is recommended that more tests are carried out for various configurations of STM and geometry of the dapped ended beams. Further research is also recommended to assess the strength of the compression struts for the STM used for design of dapped ended beams.

Conclusions

Based on the results of this experimental investigation on the shear design of dapped simply supported beams, the following conclusions are made.

1. The dapped beams of G-1 having depths of 18” (457mm) have carried about 50 % more load than the theoretical loads for which the beams were designed on the basis of Strut and Tie Model. However for beams of G-2 having depth of 12” (305mm), failure loads were lower than design loads.
2. The strength reduction factors βs for the assumed diagonal bottle shaped struts and horizontal prismatic struts at the failure loads are quite close to the values prescribed by ACI-318, only for G-1 beams of 18”(457mm) but for G-2 beams of 12”(305mm), values of βs are very lower as compared to the values prescribed by ACI-318 . This may however require further experimental evidence to be generalized.
3. Even after forming a diagonal crack, reinforced concrete dapped end beams were able to sustain additional loads due to arch action.
4. Shear strength is not only affected by the overall depth of the beam but shear span/depth ratio also plays an important role in shear strength of the beam.
5. Design of dapped end beams using Strut and Tie model gave results varied with change in the angle of strut inclination.

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