

SCG-ICA algorithm for Blind Signal SeparationM. EL-Sayed Waheed¹, H. Ahmad Khalil^{2,4}, O. Farouk Hassan^{3,4}¹. Department of Computer Science, Faculty of Computers & Informatics, Suez Canal University, Ismailia, Egypt². Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt, P.O. Box 44519³. Department of Mathematics, Faculty of Science, Suez Canal University, Ismailia, Egypt⁴. Unaizah Community College, Qassim University, Unaizah, KSA, P.O. Box 51911-4394^{3,4} ufarouk@yahoo.com

Abstract: The gradient based algorithms are the most basic independent component analysis (ICA) algorithms, used in Blind signal separation (BSS). Because these algorithms adopt fixed step size, the choice of step size affects the performance and the convergence speed of the algorithm. In this paper, we propose a new algorithm SCG-ICA for blind signal separation. The new algorithm significantly improves the convergence rate of gradient-based blind source separation. The proposed algorithm is based on the Scaled Conjugate Gradient method, which used to optimize the kurtosis contrast function in order to estimate the demixing matrix. The algorithm is robust to local extrema and shows a very high convergence speed in terms of the computational cost required to reach a given source extraction quality, particularly for short data records. The simulations have proved the efficiency and effectiveness of the proposed algorithm.

[M. El-Sayed Waheed, H. Ahmed Khalil and O. Farouk Hassan. **SCG-ICA algorithm for Blind Signal Separation**. *Life Sci J* 2013;10(2):727-733] (ISSN:1097-8135). <http://www.lifesciencesite.com>. 101

Keywords: Blind Signal Separation (BSS), Independent Component Analysis (ICA), Scaled Conjugated Gradient (SCG)

1. Introduction

Blind Signal Separation (BSS) has been used in the field of communications, audio signal processing, radar target detection fields, etc [1,2]. The core of blind separation problem is getting separation matrix. We address the signal separation problem. Making some assumptions on the statistics of the signals we use higher order statistics to separate mixed signals. Standard BSS methods use cost functions based on second- and higher-order statistics and maximization of likelihood and entropy [3]. To better facilitate the modeling of real-world systems, noisy environments and post-nonlinear mixtures have been recently studied in real domain algorithms [4].

The use of higher-order statistics is not new to the source separation problem [5]. Since Comon's seminal work [6], many contrast functions for ICA have been proposed in the literature, mainly based on information theoretical principles such as maximum likelihood, mutual information, marginal entropy and negentropy, as well as related non-Gaussianity measures [3]. Among them, the kurtosis (normalized fourth-order marginal cumulant) is arguably the most common statistics used in ICA, even if skewness has also been proposed [7]. The use of kurtosis dates back to the work of Wiggins [8], Donoho [9], and Shalvi-Weinstein [10] on blind deconvolution of seismic signals and blind equalization of single-input-single-output (SISO) digital communication channels, two problems that can be related to BSS/ICA. One of the main benefits of kurtosis lies in the absence of

spurious local extrema for infinite sample size when the noiseless observation model is fulfilled.

This attractive feature leads to globally convergent source extraction algorithms, from which full source separation can be performed by using some form of deflation procedure [11] even in the convolutive multiple-input and multiple-output (MIMO) case [12]. Although the adequacy of kurtosis as a contrast may be objected on the basis of statistical efficiency and robustness against outliers [13], its widespread use is justified by mathematical tractability, computational convenience, and robustness to finite sample effects. Theoretical evidence for its finite-sample robustness has been gathered by previous works. In [14], the sample kurtosis yields an estimate with less variance than the fourth-order moment and the fourth-order cumulant for all distributions tested, including sub-Gaussian and super-Gaussian densities. As an extension of these results, using the full expression of the fourth-order cumulant instead of the simplified form employed, e.g., in the FastICA algorithm [13], is shown to improve extraction performance. The computational convenience and finite sample robustness of kurtosis can be further improved by the optimal step-size iterative search proposed in this paper. In the presence of outliers, the performance of the conventional kurtosis estimate based on sample moments can be enhanced by means of more robust alternative estimates available in the literature see, e.g., [15].

The remaining of this paper is organized as follows. Problem formulation is introduced in section 2. In section 3 the SCG-ICA algorithm is discussed. Section 4 presents the results of our algorithm of source separation in various settings. Conclusions are drawn in section 5.

2. Problem formulation

By definition, independent component analysis (ICA) is the statistical method that searches for a linear transformation, which can effectively minimize the statistical dependence between its components [13,21,22]. Under the physically plausible assumption of mutual statistical independence between these components, the most applications of ICA are blind signal separation (BSS). In its simplest form, BSS aims to recover a set of unknown signals, the so-called original sources $\mathbf{s}(t) = [s_1(t), \dots, s_n(t)]^T \in R^n$, by relying exclusively on information that can be extracted from their linear and instantaneous mixtures $\mathbf{x}(t) = [x_1(t), \dots, x_m(t)]^T \in R^m$, given by:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n} \tag{1}$$

Where $\mathbf{A} \in R^{m \times n}$ is an unknown mixing matrix of full rank and $m \geq n$. In doing so, BSS remains truly (blind) in the sense that very little to almost nothing be known a priori for the mixing matrix or the original source signals. Often sources are assumed to be zero-mean and unit-variance signals with at most one having a Gaussian distribution. The problem of source separation then boils down to determining the unmixing matrix $\mathbf{W} \in R^{n \times m}$ such that the linear transformation of the sensor observation is

$$\mathbf{u}(t) = \mathbf{W}\mathbf{x}(t) \tag{2}$$

Where $\mathbf{u}(t) = [u_1(t), \dots, u_n(t)]^T \in R^n$, yield an estimate of vector $\mathbf{s}(t)$ corresponding to the original or true sources. In general, the majority of BSS approaches perform ICA, by essentially optimizing the objective function with respect to the unmixing matrix \mathbf{W} . A widely used contrast is the kurtosis, which is defined as the normalized fourth-order marginal cumulant, kurtosis is one of the classic measures used for estimation of non-Gaussianity of random variable for ICA. Kurtosis can be expressed as:

$$g(\mathbf{W}) = \frac{E\{|\mathbf{u}|^4\} - 2E^2\{|\mathbf{u}|^2\}}{E\{|\mathbf{u}|^2\}^2} \tag{3}$$

To derive the updates of the demixing vector \mathbf{W} , we apply the standard gradient descent method to $g(\mathbf{W})$, which given by:

$$\begin{aligned} & \mathbf{W} \\ & = \mathbf{W} \\ & + \alpha \nabla_{\mathbf{W}} g(\mathbf{W}) \end{aligned} \tag{4}$$

where

$$\begin{aligned} \nabla_{\mathbf{W}} g(\mathbf{W}) = & \frac{-|\kappa(\mathbf{u})|E\{\mathbf{u}^* \mathbf{x}\}}{(E\{|\mathbf{u}|^2\})^3} \\ & + \frac{\text{sign}(\kappa(\mathbf{u}))(E\{|\mathbf{u}|^2 \mathbf{u}^* \mathbf{x}\} - 2E\{\mathbf{u}^* \mathbf{x}\}E\{|\mathbf{u}|^2\} - E\{\mathbf{u}^{*2}\}E\{\mathbf{u}\mathbf{x}\})}{(E\{|\mathbf{u}|^2\})^2} \end{aligned} \tag{5}$$

The kurtosis maximization criterion based on contrast (3) is quite general in that it does not require the observations to be prewhitened and can be applied to real- or complex-valued sources without any modification.

Most of the optimization methods used to minimize functions are based on the same strategy. The minimization is a local iterative process in which an approximation to the function in a neighborhood of the current point in weight space is minimized. Determining the next current point in this iterative process involves two independent steps. First a *search direction* has to be determined, i.e, in what direction in weight space do we want to go in the search for a new current point [23,24,25]. Once the search direction has been found we have to decide how far to go in the specified search direction, i.e, a *step size* has to be determined. If the search direction p_k is set to the negative gradient $-f'(w)$ and the step size α to a constant ϵ , then the algorithm becomes the gradient descent algorithm [16]. But this algorithm suffers from local minimum point; the Newton's method solves this by using positive definite Hessian matrix in which Newton's method has global convergence.

Newton Method

Newton method is an efficient tool of unconstrained optimization. It often converges fast and provides quadratic rate of convergence. However, its iteration may be costly, because of the necessity to compute the Hessian matrix and solve the corresponding system of equations, other methods like natural gradient is linearly convergent, while Newton method is quadratically convergent [26,27]. Amari [3] derived a Newton-based method for optimization of a single ICA model in his stability analysis of the ICA problem. Taking the derivative of (4), we find, Hessian matrix are evaluated as

$$\begin{aligned} \nabla_w^2 g(\mathbf{W}) = & \nabla_w |\kappa(\mathbf{u})| \nabla_w \frac{1}{(E\{|\mathbf{u}|^2\})^2} \\ & + |\kappa(\mathbf{u})| \nabla_w^2 \frac{1}{(E\{|\mathbf{u}|^2\})^2} \\ & + \nabla_w \frac{1}{(E\{|\mathbf{u}|^2\})^2} \nabla_w |\kappa(\mathbf{u})| \frac{1}{(E\{|\mathbf{u}|^2\})^2} \nabla_w^2 |\kappa(\mathbf{u})| \end{aligned}$$

Where the second derivatives are given in appendix A, then the update equation is given by :

$$\mathbf{W} = \mathbf{W} + (\nabla_w^2 g(\mathbf{W}))^{-1} \nabla_w g(\mathbf{W}) \quad (7)$$

Unfortunately is it not desirable to calculate the Hessian matrix explicitly, because of the calculation complexity and memory usage involved; actually calculating the Hessian would demand $O(N^2)$ memory usage and $O(N^3)$ in calculation complexity (where N is the number of sources). To solve this problem we describe the Scaled Conjugate Gradient (SCG) algorithm [17,28,29] as illustrated in the following section.

3. SCG-ICA algorithm

The SCG-ICA algorithm is a gradient based algorithm for blind signal separation, the algorithm uses the SCG to optimize the kurtosis to maximize the non-Gaussianity of the sources. The SCG is a variation of a conjugate gradient, which avoids the line-search per learning iteration by using a Levenberg-Marquardt approach [17] in order to scale the step size. SCG chooses the search direction and the step size more carefully by using information from the second order approximation of the objective function. Compared to other gradient descent algorithms, the SCG has the advantage of requiring virtually no parameter tuning. Second-order information (the Hessian) is approximated using the gradient only as

$$f''(w)p = \frac{f'(w + \sigma p) - f'(w)}{\sigma}, \quad 0 < \sigma < 1 \quad (8)$$

Where p is the descent vector to $f(w)$, the algorithm of SCG-ICA is described in algorithm1 :

algorithm 1:

Input: $f''(w)$, $f'(w)$, and the Mixures X .
Output: w .

1. Initiate the demixing matrix w_1 and scalars $\sigma > 0$ and $\bar{\lambda}_1 = 0$.
2. Set $p_1 = r_1 = -f'(w_1)$, $k = 1$ and success = true.
3. If success = true then calculate second order information:

$$\sigma_k = \frac{\sigma}{|p_k|}$$

$$s_k = \frac{f'(w_k + \sigma_k p_k) - E f'(w_k)}{\sigma_k}$$

$$\sigma_k = p_k^T s_k.$$

4. Scale s_k :

$$s_k = s_k + (\lambda_k - \bar{\lambda}_k) p_k$$

$$\delta_k = \delta_k + (\lambda_k - \bar{\lambda}_k) |p_k|^2.$$

5. If $\delta_k \leq 0$ then make the Hessian matrix positive definite:

$$s_k = s_k + (\lambda_k - 2 \frac{\delta_k}{|p_k|^2}) p_k$$

$$\bar{\lambda}_k = 2(\lambda_k - \frac{\sigma}{|p_k|^2})$$

$$\delta_k = -\delta_k + \lambda_k |p_k|^2, \quad \lambda_k = \bar{\lambda}_k$$

6. Calculate step size :

$$\mu_k = p_k^T r_k, \quad \alpha_k = \frac{\mu_k}{\delta_k}$$

Calculate the comparison parameter :

$$\Delta_k = \frac{2\delta_k [f(w_k) - f(w_k + \alpha_k p_k)]}{\mu_k^2}$$

7. If $\Delta_k \geq 0$ then a successful reduction in error can be made :

$$w_{k+1} = w_k + \alpha_k p_k,$$

$$r_{k+1} = -E'(w_{k+1}),$$

$$\bar{\lambda}_k = 0, \text{ success} = \text{true}.$$

7a. If $k \bmod N = 0$ then restart algorithm: $p_{k+1} = r_{k+1}$
 else create new conjugate direction:

$$\beta_k = \frac{|r_{k+1}|^2 - r_{k+1}^T r_k}{\mu_k}$$

$$p_{k+1} = r_{k+1} + \beta_k p_k.$$

7b. If $\Delta_k \geq 0.75$ then reduce the scale parameter: $\lambda_k = \frac{1}{2} \lambda_k$.
 else are duction in error is not possible:

$$\bar{\lambda}_k = \lambda_k, \text{ success} = \text{false}.$$

8. If $\Delta_k < 0.25$ then increase the scale parameter : $\lambda_k = 4\lambda_k$

9. If the steepest descent direction $r_k \neq 0$ then set $k=k+1$ and go to 2
 else terminate and return w_{k+1} as the desired minimum

10. Compute the estimated signals $Y = WX$

4. Results and Analysis

The performance of separation is evaluated with the BSS EVAL toolbox, which is based on the criteria proposed in [18], using time-invariant filters of 1024 taps to represent the family of allowed distortions. The source-to-interferences ratio (SIR) is evaluated using the whole separated signals.

This experiment was performed using the mixture generation, from a group of speakers (male and female) were selected from the TIMIT speech database [19]. All mixtures are sampled at 16 kHz, in which there are a fixed number of sensors and sources that are used here, the mixing matrices are randomly chosen. Figures 1,2 show the time domain of the original sources and the mixtures of sources respectively. Figures 3,4 show the final recovered time-domain sources of our method and Newton method respectively.

To further analyses the performance of all the above methods in separating the mixed signal where the time domain of the each recovered source has been plotted in Figures 3,4. The Figures 3,4 denote the recovered sources by using the SCG-ICA and Newton's algorithm, respectively. In particular, it is noted that both algorithms exhibit good reconstruction. However, the Newton-ICA algorithm fails to identify several missing components as indicated in the red box marked area in Figure 4, Hence, less accuracy is obtained in the estimation of the sources as compared with the our algorithm which has successfully estimated sources with high accuracy. Table 1 also shows that, the proposed algorithm is better in speed of convergence measured in terms of number of iterations and has a higher SIR than Netwon method, which implies that SCG-ICA algorithm is faster and accurate than the Newton method based FASTICA .

Table 1:Comparison between SCG-ICA and Newton-ICA.

Alg.	Performance	S1	S2	S3	Iteration
SCG - ICA	Original kurtosis	5.072	3.9	-0.55571	10
	SIR	39.2412	27.9913	26.8942	
	Estimated kurtosis	5.0734	3.9149	-0.55288	
Newton-ICA	SIR	7.5623	23.5919	14.9611	46
	Estimated kurtosis	5.2746	3.8222	0.5702	

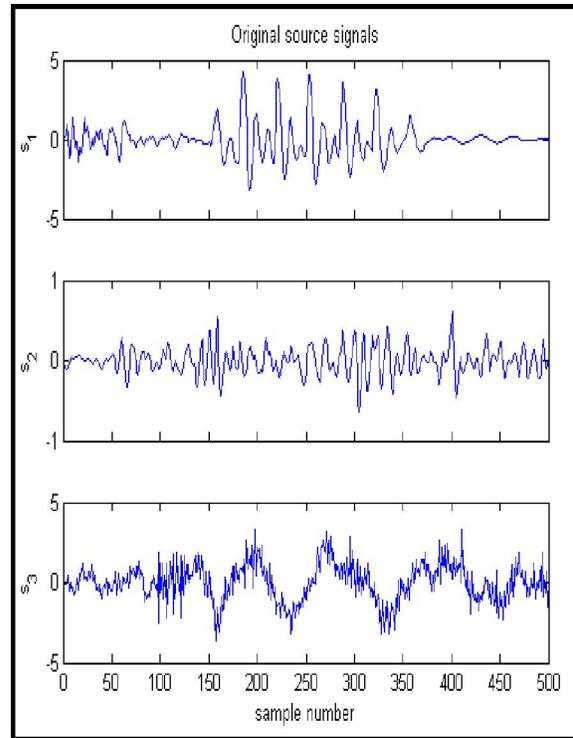


Figure 1. Original of three sources

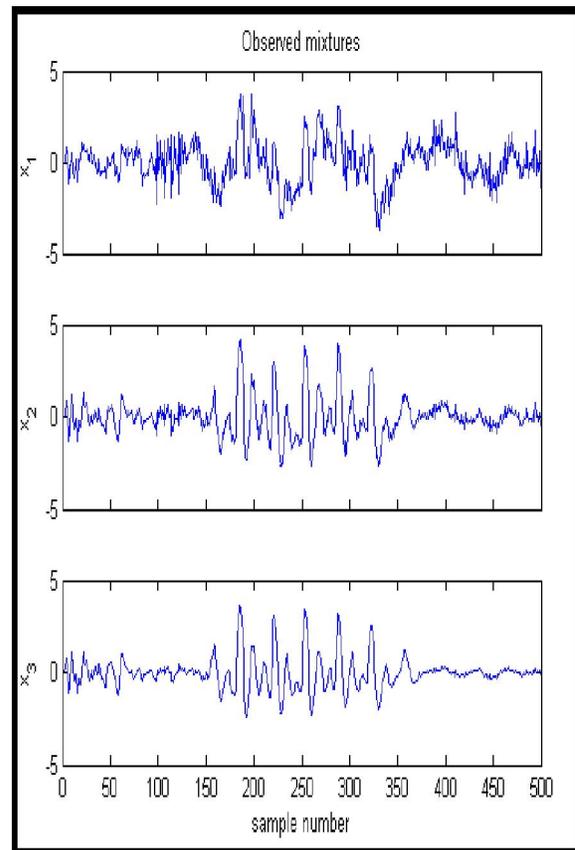


Figure 2. mixture of sources

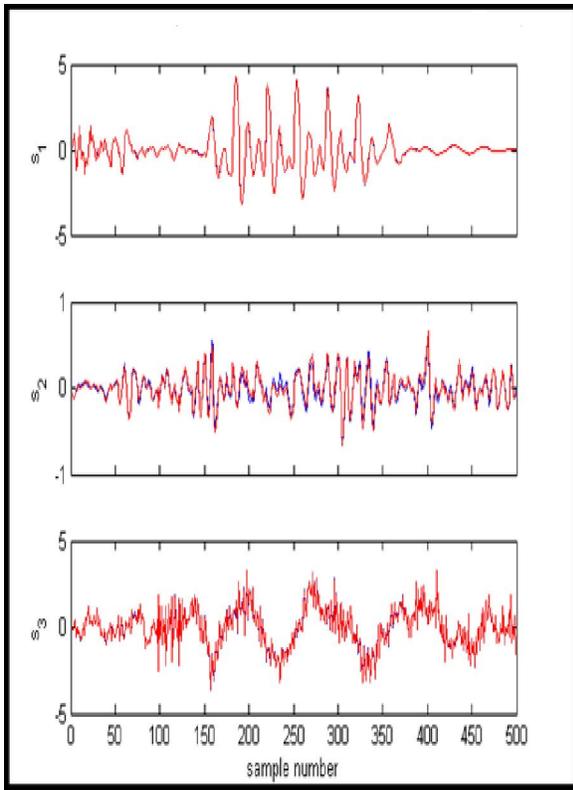


Figure 3. Estimated by SCG-ICA

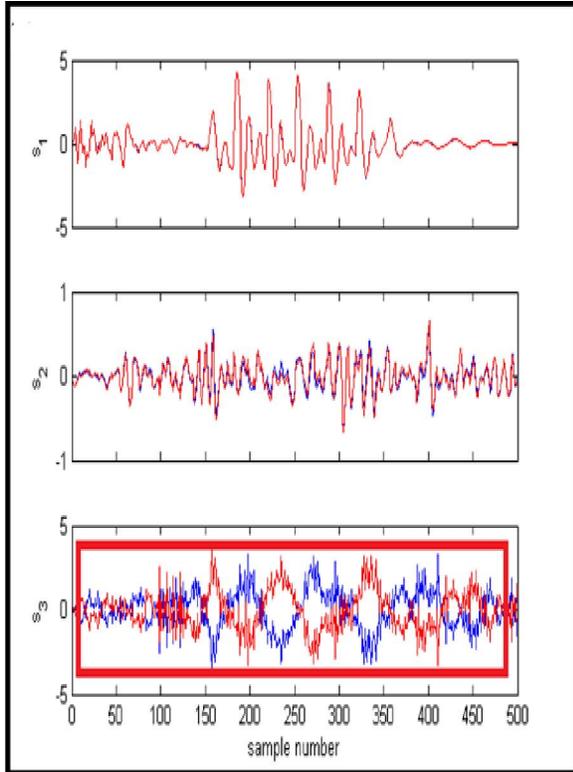


Figure 4. Estimated source by Newton-ICA

In routinely recorded ECG many types of noise and artifacts are presented. Noise is defined to be part of the signal that confuses analysis (muscle movements), an artifact is defined to be any distortion of the signal caused by the recording process, such as electrode movement. In this example we used dataset that found in [20] where Figure 5 shows the original sources and their mixtures. Figure 6 shows the estimated sources after applying the SCG-ICA, Figure 7 Shows the estimated sources by Newton-ICA.

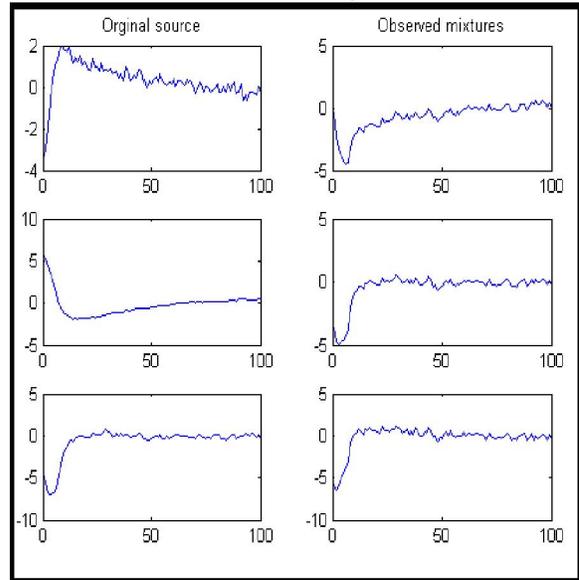


Figure 5. original source and the mixtures of these sources

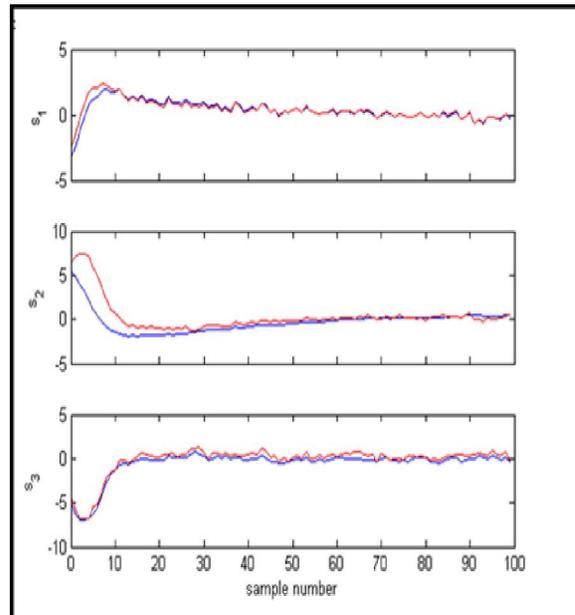


Figure 6. Estimated sources by using SCG-ICA

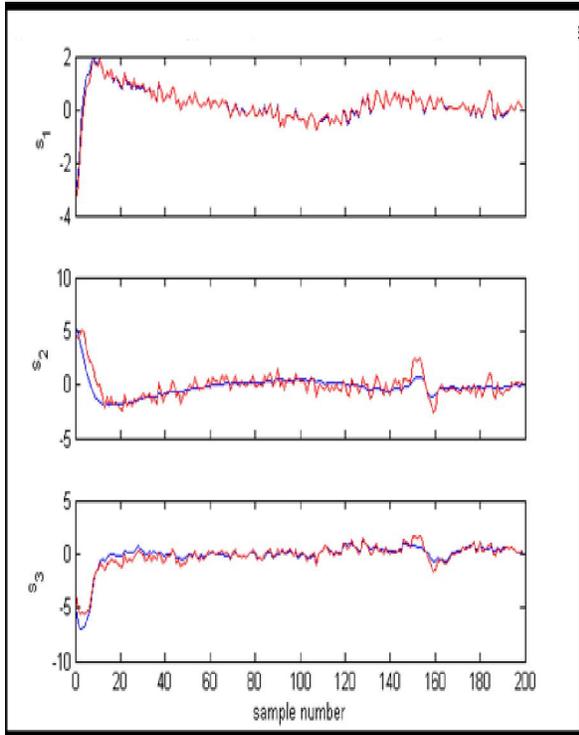


Figure 7. Estimated Sources by Newton-ICA

Table 1: Comparison between SCG-ICA and Newton-ICA for ECG

Alg.		S1	S2	S3	S4	Iteration
Newton-ICA	Original kurtosis	17.9901	-0.55209	-0.4708	0.12693	51
	SIR	4.6909	6.2522	6.6095	7.0444	
SCG-ICA	Estimated kurtosis	46.6102	-0.66686	-0.36766	0.2264	14
	SIR	16.8190	24.7950	9.9524	10.5371	
	Estimated kurtosis	18.1588	-0.7125	-0.4856	0.17111	

5- Conclusion

Blind source separation of signals based on the degree of non-Gaussianity and from noisy mixtures has been addressed. A cost function based on the normalized kurtosis is utilized to perform the blind separation, and the corresponding Scaled Conjugate Gradient algorithm (SCG-ICA) has been derived. The algorithm is shown to be convergent and variable step-size variants of to the algorithm have been discussed. It has been shown. Simulations in noise-free and noisy environments illustrate the performance of the algorithm for the successful separation of speech signals.

APPENDIX A:

In this Appendix, we provide a detailed derivation of the gradient vector.

$$\begin{aligned} \nabla_w \mathcal{G}(\mathbf{W}) &= |\kappa(\mathbf{u})| \nabla_w \frac{1}{(E\{|\mathbf{u}|^2\})^2} \\ &+ \frac{1}{(E\{|\mathbf{u}|^2\})^2} \nabla_w |\kappa(\mathbf{u})| \\ &= -\frac{2|\kappa(\mathbf{u})|}{(E\{|\mathbf{u}|^2\})^3} \nabla_w E\{|\mathbf{u}|^2\} \\ &+ \frac{\text{sign}(|\kappa(\mathbf{u})|)}{(E\{|\mathbf{u}|^2\})^2} \nabla_w |\kappa(\mathbf{u})| \end{aligned} \quad (8)$$

Where

$$\nabla_w E\{|\mathbf{u}|^2\} = 2E\{\mathbf{u}^* \mathbf{x}\} \quad (9)$$

and

$$\nabla_w |\kappa(\mathbf{u})| = (4E\{|\mathbf{u}|^2 \mathbf{u}^* \mathbf{x}\} - 8E\{\mathbf{u}^* \mathbf{x}\} E\{|\mathbf{u}|^2\} - 4E\mathbf{u}^* 2E\mathbf{u} \mathbf{x} \quad (10)$$

Combining (9) and (10), we obtained the gradient vector as in (5) of Section 1.

The second derivatives of $\mathcal{G}(\mathbf{W})$:

$$\begin{aligned} \nabla_w^2 \mathcal{G}(\mathbf{W}) &= \nabla_w |\kappa(\mathbf{u})| \nabla_w \frac{1}{(E\{|\mathbf{u}|^2\})^2} \\ &+ |\kappa(\mathbf{u})| \nabla_w^2 \frac{1}{(E\{|\mathbf{u}|^2\})^2} \\ &+ \nabla_w \frac{1}{(E\{|\mathbf{u}|^2\})^2} \nabla_w |\kappa(\mathbf{u})| \\ &+ \frac{1}{(E\{|\mathbf{u}|^2\})^2} \nabla_w^2 |\kappa(\mathbf{u})| \end{aligned} \quad (11)$$

where

$$\begin{aligned} \nabla_w^2 |\kappa(\mathbf{u})| &= \nabla_w (4E\{[w^H x x^H w] x x^H w\} \\ &- 8 [w^H E\{x x^H\} w] E\{x x^H\} w \\ &- 4E\{x x^T\} w^* w^T E\{x^* x^H\} w) \end{aligned} \quad (12)$$

$$\begin{aligned} \nabla_w E\{[w^H x x^H w] x x^H w\} \\ &= (E\{|y|^2 |x|^2\} \\ &+ 2E\{x^2 y^{*2}\}) \end{aligned} \quad (13)$$

$$\begin{aligned} \nabla_w [w^H E\{x x^H\} w] E\{x x^H\} w \\ &= 2(E\{x y^*\})^2 \\ &+ E\{|y|^2 |x|^2\} \end{aligned} \quad (14)$$

$$\begin{aligned} \nabla_w E\{x x^T\} w^* w^T E\{x^* x^H\} w \\ &= 2E\{x y\} E\{x^H y^*\} \end{aligned} \quad (15)$$

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References

[1]. Mandic D., Javidi S., Souretis G., and Goh S., "Why a complex valued solution for a real domain problem," in IEEE Workshop on Machine Learning for Signal Processing, pp. 384–389, Aug. 2007.

- [2]. Mandic D. and Goh S., *Complex Valued Nonlinear Adaptive Filters: Noncircularity, Widely Linear and Neural Models*. Wiley, 2009.
- [3]. Pham D. and Garat P., "Blind separation of mixture of independent sources through a quasi-maximum likelihood approach," *IEEE Trans.Signal Process.*, vol. 5, no. 7, pp. 1712–1725, Jul. 1997.
- [4]. Comon P., "Tensor diagonalization, a useful tool in signal processing," in *Proc. 10th IFAC Symposium on System Identification (IFAC- SYSID)*, Copenhagen, Denmark, Jul. 4–6, 1994.
- [5]. Lacoume J., Ruix P., "Source Identification: A solution based on the cumulants," *Proc. 4th ASSP Workshop Spectral Estimation Modeling*, Minneapolis, MN, pp.199-203, Aug. 1988.
- [6]. Saba, T., and Altameem, A. (2013) Analysis of vision based systems to detect real time goal events in soccer videos, *Applied Artificial Intelligence*, vol.27(7), pp. 656-667.
- [7]. Leong W. and Mandic D., "Post-nonlinear blind extraction in the presence of ill-conditioned mixing," *IEEE Transactions on Circuits and Systems I*, vol. 55, pp. 2631–2638, Oct. 2008.
- [8]. Wiggins R., "Minimum entropy deconvolution," *Geoexploration*, vol. 16, pp. 21–35, 1978.
- [9]. Donoho D., "On minimum entropy deconvolution," *Applied Time Series Anal. Symp.*, vol. 2, pp. 565–608, 1981.
- [10]. Shalvi O. and Weinstein E., "New criteria for blind deconvolution of nonminimum phase systems (channels)," *IEEE Trans. Inf. Theory*, vol. 36, no. 2, pp. 312–321, Mar. 1990
- [11]. Delfosse N. and Loubaton P., "Adaptive blind separation of independent sources: A deflation approach," *Signal Process.*, vol. 45, no. 1, pp. 59–83, Jul. 1995.
- [12]. Rehman, A., and Saba, T. (2013) An intelligent model for visual scene analysis and compression, *International Arab Journal of Information Technology*, vol.10(13), pp. 126-136.
- [13]. Hyvärinen A., Karhunen J., and Oja E., *Independent Component Analysis*. Wiley, 2001.
- [14]. Bermejo S., "Finite sample effects in higher order statistics contrast functions for sequential blind source separation," *IEEE Signal Process. Lett.*, vol. 12, no. 6, pp. 81–84, Jun. 2005.
- [15]. Nandi A., Ed., *Blind Estimation Using Higher-Order Statistics*. Boston, MA: Kluwer, 1999.
- [16]. Amari S., Douglas S., Cichocki A., and Yang H., "Novel on-line adaptive learning algorithms for blind deconvolution using the natural gradient approach," in *Proc. 11th IFAC Symp. System Identification*, vol. 3, July 1997, pp. 1057–1062.
- [17]. Møller F., "A Scaled Conjugate Gradient Algorithm for Fast Supervised Learning", neural,1993.
- [18]. Vincent E., Gribonval R, and Févotte C., "Performance measurement in blind audio source separation", *IEEE Trans. Audio, Speech, Language Process.*, vol. 14, no. 4, pp. 1462–1469, Jul. 2006.
- [19]. www ldc.upenn.edu/Catalog/LDC93S1.html.
- [20]. Tarassenko L., Townsend N. and Clifford G., "Medical signal processing using the software monitor", In proceeding of IEEE/DERA workshop on Intelligent signal processing, 2001.
- [21]. Elarbi-Boudiher, M. Rehman, A. Saba, T. (2011) Video motion perception using optimized Gabor filter, *International journal of physical sciences*, vol.6(12), pp. 2799-2806.
- [22]. Meethongjan, K. Dzulkipli, M. Rehman, A. Altameem, A. Saba, T. (2013) An intelligent fused approach for face recognition, *Journal of Intelligent Systems* vol.22(2), pp. 197-212, DOI: <https://doi.org/10.1515/jisys-2013-0010>
- [23]. Saba T, Al-Zahrani S, Rehman A. (2012) Expert system for offline clinical guidelines and treatment *Life Sci Journal*, vol.9(4), pp. 2639-2658.
- [24]. Saba, T. Rehman, A. Elarbi-Boudiher, M. (2014). Methods and Strategies on Off-Line Cursive Touched Characters Segmentation: A Directional Review, *Artificial Intelligence Review* vol. 42 (4), pp. 1047-1066. doi 10.1007/s10462-011-9271-5.
- [25]. Haron, H. Rehman, A., Wulandhari, L.A., Saba, T. (2011) Improved vertex chain code based mapping algorithm for curve length estimation, *Journal of Computer Science* vol. 7(5), pp. 736-743.
- [26]. Rehman, A. and Saba, T. (2011). Document skew estimation and correction: analysis of techniques, common problems and possible solutions *Applied Artificial Intelligence*, vol. 25(9), pp. 769-787. doi. 10.1080/08839514.2011.607009
- [27]. Rehman, A., Saba, T. (2011) Performance analysis of character segmentation approach for cursive script recognition on benchmark database, *Digital Signal Processing*, vol. 21(3), pp. 486-490, doi.10.1016/j.dsp.2011.01.016.
- [28]. Selamat, A. Phetchanchai, C. Saba, T. Rehman, A. (2010). Index financial time series based on zigzag-perceptually important points, *Journal of Computer Science*, vol. 6(12), pp. 1389-1395, doi. 10.3844/jcssp.2010.1389.1395.
- [29]. Haron, H. Rehman, A. Adi, DIS, Lim, S.P. and Saba, T. (2012). Parameterization method on B-Spline curve. *mathematical problems in engineering*, vol. 2012, doi:10.1155/2012/640472.

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