On Some Systems of Three Nonlinear Difference Equations

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Abstract: We consider in this paper, the solution of the following systems of difference equations: $x_{n+1}=\frac{x_{n-2}}{\pm 1\pm y_n z_{n-1} x_{n-2}}$, $y_{n+1}=\frac{y_{n-2}}{\pm 1\pm z_n x_{n-1} y_{n-2}}$, $z_{n+1}=\frac{z_{n-2}}{\pm 1\pm x_n y_{n-1} z_{n-2}}$, with initial conditions are nonzero real numbers.

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1. Introduction

We are interested in studying the following third order systems of rational difference equations

$$x_{n+1}=\frac{x_{n-2}}{\pm 1\pm y_n z_{n-1} x_{n-2}},\ y_{n+1}=\frac{y_{n-2}}{\pm 1\pm z_n x_{n-1} y_{n-2}},$$
$$z_{n+1}=\frac{z_{n-2}}{\pm 1\pm x_n y_{n-1} z_{n-2}},$$

with initial conditions are nonzero real numbers. We mainly focus on studying their forms of solutions and periodicity.

In the last decade, there are a number of mathematical models that describe real life, such as population biology, economics, genetics, psychology and etc. To examine these models, we need to find some means to describe these models. So, studying systems of difference equations received considerable attention from researchers.

In the references cited in [1-10], it is obvious that the investigation of behaviors of difference equations is intensely challenging. Currently, the main focus on studying rational difference equations and rational difference systems is the qualitative analysis, for further details see [11-18].

There are many papers related to the difference equations system, for example, the periodicity of the positive solutions of the rational difference equations systems

$$x_{n+1}=\frac{m}{y_n},\ y_{n+1}=\frac{py_n}{x_{n-1}y_{n-1}},$$
$$x_{n+1}=\frac{1}{z_n},\ y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}},\ z_{n+1}=\frac{1}{x_{n-1}},$$

has been obtained by Cinar in [3-4].

Elabbasy et al. [6] has obtained the solution of particular cases of the following general system of difference equations

$$x_{n+1}=\frac{a_1+a_2y_n}{a_3z_n+a_4x_{n-1}z_n},\ y_{n+1}=\frac{b_1z_{n-1}+b_2z_n}{b_3x_ny_n+b_4x_ny_{n-1}},$$
$$z_{n+1}=\frac{c_1z_{n-1}+c_2z_n}{c_3x_{n-1}y_{n-1}+c_4x_{n-1}y_n+c_5x_ny_n}.$$

Also, the behavior of the solutions of the following two systems

$$x_{n+1}=\frac{1}{y_n},\ \ y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}},$$

$$x_{n+1}=\frac{1}{y_n},\ \ y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}},\ z_{n+1}=\frac{x_{n-1}y_{n-1}}{x_{n-2}y_{n-2}z_{n-2}},$$

has been studied by Elsayed [15].

In [16], Elsayed et al. dealt with the solutions of the systems of the difference equations

$$x_{n+1}=\frac{1}{x_{n-p}y_{n-p}},\ \ y_{n+1}=\frac{x_{n-p}y_{n-p}}{x_{n-q}y_{n-q}},$$

and

$$x_{n+1}=\frac{1}{x_{n-p}y_{n-p}z_{n-p}},\ \ y_{n+1}=\frac{x_{n-p}y_{n-p}z_{n-p}}{x_{n-q}y_{n-q}z_{n-q}},$$
$$z_{n+1}=\frac{x_{n-q}y_{n-q}z_{n-q}}{x_{n-r}y_{n-r}z_{n-r}}.$$

Grove et al. [17] has studied existence and behavior of solutions of the rational system

$$x_{n+1}=\frac{a}{x_n}+\frac{b}{y_n},\ \ y_{n+1}=\frac{c}{x_n}+\frac{d}{y_n}.$$

Kurbanli [20-21] investigated the behavior of the solutions of the difference equation systems

$$x_{n+1}=\frac{x_{n-1}}{x_{n-1}y_n-1},\ y_{n+1}=\frac{y_{n-1}}{y_{n-1}x_n-1},\ \ z_{n+1}=\frac{1}{z_n y_n},$$

and

$$x_{n+1}=\frac{x_{n-1}}{x_{n-1}y_n-1},\ y_{n+1}=\frac{y_{n-1}}{y_{n-1}x_n-1},\ \ z_{n+1}=\frac{z_{n-1}}{z_{n-1}y_n-1}.$$

In [27], Papaschinopoulos and Schinas studied the oscillatory behavior, the boundedness of the solutions, and the global asymptotic stability of the positive equilibrium of the system of nonlinear difference equations

$$x_{n+1}=A+\frac{y_n}{x_{n-p}},\qquad y_{n+1}=A+\frac{x_n}{y_{n-q}}.$$

In [30], Yalçinkaya investigated the sufficient condition for the global asymptotic stability of the following system of difference equations

$$z_{n+1}=\frac{t_n z_{n-1}+a}{t_n+z_{n-1}},\ t_{n+1}=\frac{z_n t_{n-1}+a}{z_n+t_{n-1}}.$$

In [34], Zhang et al. studied the boundedness, the persistence and global asymptotic stability of the positive solutions of the system of difference equations

$$x_n=A+\frac{1}{y_{n-p}},\qquad y_n=A+\frac{y_{n-1}}{x_{n-r}y_{n-s}},$$

Similar to difference equations and nonlinear systems of rational difference equations were investigated see [1-2, 7-12, 18-33].

Definition (Periodicity)

A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p}=x_n$ for all $n\geq -k$.

2. The system:

$$x_{n+1}=\frac{x_{n-2}}{1+y_n z_{n-1}x_{n-2}},\ y_{n+1}=\frac{y_{n-2}}{1+z_n x_{n-1}y_{n-2}},\ z_{n+1}=\frac{z_{n-2}}{1+x_n y_{n-1}z_{n-2}}.$$

In this section, we obtain the form of the solutions of the system of three difference equations

$$x_{n+1}=\frac{x_{n-2}}{1+y_n z_{n-1}x_{n-2}},\ \ y_{n+1}=\frac{y_{n-2}}{1+z_n x_{n-1}y_{n-2}},\ \ z_{n+1}=\frac{z_{n-2}}{1+x_n y_{n-1}z_{n-2}},\qquad (1)$$

where $n\in\mathbb{N}_0$ and the initial conditions are arbitrary nonzero real numbers.

The following theorem is devoted to the form of the solutions of system (1).

Theorem 1. Suppose that $\{x_n, y_n, z_n\}$ are solutions of system (1). Then for $n=0,1,2,\ldots$

$$x_{3n-2}=x_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)},$$

$$x_{3n-1}=x_{-1}\prod_{i=0}^{n-1}\frac{(1+(3i+1)y_{-2}x_{-1}z_0)}{(1+(3i+2)y_{-2}x_{-1}z_0)},$$

$$x_{3n}=x_0\prod_{i=0}^{n-1}\frac{(1+(3i+2)z_{-2}y_{-1}x_0)}{(1+(3i+3)z_{-2}y_{-1}x_0)},$$

$$y_{3n-2}=y_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)},$$

$$y_{3n-1}=y_{-1}\prod_{i=0}^{n-1}\frac{(1+(3i+1)z_{-2}y_{-1}x_0)}{(1+(3i+2)z_{-2}y_{-1}x_0)},$$

$$y_{3n}=y_0\prod_{i=0}^{n-1}\frac{(1+(3i+2)x_{-2}z_{-1}y_0)}{(1+(3i+3)x_{-2}z_{-1}y_0)},$$

and

$$z_{3n-2} = z_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)},$$

$$z_{3n-1} = z_{-1}\prod_{i=0}^{n-1}\frac{(1+(3i+1)x_{-2}z_{-1}y_0)}{(1+(3i+2)x_{-2}z_{-1}y_0)},$$

$$z_{3n} = z_0\prod_{i=0}^{n-1}\frac{(1+(3i+2)y_{-2}x_{-1}z_0)}{(1+(3i+3)y_{-2}x_{-1}z_0)}.$$

Proof: For $n=0$ the result holds. Now suppose that $n>1$ and that our assumption holds for $n-1$. that is,

$$x_{3n-5} = x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)},$$

$$x_{3n-4} = x_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)y_{-2}x_{-1}z_0)}{(1+(3i+2)y_{-2}x_{-1}z_0)},$$

$$x_{3n-3} = x_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)z_{-2}y_{-1}x_0)}{(1+(3i+3)z_{-2}y_{-1}x_0)},$$

$$y_{3n-5} = y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)},$$

$$y_{3n-4} = y_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)z_{-2}y_{-1}x_0)}{(1+(3i+2)z_{-2}y_{-1}x_0)},$$

$$y_{3n-3} = y_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)x_{-2}z_{-1}y_0)}{(1+(3i+3)x_{-2}z_{-1}y_0)},$$

and

$$z_{3n-5} = z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)},$$

$$z_{3n-4} = z_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)x_{-2}z_{-1}y_0)}{(1+(3i+2)x_{-2}z_{-1}y_0)},$$

$$z_{3n-3} = z_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)y_{-2}x_{-1}z_0)}{(1+(3i+3)y_{-2}x_{-1}z_0)}.$$

It follows from Eq.(1) that

$$x_{3n-2} = \frac{x_{3n-5}}{1+y_{3n-3}z_{3n-4}x_{3n-5}}$$

$$= \frac{x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}}{1+\left(\begin{array}{c} y_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)x_{-2}z_{-1}y_0)}{(1+(3i+3)x_{-2}z_{-1}y_0)} \\ z_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)x_{-2}z_{-1}y_0)}{(1+(3i+2)x_{-2}z_{-1}y_0)} \\ x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)} \end{array}\right)}$$

$$= \frac{x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}}{\left(1+x_{-2}z_{-1}y_0\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+3)x_{-2}z_{-1}y_0)}\right)}$$

$$= x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}\frac{1}{\left(1+\frac{x_{-2}z_{-1}y_0}{(1+(3n-3)x_{-2}z_{-1}y_0)}\right)}$$

$$= x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}\frac{1}{\left(1+\frac{x_{-2}z_{-1}y_0}{(1+(3n-3)x_{-2}z_{-1}y_0)}\right)}$$

$$= x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}\left(\frac{(1+(3n-3)x_{-2}z_{-1}y_0)}{(1+(3n-3)x_{-2}z_{-1}y_0+x_{-2}z_{-1}y_0)}\right)$$

$$= x_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}\left(\frac{(1+(3n-3)x_{-2}z_{-1}y_0)}{(1+(3n-2)x_{-2}z_{-1}y_0)}\right).$$

Then, we see that

$$x_{3n-2} = x_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)x_{-2}z_{-1}y_0)}{(1+(3i+1)x_{-2}z_{-1}y_0)}.$$

Also, we see from Eq.(1) that

$$y_{3n-2} = \frac{y_{3n-5}}{1+z_{3n-3}x_{3n-4}y_{3n-5}}$$

$$= \frac{y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)}}{1+\left(\begin{array}{l} z_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)y_{-2}x_{-1}z_0)}{(1+(3i+3)y_{-2}x_{-1}z_0)} \\ x_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)y_{-2}x_{-1}z_0)}{(1+(3i+2)y_{-2}x_{-1}z_0)} \\ y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)} \end{array}\right)}$$

$$= \frac{y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)}}{\left(1+y_{-2}x_{-1}z_0\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+3)y_{-2}x_{-1}z_0)}\right)}$$

$$= y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)}\frac{1}{\left(1+\frac{y_{-2}x_{-1}z_0)}{(1+(3n-3)y_{-2}x_{-1}z_0)}\right)}$$

$$= y_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)}\frac{1+(3n-3)y_{-2}x_{-1}z_0}{(1+(3n-3)y_{-2}x_{-1}z_0+y_{-2}x_{-1}z_0)}.$$

Then,

$$y_{3n-2} = y_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)y_{-2}x_{-1}z_0)}{(1+(3i+1)y_{-2}x_{-1}z_0)}.$$

Finally, from Eq.(1), we see that

$$z_{3n-2} = \frac{z_{3n-5}}{1+x_{3n-3}y_{3n-4}z_{3n-5}}$$

$$= \frac{z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)}}{1+\left(\begin{array}{l} x_0\prod_{i=0}^{n-2}\frac{(1+(3i+2)z_{-2}y_{-1}x_0)}{(1+(3i+3)z_{-2}y_{-1}x_0)} \\ y_{-1}\prod_{i=0}^{n-2}\frac{(1+(3i+1)z_{-2}y_{-1}x_0)}{(1+(3i+2)z_{-2}y_{-1}x_0)} \\ z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)} \end{array}\right)}$$

$$= \frac{z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)}}{\left(1+z_{-2}y_{-1}x_0\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+3)z_{-2}y_{-1}x_0)}\right)}$$

$$= z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)}\frac{1}{\left(1+\frac{z_{-2}y_{-1}x_0)}{(1+(3n-3)z_{-2}y_{-1}x_0)}\right)}$$

$$= z_{-2}\prod_{i=0}^{n-2}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)}\frac{1+(3n-3)z_{-2}y_{-1}x_0}{(1+(3n-3)z_{-2}y_{-1}x_0+z_{-2}y_{-1}x_0)}.$$

Thus,

$$z_{3n-2} = z_{-2}\prod_{i=0}^{n-1}\frac{(1+(3i)z_{-2}y_{-1}x_0)}{(1+(3i+1)z_{-2}y_{-1}x_0)}.$$

Similarly, we can prove the other relations. This completes the proof.

Lemma 1. Let $\{x_n, y_n, z_n\}$ be a positive solution of system (1), then every solution of system (1) is bounded and converges to zero.

Proof: It follows from Eq.(1) that

$$x_{n+1} = \frac{x_{n-2}}{1+y_n z_{n-1}x_{n-2}} \le x_{n-2},$$

$$y_{n+1} = \frac{y_{n-2}}{1+z_n x_{n-1}y_{n-2}} \le y_{n-2},$$

$$z_{n+1} = \frac{z_{n-2}}{1+x_n y_{n-1}z_{n-2}} \le z_{n-2}.$$

Then, the subsequences $\{x_{3n-2}\}_{n=0}^{\infty}$, $\{x_{3n-1}\}_{n=0}^{\infty}$ and $\{x_{3n}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by $M = \max\{x_{-2}, x_{-1}, x_0\}$. Also, the subsequences $\{y_{3n-2}\}_{n=0}^{\infty}$, $\{y_{3n-1}\}_{n=0}^{\infty}$ and $\{y_{3n}\}_{n=0}^{\infty}$ are decreasing and so are bounded from above by $N = \max\{y_{-2}, y_{-1}, y_0\}$. Moreover, $\{z_{3n-2}\}_{n=0}^{\infty}$, $\{z_{3n-1}\}_{n=0}^{\infty}$ and $\{z_{3n}\}_{n=0}^{\infty}$ are decreasing and also bounded from above by $L = \max\{z_{-2}, z_{-1}, z_0\}$.

Lemma 2. If $x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1}, y_0, z_{-2}, z_{-1}$ and z_0 arbitrary real numbers and let $\{x_n, y_n, z_n\}$ are solutions of system (1) then the following statements are true:-

(i) If $x_{-2} = 0,\ y_0 \neq 0,\ z_{-1} \neq 0,$ then we have $x_{3n-2} = 0$ and $y_{3n} = y_0,\ z_{3n-1} = z_{-1}.$

(ii) If $x_{-1} = 0,\ y_{-2} \neq 0,\ z_0 \neq 0,$ then we have $x_{3n-1} = 0$ and $y_{3n-2} = y_{-2},\ z_{3n} = z_0.$

(iii) If $x_0 = 0,\ y_{-1} \neq 0,\ z_{-2} \neq 0,$ then we have $x_{3n} = 0$ and $y_{3n-1} = y_{-1},\ z_{3n-2} = z_{-2}.$

(iv) If $y_{-2} = 0,\ x_{-1} \neq 0,\ z_0 \neq 0,$ then we have $y_{3n-2} = 0$ and $x_{3n-1} = x_{-1},\ z_{3n} = z_0.$

(v) If $y_{-1} = 0,\ x_0 \neq 0,\ z_{-2} \neq 0,$ then we have $y_{3n-1} = 0$ and $x_{3n} = x_0,\ z_{3n-2} = z_{-2}.$

(vi) If $y_0 = 0,\ x_{-2} \neq 0,\ z_{-1} \neq 0,$ then we have $y_{3n} = 0$ and $x_{3n-2} = x_{-2},\ z_{3n-1} = z_{-1}.$

(vii) If $z_{-2} = 0,\ x_0 \neq 0,\ y_{-1} \neq 0,$ then we have $z_{3n-2} = 0$ and $x_{3n} = x_0,\ y_{3n-1} = y_{-1}.$

(viii) If $z_{-1} = 0,\ x_{-2} \neq 0,\ y_0 \neq 0,$ then we have $z_{3n-1} = 0$ and $x_{3n-2} = x_{-2},\ y_{3n} = y_0.$

(ix) If $z_0 = 0,\ x_{-1} \neq 0,\ y_{-2} \neq 0,$ then we have $z_{3n} = 0$ and $x_{3n-1} = x_{-1},\ y_{3n-2} = y_{-2}.$

Proof: The proof follows from the form of the solutions of system (1).

Example 1. We consider interesting numerical example for the difference system (1) with the initial conditions $x_{-2} = 2,\ x_{-1} = 0.4,\ x_0 = 0.3,\ y_{-2} = 0.9,$ $y_{-1} = 1.7,\ y_0 = 0.13,\ z_{-2} = 0.21,\ z_{-1} = 0.7$ and $z_0 = 1.1.$ (See Fig. 1).

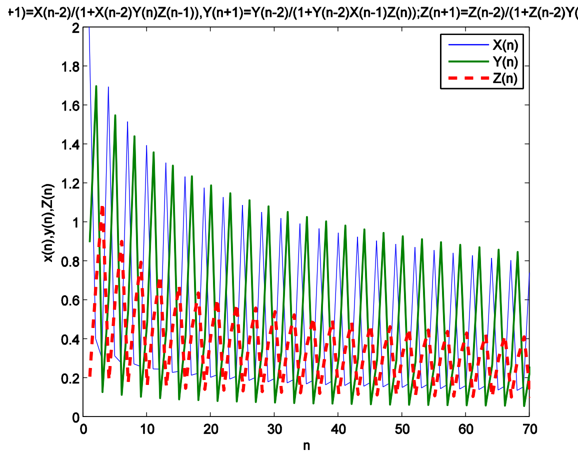

Figure (1)

Example 2. Figure (2) is an example for the system (1) with the initial values $x_{-2} = 2,$ $x_{-1} = -0.4,\ x_0 = 0.3,\ y_{-2} = 0.9,\ y_{-1} = -1.7,$ $y_0 = 0.13,\ z_{-2} = 0.21,\ z_{-1} = 0.7$ and $z_0 = 1.1.$

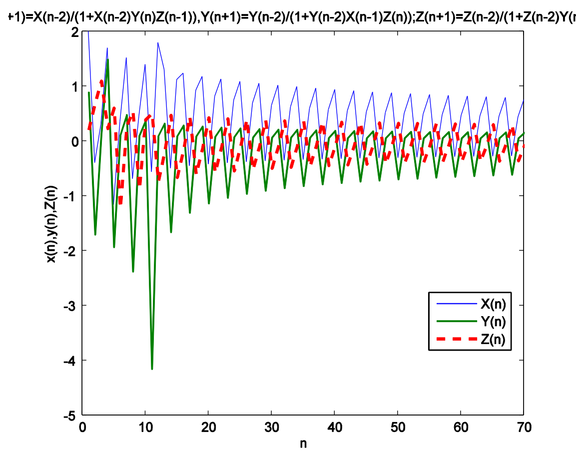

Figure (2)

3 The system:
$x_{n+1} = \frac{x_{n-2}}{1-y_n z_{n-1} x_{n-2}},\ y_{n+1} = \frac{y_{n-2}}{1-z_n x_{n-1} y_{n-2}},\ z_{n+1} = \frac{z_{n-2}}{1-x_n y_{n-1} z_{n-2}}$

In this section, we investigate the solutions of the system of three difference equations

$$x_{n+1} = \frac{x_{n-2}}{1-y_n z_{n-1} x_{n-2}},\quad y_{n+1} = \frac{y_{n-2}}{1-z_n x_{n-1} y_{n-2}},\quad z_{n+1} = \frac{z_{n-2}}{1-x_n y_{n-1} z_{n-2}}, \tag{2}$$

where $n \in N_0$ and the initial conditions are arbitrary non zero real numbers such that $y_{-2}x_{-1}z_0 \neq \pm 1$, $x_{-2}z_{-1}y_0 \neq \pm 1$.

The following theorem is devoted to the expression of the form of the solutions of system (2).

Theorem 2. Suppose that $\{x_n, y_n, z_n\}$ are solutions of system (2). Then, the solution of system (2) are given by the following formula for $n = 0,1,2,\ldots$

$$x_{3n-2} = x_{-2}\prod_{i=0}^{n-1}\frac{(1-(3i)x_{-2}z_{-1}y_0)}{(1-(3i+1)x_{-2}z_{-1}y_0)},$$

$$x_{3n-1} = x_{-1}\prod_{i=0}^{n-1}\frac{(1-(3i+1)y_{-2}x_{-1}z_0)}{(1-(3i+2)y_{-2}x_{-1}z_0)},$$

$$x_{3n} = x_0\prod_{i=0}^{n-1}\frac{(1-(3i+2)z_{-2}y_{-1}x_0)}{(1-(3i+3)z_{-2}y_{-1}x_0)},$$

$$y_{3n-2} = y_{-2}\prod_{i=0}^{n-1}\frac{(1-(3i)y_{-2}x_{-1}z_0)}{(1-(3i+1)y_{-2}x_{-1}z_0)},$$

$$y_{3n-1} = y_{-1}\prod_{i=0}^{n-1}\frac{(1-(3i+1)z_{-2}y_{-1}x_0)}{(1-(3i+2)z_{-2}y_{-1}x_0)},$$

$$y_{3n} = y_0\prod_{i=0}^{n-1}\frac{(1-(3i+2)x_{-2}z_{-1}y_0)}{(1-(3i+3)x_{-2}z_{-1}y_0)},$$

and

$$z_{3n-2} = z_{-2}\prod_{i=0}^{n-1}\frac{(1-(3i)z_{-2}y_{-1}x_0)}{(1-(3i+1)z_{-2}y_{-1}x_0)},$$

$$z_{3n-1} = z_{-1}\prod_{i=0}^{n-1}\frac{(1-(3i+1)x_{-2}z_{-1}y_0)}{(1-(3i+2)x_{-2}z_{-1}y_0)},$$

$$z_{3n} = z_0\prod_{i=0}^{n-1}\frac{(1-(3i+2)y_{-2}x_{-1}z_0)}{(1-(3i+3)y_{-2}x_{-1}z_0)}.$$

Proof: As the proof of Theorem 1 and so will be omitted.

Lemma 3. If $x_{-2}, x_{-1}, x_0, y_{-2}, y_{-1}, y_0, z_{-2}, z_{-1}$ and z_0 are arbitrary real numbers and let $\{x_n, y_n, z_n\}$ be solutions of system (1), then the following statements are true:

(i) If $x_{-2} = 0,\ y_0 \neq 0,\ z_{-1} \neq 0,$ then we have $x_{3n-2} = 0$ and $y_{3n} = y_0,\ z_{3n-1} = z_{-1}$.

(ii) If $x_{-1} = 0,\ y_{-2} \neq 0,\ z_0 \neq 0,$ then we have $x_{3n-1} = 0$ and $y_{3n-2} = y_{-2},\ z_{3n} = z_0$.

(iii) If $x_0 = 0,\ y_{-1} \neq 0,\ z_{-2} \neq 0,$ then we have $x_{3n} = 0$ and $y_{3n-1} = y_{-1},\ z_{3n-2} = z_{-2}$.

(iv) If $y_{-2} = 0,\ x_{-1} \neq 0,\ z_0 \neq 0,$ then we have $y_{3n-2} = 0$ and $x_{3n-1} = x_{-1},\ z_{3n} = z_0$.

(v) If $y_{-1} = 0,\ x_0 \neq 0,\ z_{-2} \neq 0,$ then we have $y_{3n-1} = 0$ and $x_{3n} = x_0,\ z_{3n-2} = z_{-2}$.

(vi) If $y_0 = 0,\ x_{-2} \neq 0,\ z_{-1} \neq 0,$ then we have $y_{3n} = 0$ and $x_{3n-2} = x_{-2},\ z_{3n-1} = z_{-1}$.

(vii) If $z_{-2} = 0,\ x_0 \neq 0,\ y_{-1} \neq 0,$ then we have $z_{3n-2} = 0$ and $x_{3n} = x_0,\ y_{3n-1} = y_{-1}$.

(viii) If $z_{-1} = 0,\ x_{-2} \neq 0,\ y_0 \neq 0,$ then we have $z_{3n-1} = 0$ and $x_{3n-2} = x_{-2},\ y_{3n} = y_0$.

(ix) If $z_0 = 0,\ x_{-1} \neq 0,\ y_{-2} \neq 0,$ then we have $z_{3n} = 0$ and $x_{3n-1} = x_{-1},\ y_{3n-2} = y_{-2}$.

Example 3. We assume the initial conditions $x_{-2} = 2,\ x_{-1} = 1.4,\ x_0 = 1.5,\ y_{-2} = 0.9,\ y_{-1} = 7,$ $y_0 = 3,\ z_{-2} = 1.3,\ z_{-1} = 0.7$ and $z_0 = 6,$ for the difference system (2), see Fig. 3.

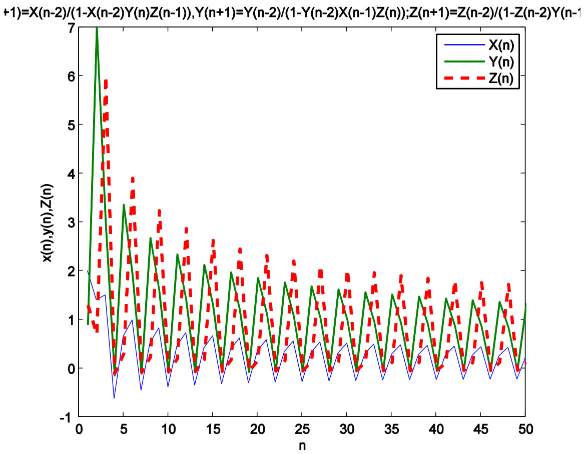

Figure (3)

4 The system:

$$x_{n+1}=\frac{x_{n-2}}{-1+y_nz_{n-1}x_{n-2}},\ y_{n+1}=\frac{y_{n-2}}{-1+z_nx_{n-1}y_{n-2}},\ z_{n+1}=\frac{z_{n-2}}{-1+x_ny_{n-1}z_{n-2}}$$

In this section, we get the solutions of the system of the following difference equations

$$x_{n+1}=\frac{x_{n-2}}{-1+y_nz_{n-1}x_{n-2}},\quad y_{n+1}=\frac{y_{n-2}}{-1+z_nx_{n-1}y_{n-2}},\quad (3)$$

$$z_{n+1}=\frac{z_{n-2}}{-1+x_ny_{n-1}z_{n-2}},$$

where $n\in\mathbb{N}_0$ and the initial conditions are arbitrary nonzero real numbers such that $x_{-2}z_{-1}y_0\neq 1$, $z_{-2}y_{-1}x_0\neq 1$ and $y_{-2}x_{-1}z_0\neq 1$.

Theorem 3. I $\{x_n,y_n,z_n\}$ are solutions of difference equation system (3). Then, every solution of system (3) are periodic with period six and takes the following form for $n=0,1,2,\ldots$

$$x_{6n-2}=x_{-2},\ x_{6n-1}=x_{-1},\ x_{6n}=x_0,$$

$$x_{6n+1}=\frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)},$$

$$x_{6n+2}=x_{-1}(-1+y_{-2}x_{-1}z_0),\quad x_{6n+3}=\frac{x_0}{(-1+z_{-2}y_{-1}x_0)},$$

$$y_{6n-2}=y_{-2},\ y_{6n-1}=y_{-1},\ y_{6n}=y_0,$$

$$y_{6n+1}=\frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)},$$

$$y_{6n+2}=y_{-1}(-1+z_{-2}y_{-1}x_0),\quad y_{6n+3}=\frac{y_0}{(-1+x_{-2}z_{-1}y_0)}$$

and

$$z_{6n-2}=z_{-2},\ z_{6n-1}=z_{-1},\ z_{6n}=z_0,$$

$$z_{6n+1}=\frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)},$$

$$z_{6n+2}=z_{-1}(-1+x_{-2}z_{-1}y_0),\quad z_{6n+3}=\frac{z_0}{(-1+y_{-2}x_{-1}z_0)}.$$

Proof: For $n=0$ the result holds. Now, suppose that $n>0$ and that our assumption holds for $n-1$. That is,

$$x_{6n-8}=x_{-2},\ x_{6n-7}=x_{-1},\ x_{6n-6}=x_0,$$

$$x_{6n-5}=\frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)},$$

$$x_{6n-4}=x_{-1}(-1+y_{-2}x_{-1}z_0),\ x_{6n-3}=\frac{x_0}{(-1+z_{-2}y_{-1}x_0)},$$

$$y_{6n-8}=y_{-2},\ y_{6n-7}=y_{-1},\ y_{6n-6}=y_0,$$

$$y_{6n-5}=\frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)},$$

$$y_{6n-4}=y_{-1}(-1+z_{-2}y_{-1}x_0),\ y_{6n-3}=\frac{y_0}{(-1+x_{-2}z_{-1}y_0)},$$

and

$$z_{6n-8}=z_{-2},\ z_{6n-7}=z_{-1},\ z_{6n-6}=z_0,$$

$$z_{6n-5}=\frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)},$$

$$z_{6n-4}=z_{-1}(-1+x_{-2}z_{-1}y_0),\ z_{6n-3}=\frac{z_0}{(-1+y_{-2}x_{-1}z_0)}.$$

Now, from Eq.(3) it follows that

$$x_{6n-2}=\frac{x_{6n-5}}{-1+x_{6n-5}z_{6n-4}y_{6n-3}}$$

$$=\frac{\frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)}}{-1+\left(\frac{\frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)}z_{-1}(-1+x_{-2}z_{-1}y_0)}{\frac{y_0}{(-1+x_{-2}z_{-1}y_0)}}\right)}$$

$$= \frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)\left(-1+\frac{x_{-2}z_{-1}y_0}{(-1+x_{-2}z_{-1}y_0)}\right)} = x_{-2},$$

$$y_{6n-2} = \frac{y_{6n-5}}{-1+y_{6n-5}x_{6n-4}z_{6n-3}}$$

$$= \frac{\frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)}}{-1+\left(\begin{array}{c}\frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)}x_{-1}(-1+y_{-2}x_{-1}z_0)\\ \frac{z_0}{(-1+y_{-2}x_{-1}z_0)}\end{array}\right)}$$

$$= \frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)\left(-1+\frac{y_{-2}x_{-1}z_0}{(-1+y_{-2}x_{-1}z_0)}\right)} = y_{-2},$$

$$z_{6n-2} = \frac{z_{6n-5}}{-1+z_{6n-5}y_{6n-4}x_{6n-3}}$$

$$= \frac{\frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)}}{-1+\left(\begin{array}{c}\frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)}y_{-1}(-1+z_{-2}y_{-1}x_0)\\ \frac{x_0}{(-1+z_{-2}y_{-1}x_0)}\end{array}\right)}$$

$$= \frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)\left(-1+\frac{z_{-2}y_{-1}x_0}{(-1+z_{-2}y_{-1}x_0)}\right)} = z_{-2},$$

and so,

$$x_{6n-1} = \frac{x_{6n-4}}{-1+x_{6n-4}z_{6n-3}y_{6n-2}}$$

$$= \frac{x_{-1}(-1+y_{-2}x_{-1}z_0)}{-1+\left(x_{-1}(-1+y_{-2}x_{-1}z_0)\frac{z_0}{(-1+y_{-2}x_{-1}z_0)}y_{-2}\right)}$$

$$= \frac{x_{-1}(-1+y_{-2}x_{-1}z_0)}{(-1+x_{-1}z_0y_{-2})} = x_{-1},$$

$$y_{6n-1} = \frac{y_{6n-4}}{-1+y_{6n-4}x_{6n-3}z_{6n-2}}$$

$$= \frac{y_{-1}(-1+z_{-2}y_{-1}x_0)}{\left(-1+y_{-1}(-1+z_{-2}y_{-1}x_0)\frac{x_0}{(-1+z_{-2}y_{-1}x_0)}z_{-2}\right)}$$

$$= y_{-1},$$

$$z_{6n-1} = \frac{z_{6n-4}}{-1+z_{6n-4}y_{6n-3}x_{6n-2}}$$

$$= \frac{z_{-1}(-1+x_{-2}z_{-1}y_0)}{\left(-1+z_{-1}(-1+x_{-2}z_{-1}y_0)\frac{y_0}{(-1+x_{-2}z_{-1}y_0)}x_{-2}\right)}$$

$$= z_{-1}.$$

Also, we can prove the other relation. The proof is complete.

Theorem 4. The system (3) has a periodic solutions of period three iff $x_{-2}z_{-1}y_0 = z_{-2}y_{-1}x_0 = y_{-2}x_{-1}z_0 = 2$ and will take the form

$$\{x_n\} = \{x_{-2}, x_{-1}, x_0, x_{-2}, x_{-1}, x_0, \ldots\},$$
$$\{y_n\} = \{y_{-2}, y_{-1}, y_0, y_{-2}, y_{-1}, y_0, \ldots\},$$
$$\{z_n\} = \{z_{-2}, z_{-1}, z_0, z_{-2}, z_{-1}, z_0, \ldots\}.$$

Proof: First, suppose that there exists a prime period three solutions

$$\{x_n\} = \{x_{-2}, x_{-1}, x_0, x_{-2}, x_{-1}, x_0, \ldots\},$$
$$\{y_n\} = \{y_{-2}, y_{-1}, y_0, y_{-2}, y_{-1}, y_0, \ldots\},$$
$$\{z_n\} = \{z_{-2}, z_{-1}, z_0, z_{-2}, z_{-1}, z_0, \ldots\}.$$

of Eq.(3), we see from Eq.(3) that

$$x_{-2} = \frac{x_{-2}}{(-1+x_{-2}z_{-1}y_0)},$$

$$x_{-1} = x_{-1}(-1+y_{-2}x_{-1}z_0),$$

$$x_0 = \frac{x_0}{(-1+z_{-2}y_{-1}x_0)},$$

$$y_{-2} = \frac{y_{-2}}{(-1+y_{-2}x_{-1}z_0)},$$

$$y_{-1} = y_{-1}(-1+z_{-2}y_{-1}x_0),$$

$$y_0 = \frac{y_0}{(-1+x_{-2}z_{-1}y_0)},$$

and

$$z_{-2} = \frac{z_{-2}}{(-1+z_{-2}y_{-1}x_0)},$$
$$z_{-1} = z_{-1}(-1+x_{-2}z_{-1}y_0),$$
$$z_0 = \frac{z_0}{(-1+y_{-2}x_{-1}z_0)}.$$

Then,

$$x_{-2}z_{-1}y_0 = z_{-2}y_{-1}x_0 = y_{-2}x_{-1}z_0 = 2.$$

Second suppose is given by

$$x_{-2}z_{-1}y_0 = z_{-2}y_{-1}x_0 = y_{-2}x_{-1}z_0 = 2.$$

Then, we see from Eq.(4) that

$$x_{6n-2} = x_{-2},\quad x_{6n-1} = x_{-1},\quad x_{6n} = x_0,$$
$$x_{6n+1} = x_{-2},\quad x_{6n+2} = x_{-1},\quad x_{6n+3} = x_0,$$

$$y_{6n-2} = y_{-2},\quad y_{6n-1} = y_{-1},\quad y_{6n} = y_0,$$
$$y_{6n+1} = y_{-2},\quad y_{6n+2} = y_{-1},\quad y_{6n+3} = y_0,$$

and

$$z_{6n-2} = z_{-2},\quad z_{6n-1} = z_{-1},\quad z_{6n} = z_0,$$
$$z_{6n+1} = z_{-2},\quad z_{6n+2} = z_{-1},\quad z_{6n+3} = z_0.$$

Thus, we have a period three solution and the proof is complete.

Example 4. Figure (4) shows the behavior of the solution of the difference system (3) with the initial condition $x_{-2} = 2,\ x_{-1} = 1.4,\ x_0 = -1.5,\ y_{-2} = 0.9,$ $y_{-1} = -0.17,\ y_0 = -0.3,\ z_{-2} = 1.3,\quad z_{-1} = 0.7$ and $z_0 = 0.6.$

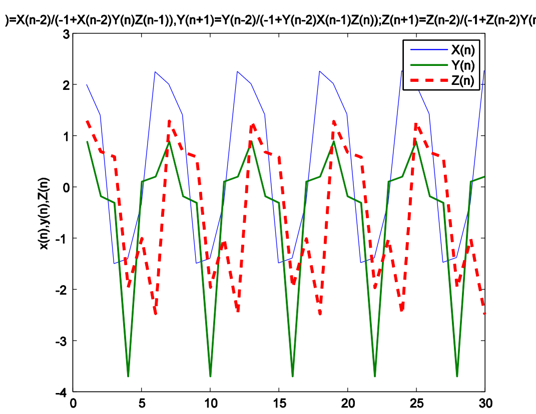

Figure (4)

Example 5. It can be seen from Figure (5) the behavior of the solution of the difference system (3) with the initial conditions $x_{-2} = 2, x_{-1} = 14,$ $x_0 = -1.5,\ y_{-2} = 0.1,\quad y_{-1} = -7,\quad y_0 = 3,$ $z_{-2} = 4/21,\ z_{-1} = -1/3$ and $z_0 = 10/7.$

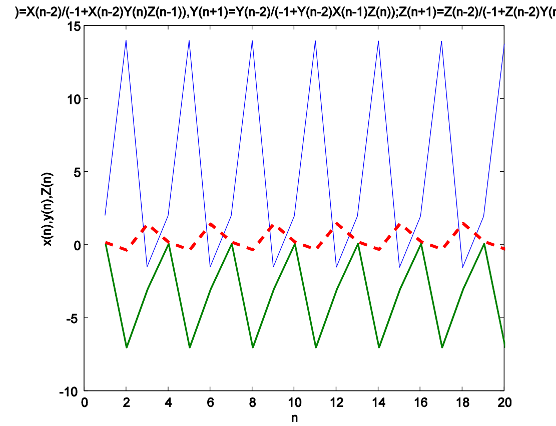

Figure (5)

5 The system: $x_{n+1} = \frac{x_{n-2}}{-1-y_nz_{n-1}x_{n-2}},\ y_{n+1} = \frac{y_{n-2}}{-1-z_nx_{n-1}y_{n-2}},\ z_{n+1} = \frac{z_{n-2}}{-1-x_ny_{n-1}z_{n-2}}$

In this section, we study the solutions of the following system of the difference equations

$$x_{n+1} = \frac{x_{n-2}}{-1-y_nz_{n-1}x_{n-2}},\ y_{n+1} = \frac{y_{n-2}}{-1-z_nx_{n-1}y_{n-2}},\quad (4)$$
$$z_{n+1} = \frac{z_{n-2}}{-1-x_ny_{n-1}z_{n-2}},$$

where $n \in \mathbb{N}_0$ and the initial conditions are arbitrary non zero real numbers with $x_{-2}z_{-1}y_0 \neq -1,\ z_{-2}y_{-1}x_0 \neq -1,$ and $y_{-2}x_{-1}z_0 \neq -1.$

Theorem 5. Assume that $\{x_n, y_n, z_n\}$ are solutions of system (4). Then, for $n = 0,1,2,...$ we see that every solutions are periodic with period six and

$$x_{6n-2} = x_{-2},\quad x_{6n-1} = x_{-1},\quad x_{6n} = x_0,$$
$$x_{6n+1} = \frac{x_{-2}}{(-1-x_{-2}z_{-1}y_0)},$$
$$x_{6n+2} = x_{-1}(-1-y_{-2}x_{-1}z_0),\quad x_{6n+3} = \frac{x_0}{(-1-z_{-2}y_{-1}x_0)},$$
$$y_{6n-2} = y_{-2},\quad y_{6n-1} = y_{-1},\quad y_{6n} = y_0,$$
$$y_{6n+1} = \frac{y_{-2}}{(-1-y_{-2}x_{-1}z_0)},$$
$$y_{6n+2} = y_{-1}(-1-z_{-2}y_{-1}x_0),\quad y_{6n+3} = \frac{y_0}{(-1-x_{-2}z_{-1}y_0)},$$

and

$$z_{6n-2}=z_{-2},\ z_{6n-1}=z_{-1},\quad z_{6n}=z_0,$$

$$z_{6n+1}=\frac{z_{-2}}{(-1-z_{-2}y_{-1}x_0)},$$

$$z_{6n+2}=z_{-1}(-1-x_{-2}z_{-1}y_0),\ z_{6n+3}=\frac{z_0}{(-1-y_{-2}x_{-1}z_0)}.$$

Proof: As the proof of Theorem 3 and so will be omitted.

Theorem 6. The system (4) has periodic solutions of period three iff $x_{-2}z_{-1}y_0=z_{-2}y_{-1}x_0=y_{-2}x_{-1}z_0=-2$ and will take the form

$$\{x_n\}=\{x_{-2},x_{-1},x_0,x_{-2},x_{-1},x_0,...\},$$
$$\{y_n\}=\{y_{-2},y_{-1},y_0,y_{-2},y_{-1},y_0,...\},$$
$$\{z_n\}=\{z_{-2},z_{-1},z_0,z_{-2},z_{-1},z_0,...\}.$$

Proof: As the proof of Theorem 4 and so will be omitted.

Example 6. See Figure (6) when we put the initial conditions
$x_{-2}=0.2,\ x_{-1}=0.4,\ x_0=-1.5,\ y_{-2}=0.5,$
$y_{-1}=0.7,\ y_0=3,\ z_{-2}=4,\ z_{-1}=1/3$ and $z_0=2$
for the difference system (4).

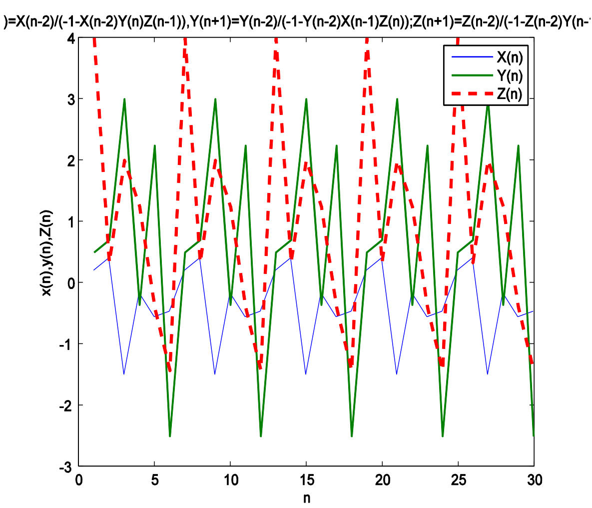

Figure (6)

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