Effects of Thermal Radiation and Hall Current on MHD Free Convection over a Vertical Plate with Thermal Diffusion

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Abstract: Simultaneous heat and mass transfer in unsteady free convection flow with thermal radiation and thermal diffusion past an impulsively started infinite vertical porous plate subjected to a strong magnetic field is presented. The dimensionless governing equations for this investigation are solved analytically using two terms harmonic and non-harmonic functions. The influence of various parameters on the convectively cooled or convectively heated plate in the laminar boundary layer are established. An analysis of the effects of the parameters on the concentration, velocity and temperature profiles, as well as skin friction and the rates of mass and heat transfer is done with the aid of graphs and tables.

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1. Introduction

In recent years, a great effort has been made in studying the heat and mass transfer in magnetohydrodynamics (MHDs) flow due to its application in many devices, like the MHD power generator and Hall accelerator. The influence of a magnetic field on the flow of an electrically conducting viscous fluid with mass transfer and radiation absorption is also useful in planetary atmosphere research [1].

Ram et al. [2] studied the heat and mass transfer of a viscous heat generating fluid with Hall current. Takhar et al. [3] investigated the hydromagnetic convective flow of a heat generating fluid past a vertical plate with Hall current and heat flux through a porous medium. Ram [4] used a finite difference scheme for solving the MHD Stokes problem for a vertical plate with Hall and ion slip currents. In [5], Kinyanjui et al. investigated the MHD Stokes problem for a vertical infinite plate in a dissipative rotating fluid with Hall current. Kinyanjui et al. [6] studied the MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption.

The effect of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and a knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Abdus Sattar and Hamid Kalim [7] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Muthucumaraswamy and Ganesan [8] studied the effect of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka et al. [9] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamkha [10] studied the problem of MHD flow of an uniformally stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Soundalgekar and Patti [11] studied the problem of the flow past an impulsively started isothermal infinite vertical plate with mass transfer effects. Chamkha [12] assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Raptis [13] investigated the steady flow of a viscous fluid through a very porous medium bounded by a porous plate subjected to a constant suction velocity in the presence of thermal radiation. Raptis and Perdikis [14] studied the unsteady free convection flow of water near 4^{o} C in the laminar boundary layer over a vertical moving porous plate. Ibrahim et al. [15] studied the effects of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Recently, Bakr [16, 17] investigated the problem of free convection heat and mass transfer adjacent to moving vertical porous infinite plate for incompressible, micropolar fluid in a rotating frame of reference in the presence of heat generation or absorption effects and a first-order chemical reactions.

The spite of all these studies, the unsteady MHD free convection heat and mass transfer for a heat generating fluid with radiation absorption has received little attention. Hence, the main objective of the present investigation is to study the effects of thermal radiation, mass diffusion, Hall current, chemical reaction, Soret effect and heat source parameter of heat generating fluid an impulsively started infinite vertical porous plate subjected to constant suction. It is also assumed that temperature over which are superimposed an exponentially varying with time.

2. Mathematical Analysis

We consider unsteady three-dimensional flow of a laminar, viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical porous plate subjected to constant suction. The temperature of the fluid and the plate are assumed to be the same initially. At time $t^* > 0$, the porous plate starts moving impulsively in its own plane with a constant velocity U, and its temperature is instantaneously raised or lowered to T_w^* which is maintained constant thereafter. The x^*-axis is taken along the plate in the vertically upward direction and the z^*-axis is taken normal to the plate. A uniform transverse magnetic field H_0 is imposed along the z^*-axis, and the plate is taken to be electrically non-conducting. The induced magnetic field is negligible for partially ionized fluids, and so the the magnetic field strength vector is given by H = (0, 0,H_0). The equation of conservation of electric charger. $\nabla J = 0$ gives $j_z^* =$ constant where J = (j_x^*, j_y^*, j_z^*) is the current density vector, j_x^* , j_y^* and j_z^* are the components of the current density along x^*, y^* and z^* directions. This constant is assumed to be zero, since $j_z^* =$ constant at the plate, which is assumed to be electrically non-conducting. Thus $j_z^* = 0$ everywhere in the flow. The chemical reactions are taking place in the flow and all thermophysical properties are assumed to be constant of the linear momentum equation which is approximated according to the Boussinesq approximation. Due to the semi-infinite plane surface assumption, the flow variables are functions of z^* and the time t^* only. Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial w^*}{\partial z^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} = \upsilon \frac{\partial^2 u^*}{\partial z^{*2}} + g_0 \beta (T - T_\infty) + g_0 \beta^* (C - C_\infty) + \frac{\mu_e H_0}{\rho} j_y^* \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} = \upsilon \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\mu_e H_0}{\rho} j_y^* \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial z^*} - \frac{Q_0}{\rho C_p}(T^* - T_\infty) \tag{4}$$

$$\frac{\partial C^*}{\partial t^*} + w^* \frac{\partial C^*}{\partial z^*} = D_m \frac{\partial^2 C^*}{\partial z^{*2}} + D_T \frac{\partial^2 T^*}{\partial z^{*2}} - k_l \left(C^* - C_\infty\right) \tag{5}$$

The initial and boundary conditions for the model are: For $t^* < 0$

$$u^*(z^*, t^*) = 0, v^*(z^*, t^*) = 0, \quad T^*(z^*, t^*) = T_\infty^*, C^*(z^*, t^*) = C_\infty^* \tag{6}$$

and for $t^* > 0$

$$\begin{aligned} & u^*(0, t^*) = U, v^*(0, t^*) = 0, \\ & T^*(0, t^*) = T_w^* + \varepsilon (T_w^* - T_\infty^*) e^{n^* t^*}, \\ & u^*(\infty, t^*) = 0,\ C^*(0, t^*) = C_w^* + \varepsilon (C_w^* - C_\infty^*) e^{n^* t^*}, \\ & v^*(\infty, t^*) = 0,\ T^*(\infty, t^*) = T_\infty^*,\ C^*(\infty, t^*) = C_\infty^* \end{aligned} \tag{7}$$

In the above equations, u^* *and* w^* are the components of dimensional velocities along x^* *and* z^* axis respectively. β and β^* are the coefficient of volume expansion and volume expansion with concentration, C_p is the specific heat at constant pressure, ρ is the density of the fluid, υ is the kinematics viscosity, g_0 is the acceleration of gravity, T^* is the dimensional temperature, μ_e is magnetic permeability, H is magnetic field intensity, $J = (j_x^*, j_y^*, j_z^*)$ is current density, D_m is the molecular diffusivity, D_T is the coefficient of thermal diffusivity, k is the thermal conductivity of the fluid, k_l is the reaction rate constant, C^* is the dimensional concentration, C_w and T_w are the concentration and temperature at the wall, respectively. C_∞ and T_∞ are the free stream dimensional concentration and temperature, q_r^* is the local radiative heat flux. The term $Q_0(T^*-T_\infty^*)$ is assumed to be the amount of heat generated or absorbed per unit volume, Q_0 is a constant, which may take on either positive or negative values. When the wall temperature T^* exceeds the free stream temperature T_∞^*, the source term $Q_0>0$ and heat sink when $Q_0<0$. It is assumed that the temperature and concentration at the wall are exponentially varying with time. We now consider a convenient solution of equation (1) to be

$$w^* = -w_0 \tag{8}$$

When the strength of the magnetic field is very large, Ohm's law must be modified to include the Hall currents as follows:

$$J+\frac{w_e\tau_e}{H_0}(J\times H)=\sigma(E+\mu_e q\times H+\frac{1}{e\eta_e}\nabla P_e), \tag{9}$$

where w_e is the cyclotron frequency, e is the electron charge, P_e is the electron pressure, $q=(u^*,v^*,w^*)$, η_e is the number density of electrons, and τ_e is the collision time of electrons. In writing Eq. (9), the ion slip and thermoelectric effects are neglected, Cowling [17]. Further, for a short circuit problem, the applied electric field $E=0$, and for partially ionized gases, the electron pressure gradient may be neglected. Equating the x^* *and* z^* components in Eq. (8) and solving for the current density components j_x^* *and* j_y^*, we have

$$\begin{aligned} j_x^* &= \frac{\sigma\mu_e H_0}{1+m^2}(mu^*-v^*),\\ j_y^* &= \frac{\sigma\mu_e H_0}{1+m^2}(mv^*-u^*) \end{aligned} \tag{10}$$

where $m=w_e\tau_e$ is the Hall current parameter. On substituting (10) into (2) and (3) we obtain

$$\begin{aligned} \frac{\partial u^*}{\partial t^*}+w^*\frac{\partial u^*}{\partial z^*} &= \upsilon\frac{\partial^2 u^*}{\partial z^{*2}}+g_0\beta(T-T_\infty)\\ &+g_0\beta^*(C-C_\infty)+\frac{\mu_e^2H_0^2}{\rho(1+m^2)}(mu^*-v^*) \end{aligned} \tag{11}$$

$$\frac{\partial v^*}{\partial t^*}+w^*\frac{\partial v^*}{\partial z^*}=\upsilon\frac{\partial^2 v^*}{\partial z^{*2}}-\frac{\mu_e^2H_0^2}{\rho(1+m^2)}(mv^*+u^*) \tag{12}$$

The quantity q_r^* in the right-hand side of equation (4) represents the radiative heat flux in the z^* direction. In order to simplify the physical problem, the optically thick radiation limit is considered in the present analysis. Thus the radiative heat flux term is simplified by using the Rosseland diffusion approximation [18] for an optically thick fluid according to, where the constant w_0 represents the normal velocity at the plate which is positive for suction and negative for blowing. We now introduce the dimensionless variables, as follows:

$$q_r^*=\frac{4\sigma^*}{3k_e}\frac{\partial T^{*4}}{\partial z^*} \tag{13}$$

where σ^* and k_e are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then Eq. (13) can be linearized by expanding T^{*4} into the Taylor series about T_∞^* and neglecting higher order terms [19], we have,

$$T^{*4}\cong 4T_\infty^{*3}T^*-3T_\infty^{*4} \tag{14}$$

By using Eqs. (13) and (14), Eq. (4) gives

$$\begin{aligned} \frac{\partial T^*}{\partial t^*}+w^*\frac{\partial T^*}{\partial z^*} &= \left(\frac{k}{\rho C_p}+\frac{16\sigma^*T_\infty^{*3}}{3\rho C_p k_e}\right)\frac{\partial^2 T^*}{\partial z^{*2}}\\ &-\frac{Q_0}{\rho C_p}(T^*-T_\infty^*) \end{aligned} \tag{15}$$

We introduce the following dimensionless quantities:

$$\begin{aligned} &z=\frac{z^*U}{\upsilon},\ u=\frac{u^*}{U},\ v=\frac{v^*}{U},\ w=\frac{w^*}{U},\\ &t=\frac{t^*U^2}{\upsilon},\ n=\frac{n^*\upsilon}{U^2},\ \theta(z,t)=\frac{T-T_\infty}{T_w-T_\infty},\\ &\phi(z,t)=\frac{C-C_\infty}{C_w-C_\infty} \end{aligned} \tag{16}$$

In view of the above non-dimensional variables, the basic field equations (11),(12) and (15) can be expressed in non-dimensional form as

$$\frac{\partial Q}{\partial t}-w_0\frac{\partial Q}{\partial z}=\frac{\partial^2 Q}{\partial z^2}-\frac{M^2(1+m)}{1+m^2}Q+Gr\,\theta+Gm\,\phi, \tag{17}$$

$$\frac{\partial\theta}{\partial t}-w_0\frac{\partial\theta}{\partial z}=\frac{1}{\Pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial z^2}-\frac{\eta}{\Pr}\theta, \quad (18)$$

$$\frac{\partial\phi}{\partial t}-w_0\frac{\partial\phi}{\partial z}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial z^2}+S_0\frac{\partial^2\theta}{\partial z^2}-\delta\phi, \quad (19)$$

subject to the boundary conditions

$$\begin{array}{lllll} Q=1, & \theta=1+\varepsilon e^{nt}, & \phi=1+\varepsilon e^{nt} & on & z=0 \\ Q\to 0, & \theta\to 0, & \phi\to 0 & as & z\to\infty \end{array}, \quad (20)$$

where

$Q(z,t)=u(z,t)+iv(z,t)$, $Gr=\dfrac{g\beta\nu(T_w^*-T_\infty^*)}{U^3}$ is the Grashof number, $Gm=\dfrac{g_0\beta^*(C_w^*-C_\infty^*)\upsilon}{U^3}$ is the solutal Grashof number, $\Pr=\dfrac{\upsilon\rho c_p}{k}$ is the Prandtl number, $R=\dfrac{4\sigma^* T_\infty^{*3}}{k\,k_e}$ is the thermal radiation prameter, $M^2=\dfrac{\sigma\mu_e^2 H_0\upsilon}{\rho U^2}$ is the magnetic field parameter, $\delta=\dfrac{k_l\,\upsilon}{U^2}$ is the chemical reaction parameter, $Sc=\dfrac{\upsilon}{D_m}$ is the Schmidt number, $\eta=\dfrac{\upsilon Q_0}{\rho U^2}$ is the heat source parameter and $S_0=\dfrac{D_T(T_w^*-T_\infty^*)}{\upsilon(C_w^*-C_\infty^*)}$ is the Soret number, $N=\dfrac{M^2(1+im)}{1+m^2}$. The mathematical statement of the problem is now complete and embodies the solution of Equations (17)-(19) subject to boundary conditions (20)

3. Method of Solution

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we perform an asymptotic analysis by representing the linear velocity,

$$\begin{aligned} Q(z,t)&=Q_0(z)+\varepsilon e^{nt}Q_1(z)+0(\varepsilon^2),\\ \theta(z,t)&=\theta_0(z)+\varepsilon e^{nt}\theta_1(z)+0(\varepsilon^2),\\ \phi(z,t)&=\phi_0(z)+\varepsilon e^{nt}\phi_1(z)+0(\varepsilon^2). \end{aligned} \quad (21)$$

By substituting the above Eqs.(21) into Eqs.(17)-(20), equating the harmonic and non-harmonic terms, and neglecting the higher order of $0(\varepsilon^2)$, and simplifying we obtains the following pairs of equations for Q_0,θ_0,ϕ_0 and Q_1,θ_1,ϕ_1

$$Q_0''+w_0Q_0'-NQ_0=-Gr\theta_0-Gm\phi_0, \quad (22)$$

$$(3+4R)\theta_0''+3\Pr w_0\theta_0'-3\eta\theta_0=0, \quad (23)$$

$$\phi_0''+Sc\,w_0\phi_0'-\delta Sc\phi_0=-S_0\,Sc\,\theta_0'', \quad (24)$$

subject to the boundary conditions

$$\begin{array}{lllll} \overline{Q}_0=1, & \theta_0=1, & \phi_0=1 & at & z=0 \\ \overline{Q}_0=0, & \theta_0=0, & \phi_0=0 & as & z\to\infty \end{array}, \quad (25)$$

for O(1) equations, and

$$Q_1''+w_0Q_1'-(n+N)Q_1=-Gr\theta_1-Gm\phi_1 \quad (26)$$

$$(3+4R)\theta_1''+3\Pr w_0\theta_1'-3(\Pr n+\eta)\theta_1=0 \quad (27)$$

$$\phi_1''+Sc\,w_0\phi_1'-(n+\delta)Sc\phi_1=-S_0\,Sc\,\theta_1'', \quad (28)$$

with the boundary conditions

$$\begin{array}{lllll} \overline{Q}_0=0, & \theta_0=1, & \phi_0=1 & at & z=0 \\ \overline{Q}_0=0, & \theta_0=0, & \phi_0=0 & as & z\to\infty \end{array}, \quad (29)$$

where the prime denotes differentiation with respect to z. Without going into detail, the solutions of Eqs. (22)-(24) and (26)-(28) subject to Eqs. (25) and (29) can be obtained as:

$$\begin{aligned} Q(z,t)&=A_1e^{-R_5z}-Z_3e^{-R_1z}-Z_4e^{-R_3z}\\ &\quad+\varepsilon e^{nt}\{A_2e^{-R_6z}-Z_5e^{-R_2z}-Z_6e^{-R_4z}\}, \end{aligned} \quad (30)$$

$$\theta(z,t)=e^{-R_1z}+\varepsilon e^{nt-R_2z}, \quad (31)$$

$$\begin{aligned} \phi(z,t)&=(1-Z_1)e^{-R_3z}+Z_1e^{-R_1z}\\ &\quad+\varepsilon e^{nt}\{Z_2e^{-R_2z}+(1-Z_2)e^{-R_4z}\}. \end{aligned} \quad (32)$$

The exponential indices and the coefficients appearing in the Eqs. (30)-(32) are given in the Appendix. The physical quantities of interest are which the local wall shear stress τ_w^*, the local surface heat q_w and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e.,skin- friction) is given by:

$$\tau_w^*=\left[\mu\frac{\partial u^*}{\partial z^*}\right]_{z^*=0}=\rho\,U^2\,Q'(0), \quad (33)$$

therefore, the local friction factor C_f is given by

$$\begin{aligned} C_f&=\frac{\tau_w^*}{\rho\,U^2}=Q'(0)=-A_1R_5+Z_3R_1+Z_4R_3\\ &\quad+\varepsilon e^{nt}(-A_2R_6+Z_5R_2+Z_6R_4) \end{aligned}$$

Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer q_w^*. This is given by

$$q_w^* = -k\left(\frac{\partial T^*}{\partial z^*}\right)_{z^*=0} - \frac{4\sigma}{3k_e}\left(\frac{\partial T^*}{\partial z^*}\right)_{z^*=0} = -\left(k + \frac{16\sigma}{3k_e}\right)\left(\frac{\partial T^*}{\partial z^*}\right)_{z^*=0}, \qquad (34)$$

which can be written in dimensionless form as:

$$q_w^* = -\frac{k(T_w - T_\infty)U}{\upsilon}\left(1 + \frac{4R}{3}\right)\left(\frac{\partial \theta}{\partial z}\right)_{z=0}$$

The definition of the local Nusselt number is given by

$$Nu_x = -\frac{q_w^* U}{(T_w - T_\infty)}\frac{x}{k}$$ One can write

$$Nu\,\mathrm{Re}_x^{-1} = -\left(1 + \frac{4R}{3}\right)\left(\frac{\partial \theta}{\partial z}\right)_{z=0} = \left(1 + \frac{4R}{3}\right)(R_1 + \varepsilon\, e^{nt}\, R_2),$$

where $\mathrm{Re}_x = U_r x/\upsilon$ is the Reynolds number. The local Sherwood number is given by the expression

$$\frac{Sh}{\mathrm{Re}_x} = -\phi'(0) = (1 - Z_1)R_3 + Z_1 R_1 + \varepsilon e^{nt}((1 - Z_2)R_4 + Z_2 R_2),$$

4 Results and Discussion

In order to get a physical understanding of the problem and for the purpose of discussing the results, numerical calculations have been performed for the concentration, velocity, temperature, rate of mass transfer, skin friction and rate of heat transfer. The results are represented graphically in (Fig. 1-7) and in Table 1. The magnetic parameter $M^2 = 5$ signifies a strong magnetic field, the Prandtl number, $\mathrm{Pr} = 0.71$ corresponds to air, and $\mathrm{Pr} = 7$, as corresponds to water at room temperature. The Grashof number, $Gr > 0$, corresponds to cooling of the plate by free convection currents, and the Grashof number, $Gr < 0$, corresponds to heating of the plate by free convection currents.

The effects of Hall parameter m on the primary and secondary velocity profiles across the boundary layer are presented in Fig. 1. We can see that, an increasing in Hall parameter m leads to an increasing in the primary velocity and secondary velocity profiles. In addition, the curves show that the peak value of secondary velocity increases rapidly near the wall of the porous plate as Hall parameter m increases, and then decays to the relevant free stream velocity.

Figure 2 show the primary velocity (u), secondary velocity (v) and concentration profiles for different values of the Soret S_0. From this figure, we see that the primary and secondary velocity profiles increase with an increasing of S_0 from which we conclude that the fluid velocity rises due to greater thermal diffusion. Also, the concentration profiles increase significantly with an increasing of Soret number.

The effect of heat generation η on the primary velocity, secondary velocity, temperature and concentration profiles is shown in Fig. 3. Physically, the presence of heat absorption effect has the tendency to reduce the fluid velocity across the momentum boundary layer. This causes the thermal buoyancy effects to decrease which results in a net reduction in the fluid velocity. These behaviors are clearly seen close to the plate. Also, an increase in heat generation η causes an decrease in the temperature profile but leads to a increase in the concentration profile.

For different values of radiation parameter R, the primary velocity, secondary velocity, temperature and concentration profiles are plotted in Fig. 4. Here we find that, as the value of R increases the primary, secondary velocity and temperature increases but leads to a decrease in the concentration profile. Increasing in the radiation parameter R will increase the velocity inside the boundary layer and, therefore, increase the thermal boundary layer thickness.

The influences of chemical reaction parameter δ and diffusion parameter Sc on the primary, secondary velocity and concentration profiles across the boundary layer are presented in Figures 5 and 6. We see that, the primary velocity, secondary velocity and concentration distribution across the boundary layer decrease with increasing of δ and Sc.

From Figures 7 and 8, we observe that, an increase in suction velocity w0 or an increase in Prandtl number Pr leads to a decrease in the primary, secondary velocity and temperature profile. Increasing in Prandtl number will increase the concentration profile but, an increase in suction velocity w_0 leads to a decrease in the concentration profile.

From Table 1, we have the following observations: (i) An increase in the Hall parameter m leads to a increase in skin friction τ_x due to the primary velocity and causes an increase in skin friction τ_z due to the secondary velocity. (ii) An increasing So, Gm and R, τ_x and τ_z increases, whereas with increasing $w_0, \delta, \eta, Sc, \tau_x$ *and* τ_z decreases. (iii) The value of the local Nusselt number increases as the suction velocity parameter w_0, heat generation η and radiation parameter R increase. (iv) The value of the local Sherwood number decreases with increase in suction velocity parameter, the Soret S0 and heat generation η whereas reverse effect is seen by increasing chemical reaction parameter, radiation parameter R and the Schmidt number Sc.

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Appendix

$$R_1 = \frac{3\Pr w_0}{2(3+4R)}\left(1+\sqrt{1+\frac{4\eta(3+4R)}{3(\Pr w_0)^2}}\right)$$

$$R_2 = \frac{3\Pr w_0}{2(3+4R)}\left(1+\sqrt{1+\frac{4(n\Pr+\eta)(3+4R)}{3(\Pr w_0)^2}}\right)$$

$$R_3 = \frac{1}{2}(\mathrm{Sc}\,w_0 + \sqrt{(\mathrm{Sc}\,w_0)^2 + 4\delta\, Sc})$$

$$R_4 = \frac{1}{2}(\mathrm{Sc}\,w_0 + \sqrt{(\mathrm{Sc}\,w_0)^2 + 4(\delta+n)\, Sc})$$

$$R_5 = \frac{1}{2}(w_0 + \sqrt{w_0^2 + 4N}$$

$$R_6 = \frac{1}{2}(w_0 + \sqrt{w_0^2 + 4(N+n)})$$

$$Z_2 = \frac{-Sc\, S_0\, R_2^2}{R_2^2 - Sc\, w_0 R_2 - (n+\delta)Sc}$$

$$Z_1 = \frac{-Sc\, S_0\, R_1^2}{R_1^2 - Sc\, w_0 R_1 - \delta\, Sc}$$

$$Z_3 = \frac{(Gr + Gm\, Z_1)}{R_1^2 - w_0 R_1 - N}$$

$$Z_4 = \frac{(1-Z_1)Gm}{R_3^2 - w_0 R_3 - N}$$

$$Z_5 = \frac{(Gr + Gm\, Z_2)}{R_2^2 - w_0 R_2 - (N+n)}$$

$$Z_4 = \frac{(1-Z_2)Gm}{R_4^2 - w_0 R_4 - (N+n)}$$

$$A_1 = 1 + Z_3 + Z_4$$

$$A_2 = Z_5 + Z_6$$

Table 1.

R	δ	Sc	m	ω	S	η	Gm	τ_x	τ_z	Nu	Sh
0.1	5	0.22	0.5	0.5	5	1	1.5	1.686	1.16	1.286	0.472
0.1	5	0.22	1	0.5	5	1	1.5	2.301	1.925	1.286	0.472
0.1	5	0.22	2	0.5	5	1	1.5	3.445	2.641	1.286	0.472
0.1	5	0.22	0.5	0	5	1	1.5	3.126	1.671	0	1.072
0.1	5	0.78	0.5	0.5	5	1	1.5	1.652	1.145	1.286	0.535
0.1	10	0.22	0.5	0.5	5	1	1.5	1.595	1.129	1.286	1.028
0.1	15	0.22	0.5	0.5	5	1	1.5	1.541	1.113	1.286	1.432
0.3	5	0.22	0.5	0.5	5	1	1.5	1.818	1.207	1.406	0.581
0.5	5	0.22	0.5	0.5	5	1	1.5	1.922	1.245	1.515	0.658
0.1	5	0.22	0.5	0.5	0	1	1.5	1.58	1.125	1.286	1.13
0.1	5	0.22	0.5	0.5	3	1	1.5	1.644	1.146	1.286	0.735
0.1	5	0.22	0.5	0.5	5	0.5	1.5	1.979	1.267	0.973	0.697
0.1	5	0.22	0.5	0.5	5	2	1.5	1.349	1.051	1.732	0.121
0.1	5	0.22	0.5	0.5	5	1	2	1.892	1.2	1.286	0.472
0.1	5	0.22	0.5	0.5	5	1	5	3.123	1.44		0.472

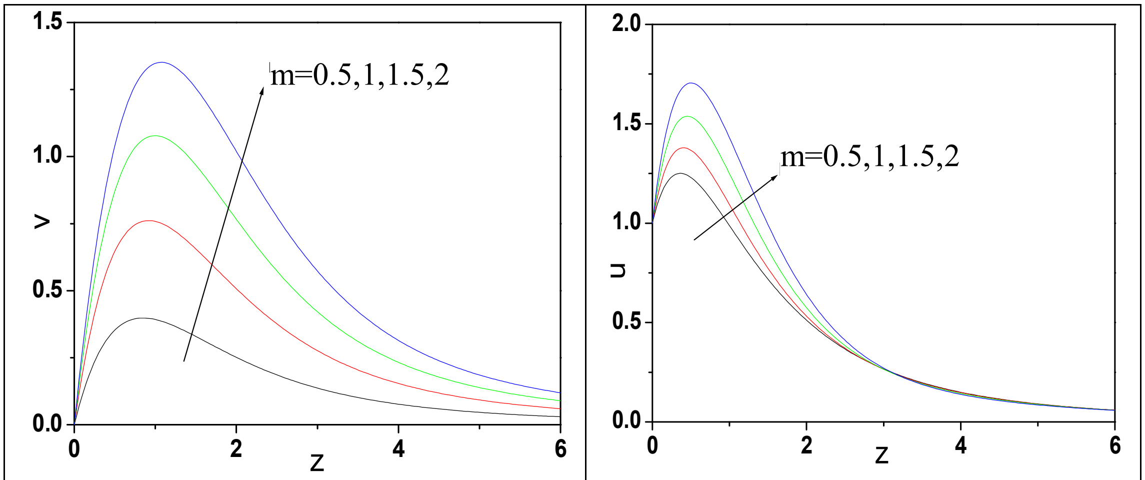


Figure 1: Effect of m on velocity profiles for $\varepsilon = 0.02$, $t = 1$, $n = 0.1$, $\omega = 1$, $M^2 = 5$, $\eta = 1$, $Gr = 10$, $Gm = 5$, $R = 0.1$, $\Pr = 0.71, Sc = 0.22$, $S_o = 5$, $\delta = 5$

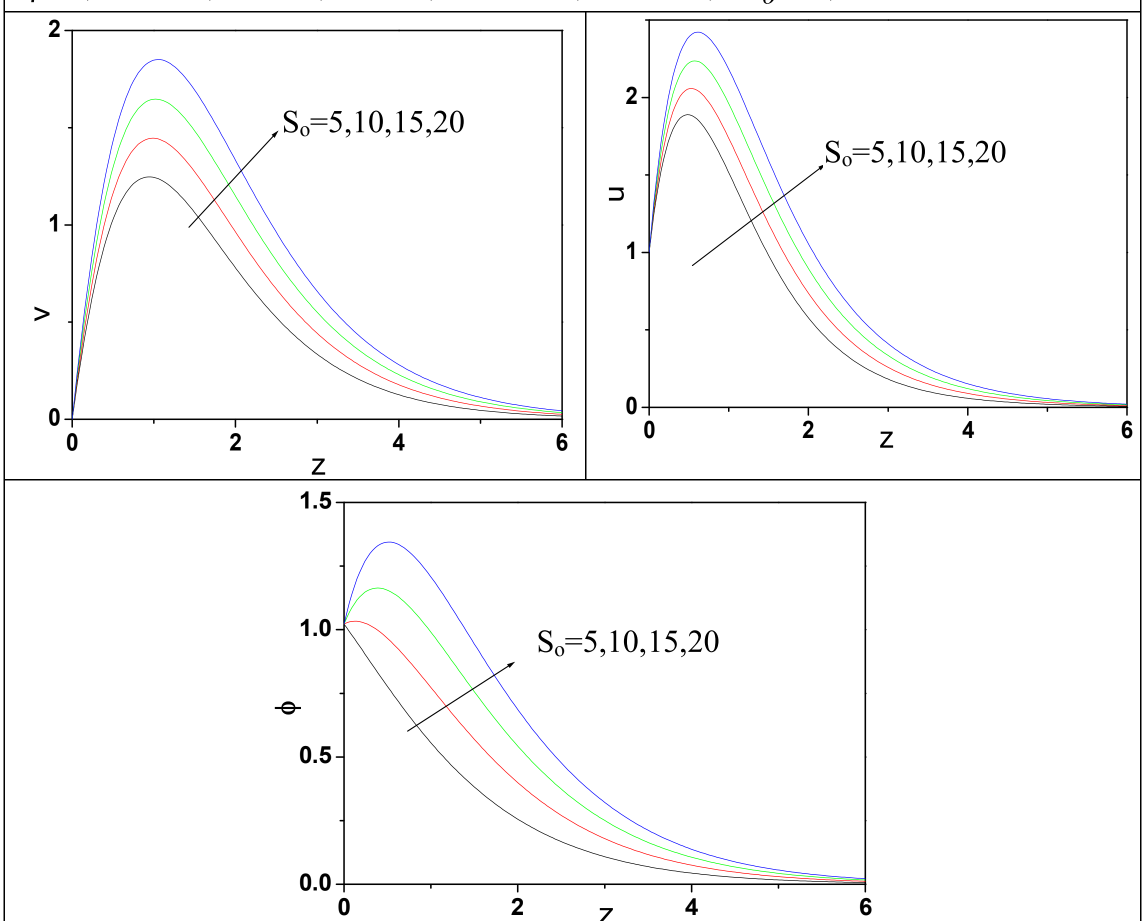


Figure 2: Effect of S_0 on velocity and concentration profiles for $\varepsilon = 0.02, t = 1, n = 0.1,$ $\omega = 1, M^2 = 5, \eta = 1, Gr = 10, Gm = 5, R = 0.1,$ $\Pr = 0.71, Sc = 0.22, S_o = 5,$ $m = 1.5$

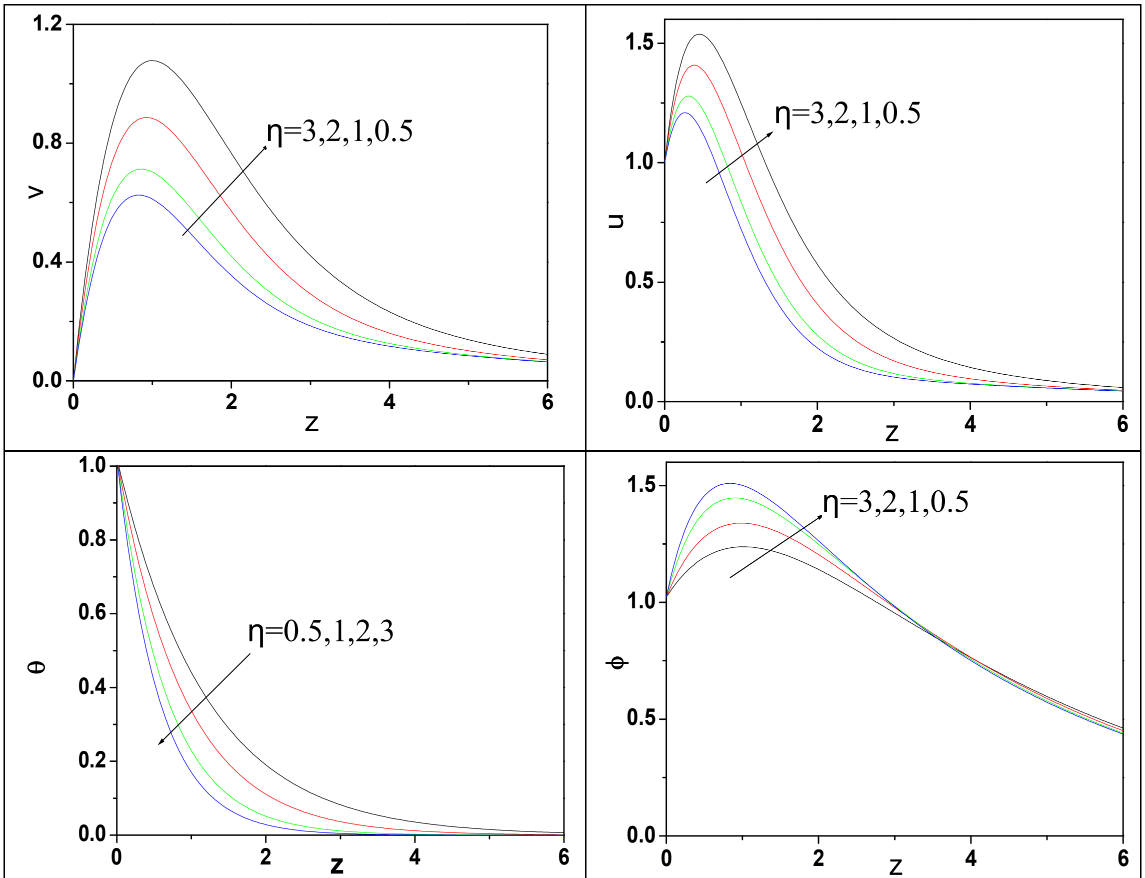
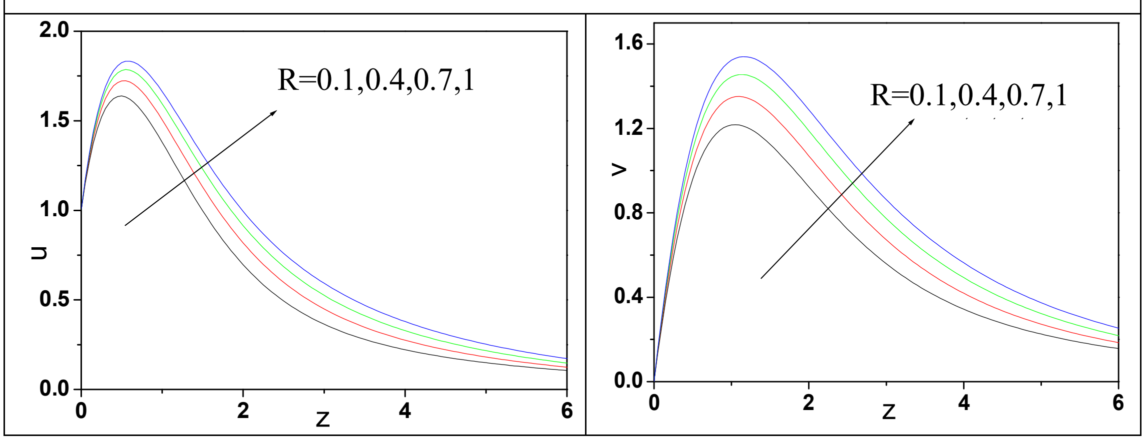


Figure 3: Effect of η on velocity, temperature and concentration profiles for $\varepsilon = 0.02, t = 1,$
$n = 0.1, \omega = 1, M^2 = 5, \eta = 1, Gr = 10, Gm = 5, R = 0.1, \Pr = 0.71, Sc = 0.22, \delta = 5, m = 1.5$



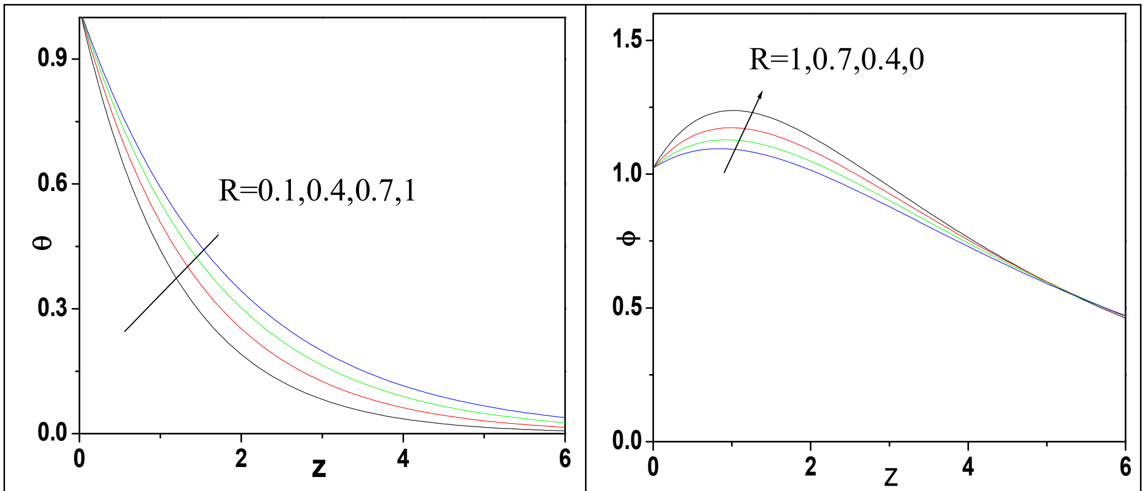


Figure 4: Effect of m on velocity, temperature and concentration profiles for $t = 1, n = 0.1$
$\omega = 1, M^2 = 5, \eta = 0.5, Gr = 10, Gm = 1, \delta = 0.2, \Pr = 0.71, Sc = 0.22, S_o = 5, m = 1.5$

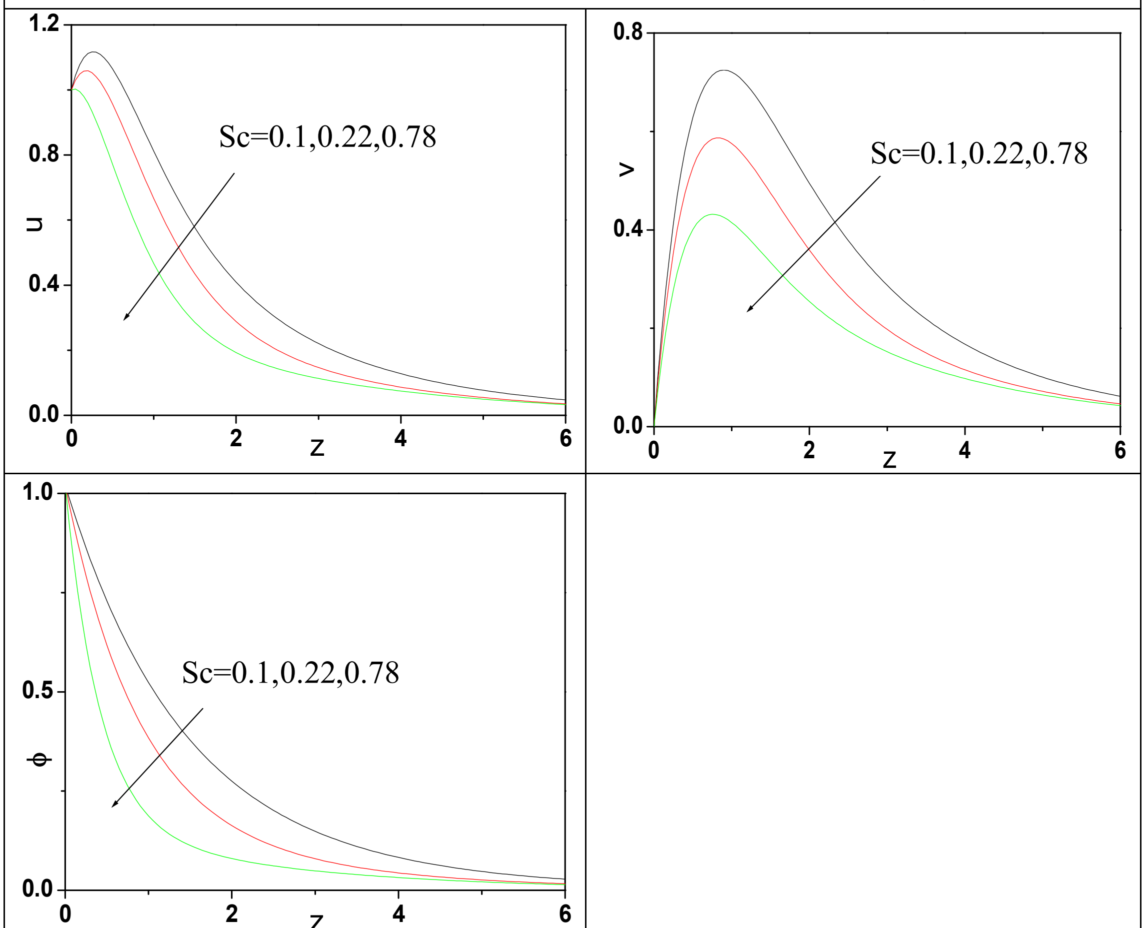


Figure 5: Effect of m on velocity and concentration profiles for $\varepsilon = 0.02, t = 1, n = 0.1$
$\omega = 1, M^2 = 5, \eta = 1, Gr = 10, Gm = 5, R = 5, \Pr = 0.71, S_o = 5, \delta = 5, m = 1.5$

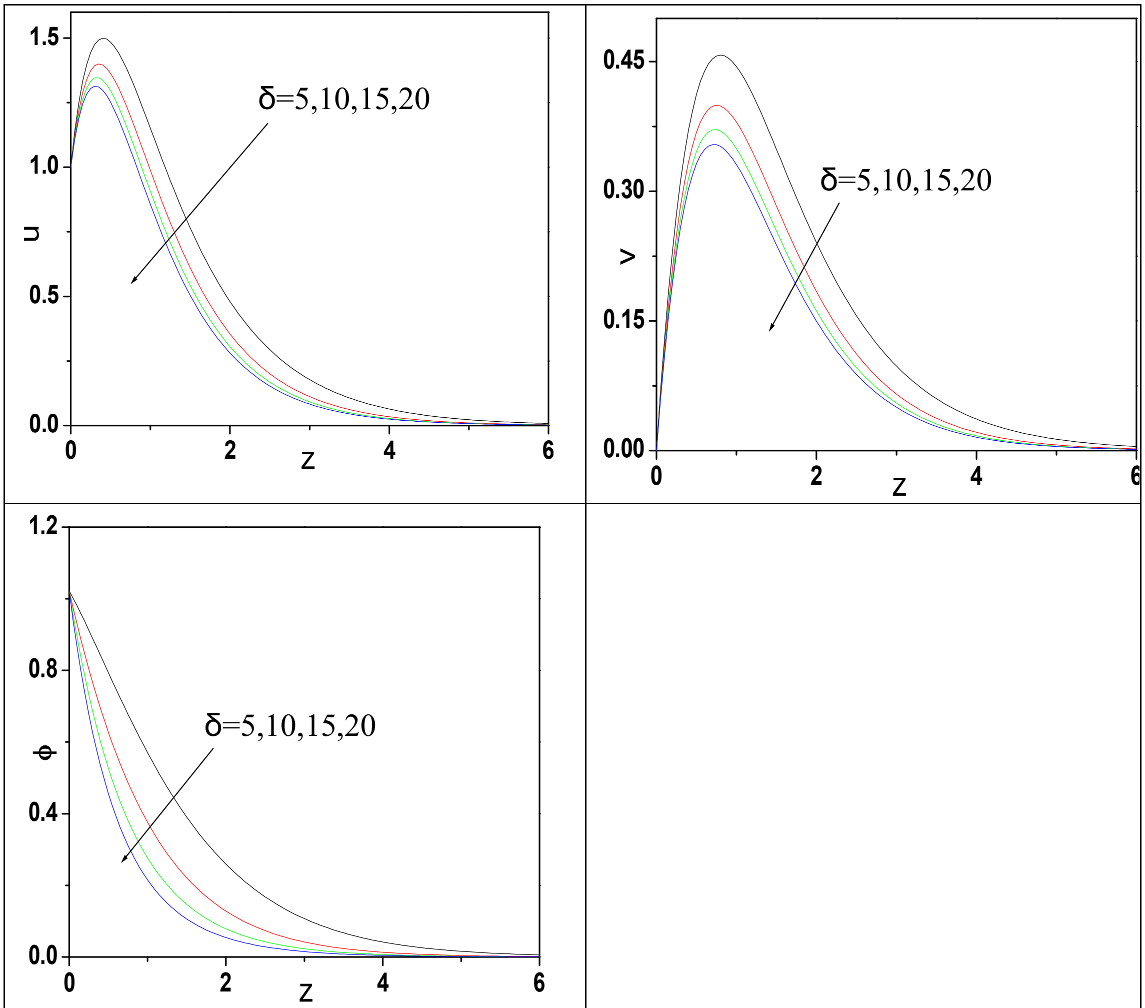
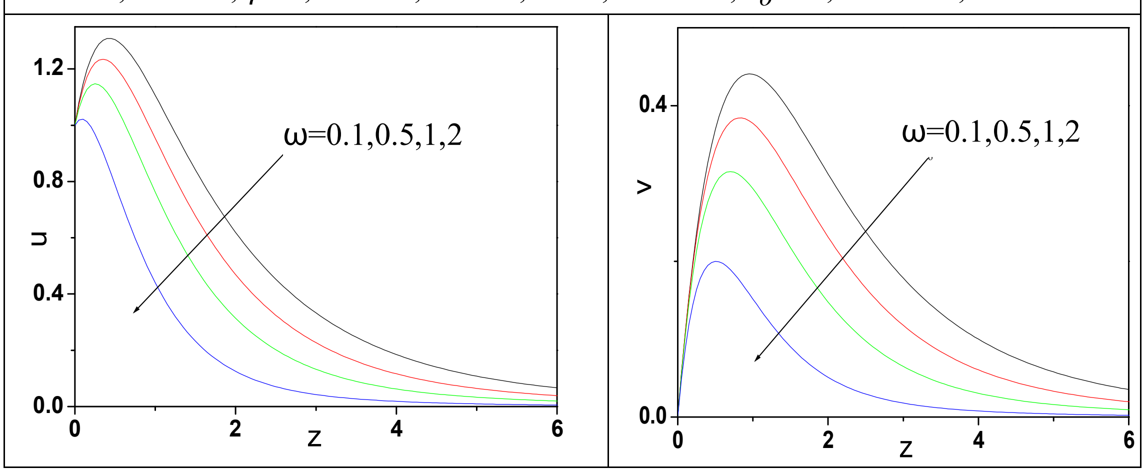


Figure 6: Effect of m on velocity and concentration profiles for $\varepsilon = 0.02, t = 1, n = 0.1$ $\omega = 0.5, M^2 = 5, \eta = 1, Gr = 10, Gm = 5, R = 0, \Pr = 0.71, S_o = 5, Sc = 0.22, m = 0.5$



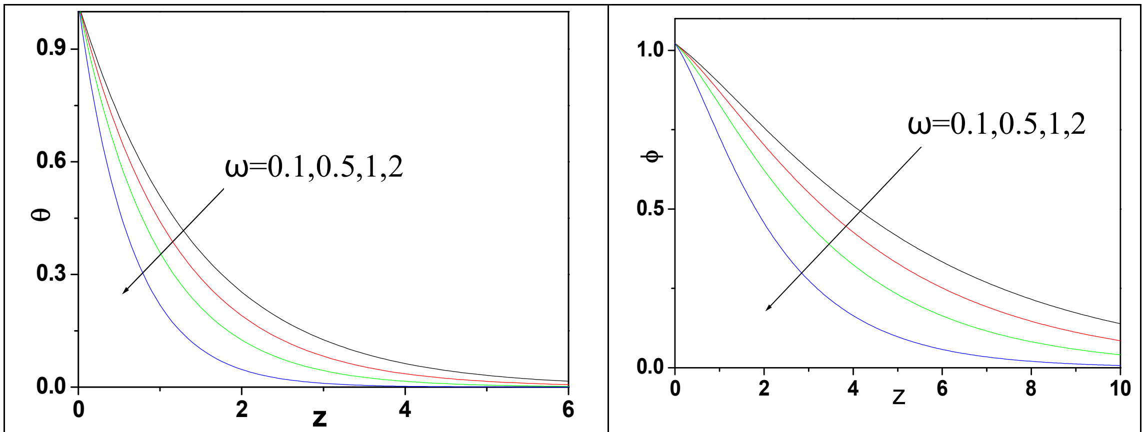


Figure 7: Effect of m on velocity, temperature and concentration profiles for $\varepsilon = 0.02, t = 1,$ $n=0.1, \delta=0.2, M^2=5, \eta=0.5, Gr=10, Gm=0.5, R=0.1, \Pr=0.71, S_o=5, Sc=0.22, m=0.5$

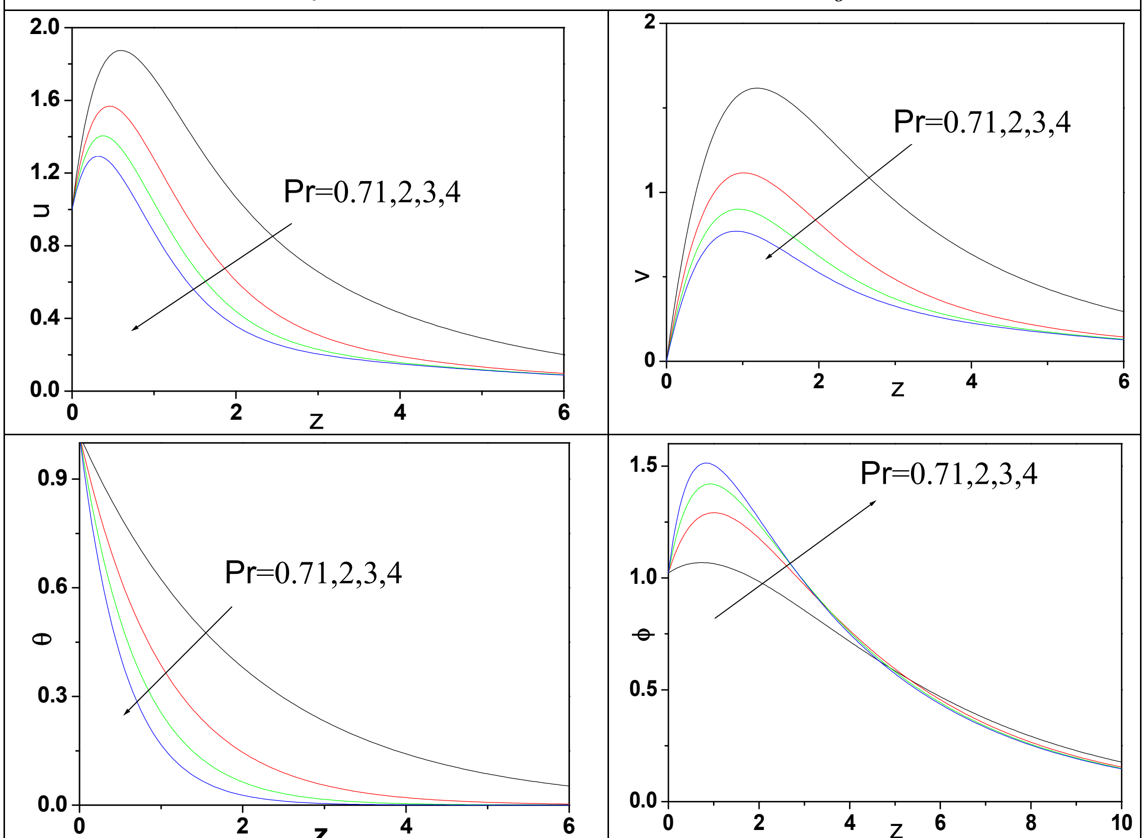


Figure 8: Effect of m on velocity, temperature and concentration profiles for $\varepsilon = 0.02, t = 1,$ $n=0.1, \delta=0.2, M^2=5, \eta=0.1, Gr=10, Gm=1, R=0.1, \omega=0.5, S_o=5, Sc=0.22, m=1.5$

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