A theoretical analysis of large scale water pollution of oceans and lakes

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Abstract: Water contamination (sea, ocean) has been a seriously challenge for decades. This paper provides a mathematical framework for understanding water pollution. We make use of a hydrodynamic advection dispersion to fully describe the phenomenon. The equation is solved via two analytical technique; the homotopy decomposition method and the differential transform method. The numerical simulations of the approximated solutions are presented. [Oukouomi Noutchie, SC. **A theoretical analysis of large scale water pollution of oceans and lakes**. *Life Sci J* 2013;10(3):615-621] (ISSN:1097-8135). http://www.lifesciencesite.com. 91

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1. Introduction [2]

. Water pollution is the contamination of water bodies (e.g. lakes, rivers, oceans, aquifers and groundwater). Water pollution occurs when pollutants are directly or indirectly discharged into water bodies without adequate treatment to remove harmful compounds. Water pollution affects plants and organisms living in these bodies of water. In almost all cases the effect is damaging not only to individual species and populations, but also to the natural biological communities. Water is typically referred to as polluted when it is impaired by anthropogenic contaminants and either does not support a human use, such as drinking water, and/or undergoes a marked shift in its ability to support its constituent biotic communities, such as fish. Natural phenomena such as volcanoes, algae blooms, storms, and earthquakes also cause major changes in water quality and the ecological status of water.

Fig.1 A polluted river draining an abandoned copper mine on Anglesey[2].

2. Mathematical formulation

In recent years functional derivatives has been used to model physical and engineering processes. Areas of considerable interest include electrical networks, control theory of dynamical systems, probability and statistics, electrochemistry of corrosion, chemical physics, optics, engineering, acoustics, material science and signal processing. Space - time fractional differential equations obtained by replacing the first order time derivative and/or second-order space derivative in the standard diffusion equation by a generalized derivative of fractional order, respectively, were successfully used for modelling relevant physical processes [1-5]. These fractional diffusion equations arise quite naturally in continuous-time random walks. In this paper we extend the analysis by inserting fractional convection and fractional heat loss into the heat equations. Fractional derivatives may be introduced by different definitions. We consider the Caputo time fractional derivative for the diffusion convection equation with lateral heat loss

(1.1)

$$\begin{cases} D_t^{\alpha(x,t)}\mathrm{u}(x,t) = D_{xx}\mathrm{u}(x,t) + D_x\mathrm{u}(x,t) + u(x,t) \\ \quad 0 < \alpha(x,t) \le 1 \end{cases}$$

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We introduce the Crank–Nicholson scheme [11] as follows. Firstly, the discretization of first and second order space derivative is stated as:

$$\frac{\partial u}{\partial x}=\frac{1}{2}\left(\left(\frac{u(x_{l+1},t_{k+1})-u(x_{l-1},t_{k+1})}{2(h)}\right)+\left(\frac{u(x_{l+1},t_k)-u(x_{l-1},t_k)}{2(h)}\right)\right)+O(h)$$

(2.1)

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{2}\left(\left(\frac{u(x_{l+1},t_{k+1})-2u(x_l,t_{k+1})+u(x_{l-1},t_{k+1})}{(h)^2}\right)+\left(\frac{u(x_{l+1},t_k)-2u(x_l,t_k)+u(x_{l-1},t_k)}{(h)^2}\right)\right)+O(h^2)$$

$$u=\frac{1}{2}\left(u(x_l,t_{k+1})+u(x_l,t_k)\right)$$

The Crank–Nicholson scheme for the time fractional diffusion with convection and lateral heat loss model can be stated as follows:

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$$\frac{\partial^{\alpha_l^{k+1}} u(x_l, t_{k+1})}{\partial t^{\alpha_l^{k+1}}} = \frac{\tau^{-\alpha_l^{k+1}}}{\Gamma(2-\alpha_l^{k+1})}\Bigg(u(x_l, t_{k+1}) - u(x_l, t_k) + \sum_{j=1}^{k}\left[u(x_l, t_{k+1-j}) - u(x_l, t_{k-j})\right]\left[(j+1)^{1-\alpha_l^{k+1}} - (j)^{1-\alpha_l^{k+1}}\right] + \sum_{j=1}^{k}\left[u(x_{l+1}, t_{k+1-j}) - u(x_{l-1}, t_{k-j})\right]\left[(j+1)^{2+k} - (j)^{1+2k}\right]\Bigg).$$

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$$\left[\frac{S\tau^{-\alpha_l^{k+1}}}{\Gamma(2-\alpha_l^{k+1})}\left(u(x_l, t_{k+1}) - u(x_l, t_k) + \sum_{j=1}^{k}\left[u(x_l, t_{k+1-j}) - u(x_l, t_{k-j})\right]\left[(j+1)^{1-\alpha_l^{k+1}} - (j)^{1-\alpha_l^{k+1}}\right]\right)\right]$$
$$= T\left[\frac{1}{2}\left(\left(\frac{u(x_{l+1}, t_{k+1}) - 2u(x_l, t_{k+1}) + u(x_{l-1}, t_{k+1})}{(h)^2}\right) + \left(\frac{u(x_{l+1}, t_k) - 2u(x_l, t_k) + u(x_{l-1}, t_k)}{(h)^2}\right)\right)\right]$$
$$+ \frac{1}{r_l}\left[\frac{1}{2}\left(\left(\frac{u(x_{l+1}, t_{k+1}) - u(x_{l-1}, t_{k+1})}{2(h)}\right) + \left(\frac{u(x_{l+1}, t_k) - u(x_{l-1}, t_k)}{2(h)}\right)\right)\right]$$
$$+ \frac{3}{r_l}\left[\frac{1}{2}\left(\left(\frac{u(x_l, t_{k+1}) - u(x_l, t_{k+1})}{2(h)}\right) + \left(\frac{u(x_l, t_k) - u(x_l, t_k)}{2(h)}\right)\right)\right]$$
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Next we set

$b_j^{l,k+1} = (j+1)^{1-\alpha_l^{k+1}} - (j)^{1-\alpha_l^{k+1}}$; $T_l^{k+1} = \frac{\Gamma(2-\alpha_l^{k+1})\tau^{\alpha_l^{k+1}}}{Sh^2}T$; $G_l^{k+1} = \frac{\Gamma(2-\alpha_l^{k+1})\tau^{\alpha_l^{k+1}}}{Sh}$

and $\lambda_j^{l,k+1} = b_{j-1}^{l,k+1} - b_j^{l,k+1}$

Equation (2.3) becomes:

$$u_l^{k+1}(1+2T_l^{k+1}) = u_{l+1}^{k+1}\left(T_l^{k+1} + \frac{G_l^{k+1}}{r_l}\right) + u_{l-1}^{k+1}\left(T_l^{k+1} - \frac{G_l^{k+1}}{r_l}\right) + u\left(T_l^{k+1} - \frac{G_l^{k+1}}{r_l}\right) + u_l^{k+1}(1+2T_l^{k+1}) + \sum_{j=1}^{k}\left(u_l^{k+1-j} - u\right)\lambda_j^{l,k+1} G_l^{k+1}$$

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$$u=\frac{1}{2}\left(u(x_l,t_{k+1})+u(x_l,t_k)\right)$$

The Crank–Nicholson scheme for the time fractional diffusion with convection and lateral heat loss model can be stated as follows:

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$$\frac{\partial^{\alpha_l^{k+1}}u(x_l,t_{k+1})}{\partial t^{\alpha_l^{k+1}}}=\frac{\tau^{-\alpha_l^{k+1}}}{\Gamma\left(2-\alpha_l^{k+1}\right)}\left(u(x_l,t_{k+1})-u(x_l,t_k)+\sum_{j=1}^{k}\left[u(x_l,t_{k+1-j})-u(x_l,t_{k-j})\right]\left[(j+1)^{1-\alpha_l^{k+1}}-(j)^{1-\alpha_l^{k+1}}\right]+\sum_{j=1}^{k}\left[u(x_{l+1},t_{k+1-j})-u(x_{l-1},t_{k-j})\right]\left[(j+1)^{2+k}-(j)^{1+2k}\right]\right).$$

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3. Mathematical formulation

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$$
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$$

$$
\left.\left.+\sum_{j=1}^{k}\left[u\left(x_l,t_{k+1-j}\right)-u\left(x_l,t_{k-j}\right)\right]\left[(j+1)^{1-\alpha_l^{k+1}}-(j)^{1-\alpha_l^{k+1}}\right]\right)\right]
$$

$$
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$$

$$
+\frac{1}{r_l}\left[\frac{1}{2}\left(\left(\frac{u\left(x_{l+1},t_{k+1}\right)-u\left(x_{l-1},t_{k+1}\right)}{2(h)}\right)+\left(\frac{u\left(x_{l+1},t_k\right)-u\left(x_{l-1},t_k\right)}{2(h)}\right)\right)\right]
$$

$$
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$$

$$
\frac{\partial^2 u}{\partial x^2}=\frac{1}{2}\left(\left(\frac{u\left(x_{l+1},t_{k+1}\right)-2u\left(x_l,t_{k+1}\right)+u\left(x_{l-1},t_{k+1}\right)}{(h)^2}\right)+\left(\frac{u\left(x_{l+1},t_k\right)-2u\left(x_l,t_k\right)+u\left(x_{l-1},t_k\right)}{(h)^2}\right)\right)+O\left(h^2\right)
$$

4. Conclusion

The Crank–Nicholson scheme for the time fractional diffusion with convection and lateral heat loss model can be stated as follows:

Areas of considerable interest include electrical networks, control theory of dynamical systems, probability and statistics, electrochemistry of corrosion, chemical physics, optics, engineering, acoustics, material science and signal processing. Space - time fractional differential equations obtained by replacing the first order time derivative and/or second-order space derivative in the standard diffusion equation by a generalized derivative of fractional order, respectively, were successfully used for modelling relevant physical processes.

5. References

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