An Active 3-D Scanning System Based on Volumetric and Photometric Calibration

Irfanud Din¹, Hafeez Anwar² and Kang Park³
¹ Dept. of Electronics Engineering, Incheon National University, Incheon, South Korea
² Ph.D. School of Informatics, Vienna University of Technology, Vienna, 1040, Austria
³ Dept. of Mechanical Engineering, Myongji University, Yongin, Gyeonggi-Do, South Korea
Email: kang@miu.ac.kr

Abstract: In this paper, we present a simple three dimensional (3-D) shape measurement system based on simple volumetric and photometric calibration. A liquid crystal display (LCD) projector is used to project a set of fringe patterns on to the surface of an object. The projected patterns are captured with a charge couple device (CCD) camera. The depth of the object causes phase variations in the projected patterns. These phase variations are used to construct a dense 3-D point cloud representing the surface of the object using homography, phase unwrapping, and phase map to coordinate conversion. The measuring volume is defined and calibrated using calibration boards. This measuring volume reduces the complexity of the system and enables it to measure 3-D objects without projector calibration. Photometric calibration is used to construct a lookup table that maps the captured intensity to the projected intensity value. The experimental results described are obtained using an HS200 LG LCD projector and a UEYE CCD camera with 1024 × 768 resolutions.

[1] [2] [3] [4]

INTRODUCTION

The importance of three dimensional (3-D) object scanning has increased tremendously over the past decade as a result of its many valuable applications in the fields of medical diagnostics, dentistry, military, machine vision, robotics, large infrastructure inspection, etc. All of these applications are based on the 3-D point cloud calculated from the object. The quality of the scanning method is strongly dependent on the accuracy and speed. Even though there is a tradeoff between speed and accuracy, an accurate 3-D coordinate reconstruction within a reasonable scanning time is needed. An accurate point cloud can be obtained using the coordinate measurement machine (CMM). However, because of time-consuming nature of point-by-point scanning, this method is unacceptable for various applications.

Optical noncontact 3-D shape measurement techniques are used to extract the geometry information of an object with high speed and high accuracy. These techniques can be divided into two types: active 3-D scanning [1] [2], and passive 3-D scanning [3] [4]. Active scanning methods use structured illuminations or laser scanning whereas passive scanning methods are based on a stereo vision system. The major problem with stereo vision is its low accuracy. Because of their noncontact nature, laser scanners are widely used in dental and others applications. Line-by-line scanning remains a drawback of laser scanners, however. In most conventional vision systems, the accuracy has been fairly low but these systems are very sensitive to environmental changes [5] [6].

The digital fringe pattern projection method is advantageous because it offers high precision, high resolution, full field measurement, and easy implementation [3]. To scan the 3-D profile of an object, different methods have been proposed by researchers. [7] used gray-scale structured lighting for recovery of 3-D surface shape. He used 32 phase-shifted images to scan a single object. Because each image consists of a single sinusoidal pattern, the extracted phase need not be unwrapped, but projecting and capturing 32 images makes this method very time consuming. The coded structured light technique [8], which is based on projecting coded pattern and capturing the illuminated scene from a different view, is another reliable 3-D scanning technique. Patterns are designed in a special way and a set of pixels is assigned a code word. Encoding the patterns and detecting the code words makes this process more complex. [9] used a strip shifting pattern method for 3-D reconstruction. The distortion of the strip shadow on the object is used to obtain 3-D information on the object surface. Li’s method is fast and low cost but the camera and the projector both need to be calibrated. The moiré techniques [10] [12] [12] are useful for objects with relatively large flat surfaces and small depth variations. They are mostly used for repeated measurements and they provide information regarding the whole surface. A prior calibration must, however, be carried out. The limitations of moiré methods include implementation complexity, the need for a high power light source, and the inability to measure large...
objects.

**Fringe Projection**

- Projecting the generated phase shifted patterns to the reference plane and on to the object

**Fringe Acquisition**

- Capturing the projected patterns with a camera

**Phase Extraction**

- Extracting the phase information from the captured patterns using fringe analysis techniques

**Phase Unwrapping**

- Extracting the continuous phase map from the wrapped phase

**Phase Modulation Extraction**

- Obtaining the phase modulation information caused by the object height

**Phase to Height Conversion**

- Calculating the object height using the phase modulation information

**Figure 1: Flow Chart**

In the digital fringe projection technique, the simple triangulation principle establishes the relationship between the unwrapped phase distribution and the geometric coordinates of the 3-D object. Using this relationship, the height can be obtained as a product of the calibration constant and the object shape-related phase component. The phase component can be obtained simply by subtracting the object phase from the reference phase. The calibration constant is a phase-to-height conversion constant that can be calculated from the parameters of the optical setup given below:

a. Distance between the centre of the lens of the camera and that of the projector.

b. Distance between the camera projector plane and the reference plane.

c. Pitch of the pattern on the reference plane.

In practical conditions, however, it is very difficult to precisely measure system parameters, especially the fringe pitch, as it does not remain constant over the entire image plane due to the non-telecentric projection [13]. In order to calculate the optical parameters precisely, the camera and the projector should be calibrated accurately but the projector calibration is usually a very complex and time-consuming task. To avoid this complexity, we introduced volumetric calibration, which makes use of calibration boards to calculate the calibration constant. The method is not only simple but it also produces accurate reconstructed 3-D geometry. Figure 1 shows a flowchart that describes the different steps involved in the measurement of the object’s height using the digital fringe projection technique. A short description of each step is also given.

Section 2 presents the digital fringe projection method in details, including the photometric and volumetric calibration procedure and the height calculation of our scanning system. Section 3 describes the experiments and the results.

**DIGITAL FRINGE PROJECTION**

**Phase-shifting technique:** Many different phase-shifting algorithms have been developed by researchers, including the double three-step algorithm, three-step algorithm, four-step algorithm, least square algorithms, and Carré algorithm [14] [15] [16]. In this paper, the four-step phase-shifting algorithm is used. Four phase-shifted sinusoidal patterns with phase shift of 0, π/2, π, and 3π/2 are projected on to the object surface. Intensities of four images with phase shift of π/2 are given by

\[ I_1(x, y) = a(x, y) + b(x, y)\cos \varphi(x, y) \]  
\[ I_2(x, y) = a(x, y) + b(x, y)\cos \left( \frac{\pi}{2} + \varphi(x, y) \right) \]  
\[ I_3(x, y) = a(x, y) + b(x, y)\cos \left( \pi + \varphi(x, y) \right) \]  
\[ I_4(x, y) = a(x, y) + b(x, y)\cos \left( \frac{3\pi}{2} + \varphi(x, y) \right) \]  

where \( a(x, y) \) is the average intensity, \( b(x, y) \) is the intensity modulation, and \( \varphi(x, y) \) is the phase to be determined. Solving (1) to (4) simultaneously, the phase is calculated as

\[ \varphi(x, y) = \arctan \frac{I_4(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)} \]

**Phase unwrapping:** The arc tangent function extracts the required phase information, but this information is in the range from -π to π along with discontinuities. In order to obtain the actual phase that is proportional to distance, these discontinuities need to be removed and the continuous phase distribution must be obtained. This process is known as phase unwrapping. Phase unwrapping appears simple but it is actually an extremely challenging task because of the errors in intensities that are caused by the electronic noise, background noise during data acquisition, abrupt phase changes caused by shadow or surface discontinuities, and low data modulation point caused by low surface reflectivity [17]. Also it is difficult to distinguish between genuine phase jumps and steps (or breaks) caused by object discontinuities. As a result of these unavoidable error sources, a simple unwrapping algorithm is unable to determine the continuous phase distribution. Different unwrapping algorithms have therefore been developed by researchers [18] [19] [20]. In this paper, the quality-guided phase unwrapping algorithm is used for unwrapping. The quality map that is required to guide the phase unwrapping algorithm is
obtained from the phase data itself. It guides the identification of significant discontinuities [21].

**Phase-to-height conversion:** Figure 3 depicts the optical geometry of the system used in the experiments. Point P and I are the centers of the lens of the beam projector and the charge-coupled device (CCD) camera. O is the point of intersection between the optical axis of the projector and the camera. From the CCD point of view, point D on the object has the same phase as point A on the reference plane. From the projector view-point, point D on the object has the same value as point C on the reference plane. It is assumed that P and I lie on the same plane, parallel to the reference plane. L is the distance between the two planes and d is the distance between P and I. Thus ∆ACD and ∆IPD are similar and the following relationship can be established [23].

\[
\frac{d}{AC} = \frac{L-DB}{DB} = \frac{L}{DB} - 1
\]

If the distance between the two planes is large enough compared to the pitch of the projected pattern under normal view conditions,

\[
DB = \frac{AC}{d} (L - DB) = \frac{LAC}{d} - \frac{DB AC}{d}
\]

\(AC\) is much smaller than \(d\) so we can neglect the last term of (7), and we get

\[
DB \approx \frac{LAC}{d}
\]

The relationship between distance \(AC\) on the reference plane and the phase change is given by (8)

\[
\frac{AC}{p} = \frac{\Phi_{AC}}{2\pi}
\]
Replacing the distance $AC$ by the corresponding phase change value, the height of the object is calculated as

$$z(x, y) = DB = \frac{pl}{2\pi} \phi_{AC} = K\phi_{AC}$$  \hspace{1cm} (9)

where $z(x, y)$ is the height of the object at point D. (9) shows a phase-to-height conversion relationship where $p$ is the pitch of the pattern on the reference plane, $K$ is a calibration constant, and $\phi_{AC}$ is the phase change or phase modulation that is caused by the height of the object. This phase modulation can be expressed according to (8) as

$$\phi_{AC} = \frac{2\pi}{p} \pi$$

According to (5), let the calculated phase for the reference plane and object plane be $\phi_r(x, y)$ and $\phi_o(x, y)$. Using these two phase maps, the phase modulation information is calculated using the following equation.

$$\phi_{AC} = \phi_o(x, y) - \phi_r(x, y)$$  \hspace{1cm} (10)

**Phase-to-height conversion:** In order to calculate the 3-D geometry of an object using an uncalibrated projector, a measuring volume is defined. Figure 4 shows the basic setup of the 3-D scanning system, which consists of a CCD camera, a beam projector, and measuring volume. Measuring volume is defined in front of the reference plane. The phase-shifted patterns generated in the personal computer (PC) are projected by the projector, and captured by the CCD camera. On the reference plane, the projected fringe patterns act like the carrier pattern. The carrier pattern is the basic pattern that is used to carry the spatial deformation in a certain direction. When an object is placed on the reference plane, the phase of the carrier pattern is modulated by the height of the object. This modulated phase contains the object’s depth information. The phase modulation information can be calculated using (10). The height of the object is reconstructed using the phase modulation information and the calibration constant. The calibration constant can be calculated from the parameters of the optical setup. These parameters are obtained by calibrating the projector, which is a complicated and time-consuming procedure. To avoid this complexity, the measuring volume is calibrated, which involves calculation of calibration constant. This is carried out using white calibration boards with known heights, as follows:

a. Four sinusoidal phase-shifted patterns are projected on to the reference plane. The phase at the reference plane is calculated from the captured patterns using (5). Using the unwrapping algorithm, the continuous phase map is obtained.

b. A flat board with a known height $h_1$ (calibration board) is placed on the reference plane. The four phase-shifted patterns are projected on to this flat board. The continuous phase map for this board is then obtained using (5) and phase unwrapping.

c. Calibration boards with heights $h_2, h_3, .., h_n$ ($n = 6$ in our experimental setup) are placed on the reference plane. The continuous phase map is calculated in the same way for each board.

d. Phase modulation information is calculated by subtracting the reference plane’s phase from the measured calibration boards’ phase according to (10).

e. In this way the phase modulation information and the height of objects (calibration boards) is determined. Using these two known values, we can calculate the unknown value (calibration constant) in (9) as $K = z(x, y)/\phi_{AC}$. For $n$ boards, the average value of the calibration constant can be calculated as $K = \frac{1}{n} \sum_{i=1}^{n} K_i$ where $K_i$ is the calibration constant for the $i^{th}$ calibration board.

![Figure 4. 3-D measurement system setup](image)

**Figure 5. Relationship of image point to object point**

The object to be measured is placed on the reference plane. Four phase-shifted patterns are projected on to the surface of the object. These patterns are captured with a camera and the phase value is calculated using (5). Phase modulation is calculated using (10). The height of the object is then calculated using the calibration constant and the phase modulation according to (9).

In order to calculate the x and y coordinates of the object, basic homography is used. As shown in Figure 5, point $\tilde{Q}$ is an object point on the reference plane that is seen as point $\tilde{q}$ on the camera image plane. Homography is induced between the reference plane
and the camera image plane. This linear mapping relates an object point to an image point as follows:

\[ \bar{q} = sh\bar{Q} \]  

(11)

where \( s \) is the scaling factor and \( H \) is homography, which maps the object points to image points [24]. Homography can be calculated from the camera’s intrinsic and extrinsic parameters. The camera is accurately calibrated using Zhang’s method [25], (11) can be expressed as

\[ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} sH \]  

(12)

Thus, in order to extract the world coordinates of a point from a known image point data, the inverse homography matrix should be multiplied on both sides. The resultant matrix is then scaled down according to (14) to obtain the world coordinates

\[ H^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sX \\ sY \\ s \end{bmatrix} \]  

(13)

\[ \bar{Q} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} sX \\ sY \\ s \end{bmatrix} \]  

(14)

Conversion from image coordinates to world coordinates is carried out using (12) to (14), where \( H^{-1} \) the inverse homography matrix and \( s \) is the scaling factor.

**Measuring Volume Limitation:**

The measuring volume (maximum depth that the system can measure) has some limitations. In order to obtain the height from the phase modulation, the pitch \( p \) must be greater than \( \overline{AC} \) in Figure 3 on the reference plane. If the pitch is smaller than \( \overline{AC} \) we obtain the same phase values for two different points that are separated by \( p \), and for two points at different heights, the calculated height will be the same. This phenomenon is explained in Figure 6. Consider three surfaces, \( S_1 \), \( S_2 \), and \( S_3 \). Let \( D_1 \), \( D_2 \), and \( D_3 \) be arbitrary points on surfaces \( S_1 \), \( S_2 \), and \( S_3 \) respectively. Let ‘A’ be a point of the reference plane. In the absence of any surface, it is captured as point A. Because of the heights of surfaces, instead of point A, points \( D_1 \), \( D_2 \), and \( D_3 \) will be projected on to a single pixel in the camera image plane. In order to calculate the height information at these arbitrary points, the phase modulation value at each point is calculated. Let \( \phi_A(x, y) \), \( \phi_{D1}(x, y) \), \( \phi_{D2}(x, y) \), and \( \phi_{D3}(x, y) \), be the phase value at point A, \( D_1 \), \( D_2 \), and \( D_3 \) respectively. According to (10), phase modulation for each point can be calculated as

At point \( D_1 \): \( \phi_{AC1} = \phi_{D1}(x, y) - \phi_A(x, y) \)

At point \( D_2 \): \( \phi_{AC2} = \phi_{D2}(x, y) - \phi_A(x, y) \)

At point \( D_3 \): \( \phi_{AC3} = \phi_{D3}(x, y) - \phi_A(x, y) \)

The obtained phase modulation value must be in the range \([0, 2\pi]\) and this value is repeated every \( 2\pi \). This is due to the periodic nature of the sinusoidal pattern. If the distance \( \overline{AC} \) on the reference plane exceeds the fringe period \( p \), then it creates ambiguity. This phenomenon can be observed at point \( D_3 \) when the distance \( \overline{AC_3} \) exceeds \( p \). It can also be observed that point \( C_1 \) and \( C_3 \) are separated by \( p \), i.e., \( \overline{AC}_3 = p + \overline{AC}_1 \). This will cause the value at \( D_1 \) to repeat at \( D_3 \), and we will obtain the same phase modulation value for both points \( D_1 \) and \( D_3 \). As a result, the calculated height at both points will be the same. This phenomenon is also known as \( 2\pi \) ambiguity. The height of an object obtained using (9) is limited by \( K \). As \( \phi_{AC_1} \) cannot exceed \( 2\pi \), the maximum depth that can be measured is given by

\[ z_{max}(x, y) = K\phi_{AC_1}(\text{max}) < 2\pi K \]  

(15)

**EXPERIMENTS AND RESULTS**

In the experimental demonstration, different objects were measured by the system to demonstrate the performance. Fringe patterns were generated by a computer and projected by an HS200 LG DLP projector. A CCD camera (UEYE) with 1024 × 768 resolutions and a 16 mm lens was used to capture the patterns. The software is written in visual C++ with the OpenCV library. An average value for the calibration constant is used for reconstruction.

A flat white board of 6 mm height was used to verify the system accuracy. The mean value of the measured flat surface is 6.02 mm and the standard deviation is approximately 0.318 mm.

![Figure 6. 2π ambiguity](http://www.lifesciencesite.com)

![Figure 7. Measurement error](http://www.lifesciencesite.com)
observed that the higher the density of the projected pattern (i.e., the smaller the pitch), the more sensitive the measurement system. The quality of the fringe pattern also affects the accuracy and the phase errors can be effectively reduced by increasing the number of phase-shifted images.

![Figure 8. Projected and captured pattern on to the object surface](image)

![Figure 9. (a) Extracted phases (wrapped) (b) Unwrapped phase map](image)

![Figure 10. Reconstructed 3-D surface of the object](image)

CONCLUSION

A simple method based on fringe projection for reconstruction of the 3-D geometry of an object is proposed. This method provides easy implementation by avoiding the complex and time-consuming process of projector calibration. The measuring volume is calibrated using flat white boards. 3-D geometry is reconstructed using an un calibrated projector and calibration boards with known depth. This method requires a video projector and a calibrated camera only. This method does not require a cumbersome optical setup. The 3-D measuring process is fast and object coordinates are reconstructed with high resolution. Various object surfaces were used for the experiments. The standard deviation of the measuring error is approximately 0.3 mm. This system can be used as a fundamental system and starting point for students and researchers in the field of 3-D reconstruction.

ACKNOWLEDGMENT

This work was supported in part by the Korea Ministry of Knowledge Economy, under Grant of the Strategic Technology Development Project on Biomedical Supplier (Development of the Digital Fusion Artificial Tooth Treatment Supporting System). This paper is a result of research during the 1st and 2nd author’s Master degree program at Myongji University.

Corresponding Author:
Kang Park, Dept. of Mechanical Engineering, Myongji University, Yongin,Gyeonggi-Do, South Korea
Email: kang@mju.ac.kr
REFERENCES


