Inventory Control System For Determining Optimal; Quantity, Cost And Cycle Time Under Retroactive Holding Cost.

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Abstract: This work addresses the problem of inventory management of an organization. The objectives of this study includes: (a) to model an inventory as a Retroactive Holding Cost problem, (b) to determine optimal order quantity, optimal total inventory cost and cycle time using Retroactive solution algorithm. We use real data (processing times, random yield factors, etc) from a poultry feed manufacturing company, providing simultaneously the model validation and the evaluation of the relative performance of the company. The holding cost per unit of the item per unit time is assumed to be an increasing step function of the time spent in storage. Retroactive holding cost increase as a time-dependent holding cost increase step function model is considered. Procedures were used for determining the optimal order quantity and the optimal cycle time using retroactive solution algorithm.

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1. Introduction

Any stored resource used to satisfy a current or future need (raw materials, work-in-process, finished goods, etc.) is what we called inventory. Inventory has been defined as idle resources that possess economic value (Monks, 1987). Usually, it is an important component of the investment portfolio of any productive system. Literature reveals that sometimes up to 60% of the annual production budget is spent on material and other inventories (Lucey, 1988; Datta, 1989) It cannot be overemphasized that better inventory management would invariably improve organizational profitability, reduce costs and lead to prudent use of scarce capital (Wemmerlov, 1982). We do not know since when ants, squirrels, rats etc are keeping inventories of their food supplies. And we do not know how they learned to keep an account of these inventories. Not only wildlife but also humans have been smart enough to realize the importance of inventories. Since stone-ages we have been keeping inventories and controlling them. But, the development of modern inventory management principles began when Harris (1913) derived the Economic Order Quantity (EOQ) formula. EOQ assumes that demand occurs at known, constant rate and supply fulfils the replenishment order after a fixed lead time. Unfortunately, the real world is not as ideal as that. In reality, demand rate is rarely constant; hard-to-predict market is common in most practical situations.

The occurrence of stock out in an inventory system is a phenomenon in real life situation. These situations are common, and answers one gets from deterministic analysis very often are not satisfactory when uncertainty is present. The decision maker faced with uncertainty does not act in the same way as the one who operates with perfect knowledge of the future. If the problem of inventory exists, then there are two main questions which generally arise and face any organization, namely: *how many to order* and *when to order*. Answering these two questions will lead to the optimal level of inventory for any organization, which minimizes its total inventory cost, while meeting the requisite requirements.

Due to ranging peculiarities of the production inventory, no particular inventory model has general application to the entire variants inventory situations. Consequently, a variety of inventory models have emerged, which address specific inventory problems (Trigg and Pitts, 1962; Naddor, 1966; Lev *et al.*, 1981; Silver, 1981; Harvey, 1987; Tersine, 1994). Usually, the problem is that of balancing the costs of less than adequate inventory (Under-stocking) and that of cost of more than adequate inventory (Over-stocking). The goal is to have adequate items at all times at minimal cost (Taha, 1982; Datta, 1989; Harris, 1990). Solution methods used to solving these problems are basically analytical techniques and the sophisticated application of mathematical programming. However, the mathematical complexity of the resulting models increases as we move away from the assumption of deterministic to probabilistic non-stationary demand (Taha, 1982; Lev and Soyster, 1979). Silver (1981) reviewed many classifications of the inventory problem, highlighting the limitations, while also advocating the bridging of the gap between theory and practice.

In recent years, substantially more research has been done on demand variation in response to inventory level assume that the holding cost is constant for the whole inventory cycle. In this paper, we also presents an inventory model with a stock-level dependent demand rate and a variable holding cost, specifically using retroactive holding cost model. Inventory level for time dependent holding cost and variable demand rate is described as a retroactive cost problem. Our main objectives for this paper are to:

- Model poultry feed inventory cost as retroactive holding cost problem.
- Determine optimal order quantity and optimal total cost and cycle time using Retroactive solution Algorithm.

This article is organised as follows: **Section1-** introduces inventory and early model. **Section 2-** is presented to review the related literature, focusing on the issues of inventory control using stock-level dependent demand rate and variable holding cost. **Section 3-** introduces the formulation of retroactive holding cost model focusing on three cost effects. That is ordering cost, holding cost and product cost. **Section 4-** describes the data and the numerical computation using the solution algorithms. The results of the numerical computations for optimal quantity, the cycle time is discussed are presented in **Sections 5**.

2. Related Works

If we observe closely, inventories can be found everywhere. We don't know since when ants and squirrels are keeping inventories of their food supplies. And we do not know how they learned to keep an account of these inventories. Not only wildlife but also humans have been smart enough to realize the benefits of inventories.

Various models have been proposed for stock-level dependent inventory system. Baker and Urban (1988a) investigated a deterministic system in which the demand rate dependence on the inventory level is described by a polynomial function. A non-linear programming algorithm is utilized to determine the optimal order size and recorder point. Urban (1995) investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and instantaneous stock. Urban formulated a profit-maximizing model and develops a closed-form solution.

Many authors have also investigated inventory system with a two-stage demand rate. Baker and Urban (1988b) considered an inventory system with an initial period of level-dependent demand followed a period of level-dependent demand. The analysis conducted on this model imposes a terminal condition of zero inventories at the end of the order cycle.

Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et.al (1996) investigated a deterministic inventory system in which shortage are allowed and fully back logged. The demand is stock dependent to certain level and then constant for the remaining periods. A flow chart is provided to solve the general solution.

One of the terminal conditions used in the development of the Datta and Pal model was that the inventory level fall to zero at the end of the order cycle (i.e. $i = 0 \ when t = T$). In an inventory system that possesses an inventory-level-dependent demand rate, this may not provide the optimal solution. It may be desirable to order large quantities, resulting in stock remaining at the end of the cycle, due to the potential profits resulting from the increased demand. This phenomenon is discussed in Baker and Urban.

Hon (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time-discounting over a finite planning horizon, where the solution is obtained by minimizing the total cost function. In this connection mention may be made of the work of Kabak and Weinberg (1972), which is an extension of classical newsboy problem considering supply as a random variable, but newsboy suffers no decrease in expected revenue.

Goh (1994) apparently provided the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a non-linear function of storage time and (b) a non-linear function of storage level. Alfares H.K (2007) presented different functional form of holding cost time dependence. Two types of discontinuous steps functions were considered. The storage time was divided into a number of distinct periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods) or incrementally (to all new periods).

The methodology for this work was based on inventory models with stock-level dependent demand rate variable holding cost in the International Journal of Production Economics Research 108 (2008) 259-265 , (Alfares H.K (2007)).

3. Formulation of model

3.1Notations:

The following notations are adopted from Goh (1992) for the model under consideration for inventory system.

$q(t)$: the on-hand inventory at time, t

D : constant demand rate

p : purchase cost per unit

n : number of distinct time periods with different holding costs rate

$h(t)$: holding cost of the item at time, t; $h(t) = h_i$ if $t_{i-1} \le t \le t_i$

t : time from the start of the cycle at $t = 0$

k : ordering cost per order

t_i : end time of the period i, where $i = 1,2,3,..n,\ t_0 = 0$

h_i : holding cost of the item in period i

β : the demand parameter indicating elasticity in relation to the inventory level

T : cycle time

3.2 Assumption and limitation

The following assumption and limitation are considered for the retroactive holding cost model:

- A single item is considered
- Replenishment Q is instantaneous
- Shortages are not allowed
- The holding
- The demand rate satisfies $R_1(q_1) \le R_2(q_2) \le R = R_i(q_i)$
- The dependence of the demand rate R on the inventory level (q) is express: $R(q) = Dq^{\beta}, \quad D > 0, \quad 0 < \beta < 1,$ $q \ge 0$ where $q =$ inventory level

A major problem of inventory is how we can establish optimal stock levels and this is difficult because of the uncertainty of supply and demand for the commodity. Using inventory models we could formulate policies to control the system.

In some cases such as retailer, wholesaler distributor, where items are purchased externally, if the problem of inventory exists, then there are two main questions, which generally arise and face any organization. These are how many to order and when to order. Having too much inventory reduces both purchase and /or ordering costs, but it may tie up capital, which may lead to unnecessary holding cost and possibility of deteriorating items.

Whereas having too little inventory reduces the holding cost, but it can result in lost of customers, which may affect the reliability of the organization.

Answering these two questions will lead to the optimal level of inventory for any organization, which minimizes its total inventory cost.

Inventory costs, which are related to the operation of an inventory system, are caused by the actions or lack of actions that the organization is establishing.

The most common costs to an inventory system may include:

- The purchase cost of an item obtained from an external source.
- The order cost of issuing a purchase order to an outside source.
- The holding /carrying cost for keeping items in storage.

The holding cost step function

The holding cost (h_i) for successive periods is assumed to be an increasing step function of the storage period (i) satisfying:

$h_1 < h_2 < h_3 < h_i ... < h_n$ (Retroactive increase)

The ordering cost per cycle is given by $\frac{k}{T}$.

Product cost per cycle

$$= \frac{unit\ price \times demand\ per\ cycle}{T}$$

$$\text{The total holding cost per cycle} = \frac{1}{T}\int_0^T h(t)q(t)dt$$

Total Inventory Cost (T_{IC})

$$= product\ \text{cost} + ordering\ \text{cost} + holding\ \text{cost}$$

$$T_{IC} = \frac{p}{T} + \frac{k}{T} + \frac{1}{T}\int_0^T h(t)q(t)dt$$

The demand rate is equal to the rate of decrease of inventory level. Hence;

$$\frac{dq(t)}{dt} = -D\big(q(t)\big)^{\beta}$$

And integrating $\int_0^t q^{-\beta} dq = -Ddt$ where $0 \le t \le T$,

$$q(t) = \left[-D(1-\beta)t + Q^{(1-\beta)}\right]^{\frac{1}{(1-\beta)}}$$

Where $q(0) = Q$

Using $Q(T) = 0$ (no shortages are allowed)

$$q(T) = \left[-D(1-\beta)T + Q^{(1-\beta)}\right]^{\frac{1}{(1-\beta)}} = 0$$

And

$$T = \frac{Q^{(1-\beta)}}{D(1-\beta)}$$

With

$$Q = \left[-D(1-\beta)T + q^{(1-\beta)}\right]^{\frac{1}{(1-\beta)}}$$

Combining $q(t)$ and T_{IC};

$$T_{IC} = \frac{p}{T} + \frac{k}{T} + \frac{1}{T}\int_0^t \left[-D(1-\beta)t + Q^{(1-\beta)}\right]^{\frac{1}{(1-\beta)}} dt$$

Therefore the formula for optimal total inventory cost is given by:

$$T_{IC}^{*} = \frac{D(p+k)(1-\beta)}{Q^{(1-\beta)}} + \frac{Qhi(1-\beta)}{(2-\beta)}$$

Where $t_{i-1} \le T \le t_i$

And

$$Q^{*} = \left[\frac{D(p+k)(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$$

3.3 Solution Algorithm for Retroactive Holding Cost Increase

Step1 (i) Begin with the lowest holding cost using

$$Q^{*} = \left[\frac{D(p+k)(1-\beta)(2-\beta)}{h_i}\right]^{\frac{1}{(2-\beta)}}$$

, $t_{i-1} \le T \le t_i$

(ii) Use Q to determine $T = \frac{Q^{(1-\beta)}}{D(1-\beta)}$ for each h_i values. If $t_{i-1} \le T \le t_i$ then are realizable otherwise, discard Q, T and continue with $i+1 \rightarrow i$ call these values Q_R and T_R

Step2. For each t_i, put $T = t_i$ and use $Q = [D(1-\beta)]^{\frac{1}{(1-\beta)}}$ to calculate all break-point values of Q, $Q_i = Q(t_i)$, $t_1 \le T < T_R$.

Select $\min Q_i$ and add to Q_R realizable(s) and call them Q_S.

Step 3 Substitute Q_S into

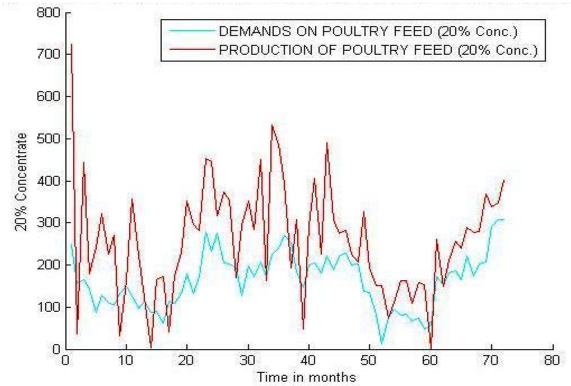
$$T_{IC}^{*} = \frac{D(p+k)(1-\beta)}{Q^{(1-\beta)}} + \frac{Qhi(1-\beta)}{(2-\beta)},$$

$t_{i-1} \le T \le t_i$ and calculate T_{IC} for each Q_S.

Step 4 Choose the value of Q that gives the lowest T_{IC} and the corresponding T is the cycle time.

4. Discussions

The production and demand data was studied at poultry feed company with retroactive holding cost model for a particular feed product from 2008 to 2012 and the trajectory is:



The trajectories of Production and Demand data of Poultry Company are both periodic and are linearly related. The quantity of poultry feed ordered from the company at any time t depends on the number of poultry feeds produced. The higher the poultry feeds that are produced in the company, the higher the demands and vice-versa. The optimal total inventory cost often depend on the availability of raw materials, type of feed formulation, lead time, environmental conditions that is festivity periods that dictate the amount of items demanded by customers.

Conclusion

- A Retroactive Holding Cost Model was used to model the 20% concentrate poultry feed of poultry feed company.
- With the model, the optimal order quantity Q^* which minimises the Total Inventory Cost (TIC) of 20% concentrate poultry feed was determined to be 633 tonnes. The cycle period T* for the quantity Q^* to be produced per cycle is 0.25years.

- The company could release 633units per each order within the cycle period of 0.25 year. This would minimise the Total Inventory Cost (TIC) to R4.4326e+003.
- It is clear that both the optimal order quantity and the cycle time experienced a reduction I value when the holding cost rises in value whenever the As the demand parameter β increases resulting in increasing the optimal order quantity and the cycle time decreases.
- The model can also be modelled to cater for variable demand where the hold cost is fixed and so on.

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