Septic B-Spline Collocation method for numerical solution of the Equal Width Wave (EW) equation

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Abstract: Numerical solutions of the Equal Width Wave (EW) equation are obtained by Septic B-Spline collocation method using Rubin and Graves linearization technique [16]. The motion of a single solitary wave and interaction of two solitary waves are studied to validate the accuracy and efficiency of the proposed method. Accuracy of the method is discussed by computing the errors norms L_2, L_∞ and conservative quantities. The analytic values of invariants C_1 C_2 and C_3 along with other results show that the present method is a successful numerical technique for solving EW equation. This numerical scheme is based on forward difference scheme in time and theta-weighted scheme in space and is unconditionally stable.
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1. INTRODUCTION

This paper is concerned with the numerical solution of the Equal Width Wave (EW) equation based on collocation method using septic B-Spline. This equation was first introduced by Morrison et. al. [12] as model equation to describe the non-linear dispersive waves. Many methods have been proposed to solve the EW and Modified Equal Width Wave (MEW) equations. Garacia-Archilla [5] used the spectral method for the solution of EW equation. Dag and Saka [1] solved the EW equation by using cubic B-Spline collocation method . Saka [17] used a finite elemet method for numerical solution of EW equation. Ramos [14] investigated solitary waves of EW and Regularized Long Wave (RLW) equations. Zaki [23] worked on solitary waves introduced by the boundary forced EW equation. Khalifa and Raslan [10] used the finite difference methods for EW equation. Gardner and G.A Gardner [6] solved the EW equation with the Galerkin method using Cubic B-Spline as a trial and test function. Zaki [21] obtained the numerical solution of the EW equation by using least-squares method. Esen applied a Lumped Galerkin method [4] on quadratic B-splines finite element for solving EW and MEW equations. Raslan [15] studied the Generalized Equal Width Wave (GEW) equation by using collocation method based on quadratic B-spine to obtain the numerical solution of a single solitary wave. Dag et. al. [18] obtained the numerical solutions of the EW equation by three different methods. Khalifa et. al. [9] studied the numerical analysis for the EW Equation. Evans and Raslan [2] obtained the solution of solitary wave for the GEW equation. Wazwaz [20] used the tanh and sine-cosine methods to obtained numerical approximation of MEW equation and its invariants. Hamdi et al. [19] obtained the exact solutions of the GEW equation. Zaki [22] worked on solitary wave interactions for the modified equal width equation. The Equal Width Wave Equation has the form

$$U_t + UU_x - \mu U_{xxt} = 0, \qquad (1)$$

where U is the amplitude, μ is a positive parameter, the subscripts x and t denote the space and time partial differentiation. EW equation represents an alternative to the RLW equation [8]. We seek numerical solution subject to the following initial and boundary conditions $U \to 0$ as $x \to \pm\infty$, $U(x,0) = f(x)$, and $u_x(a,t) = 0$, $u_x(b,t) = 0$,

$$u_{xx}(a,t) = 0, u_{xx}(b,t) = 0,$$
$$u_{xxx}(a,t) = 0, \ u_{xxx}(b,t) = 0,$$

where $a \le x \le b$ and *f(x)* is a localized disturbance inside the closed interval [a,b]. The organization of this paper is as follows. Septic B-splines are explained in section 2. Numerical stability is given in section 3. Numerical results are provided in section 4. Finally some conclusions are drawn.

2. Septic B-spline Collocation Method

The interval [a,b] is partitioned into N finite elements of uniformly equal length h by the knots x_m, m = 0,1,2,3…N such that $a = x_0 < x_1 < x_2 \ldots\ldots\ldots. < x_N = b$ and $h = \frac{b-a}{N}$. The septic B-spline function $B_m(x), m = -3, -2, -1 \ldots\ldots., N+3$ at these knots is defined as

$$B_m(x)=\frac{1}{h^7}\begin{cases}(x-x_{m-4})^7, & [x_{m-4},x_{m-3}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7, & [x_{m-3},x_{m-2}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7+28(x-x_{m-2})^7, & [x_{m-2},x_{m-1}],\\ (x-x_{m-4})^7-8(x-x_{m-3})^7+28(x-x_{m-2})^7-56(x-x_{m-1})^7, & [x_{m-1},x_m],\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7+28(x_{m+2}-x)^7-56(x_{m+1}-x)^7, & [x_m,x_{m+1}],\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7+28(x_{m+2}-x)^7, & [x_{m+1},x_{m+2}]\\ (x_{m+4}-x)^7-8(x_{m+3}-x)^7, & [x_{m+2},x_{m+3}],\\ (x_{m+4}-x)^7, & [x_{m+3},x_{m+4}],\\ 0, & otherwise.\end{cases} \quad (2)$$

The set of B-splines $\{B_{-3},B_{-2},\dots.B_{N+3}\}$ forms a basia for the functions over theinterval [a,b]. A global approximation $U_N(x,t)$ to the exact solution u(x,t) takes the form

$$U_N(x,t)=\sum_{m=-3}^{N+3}\delta_m(t)B_m(x), \quad (3)$$

where $\delta_m(t)$ are unknown time dependent quantities which are determined from collocation boundary and initial conditions. The nodal values U_m, U'_m, U''_m, U'''_m at the knots x_m are obtained from Eqs (2) and (3) in the following form

$$U_m=U(x_m)=\delta_{m-3}+120\delta_{m-2}+1191\delta_{m-1}+2416\delta_m+1191\delta_{m+1}+120\delta_{m+2}+\delta_{m+3},$$

$$U'_m=U'(x_m)=\frac{7}{h}(-\delta_{m-3}-56\delta_{m-2}-245\delta_{m-1}+245\delta_{m+1}+56\delta_{m+2}+\delta_{m+3}),$$

$$U''_m=U''(x_m)=\frac{42}{h^2}(\delta_{m-3}+24\delta_{m-2}+15\delta_{m-1}-80\delta_m+15\delta_{m+1}+24\delta_{m+2}+\delta_{m+3}),$$

$$U'''_m=U'''(x_m)=\frac{210}{h^3}(-\delta_{m-3}-8\delta_{m-2}+19\delta_{m-1}-\delta_{m+1}+8\delta_{m+2}+\delta_{m+3}), \quad (4)$$

where dashes represent differentiation with respect to the space variable.

Eq. (1) can be written as

$$(U-\mu U_{xx})_t+UU_x=0. \quad (5)$$

The time derivative of Eq. (5) is discretized by a first order accurate forward difference formula and by using the θ-weighted $(0\le\theta\le1)$, scheme to the space derivative at two time levels to get the equation

$$\frac{(U^{n+1}-\mu U^{n+1}_{xx})-(U^n-\mu U^n_{xx})}{\Delta t}+\theta(UU_x)^{n+1}+(1-\theta)(UU_x)^n=0, \quad (6)$$

where, Δt is time step and the superscripts n and n+1 are successive time levels. In this work we take $\theta=\frac{1}{2}$. Hence Eq. (6) is written as

$$\frac{(U^{n+1}-\mu U^{n+1}_{xx})-(U^n-\mu U^n_{xx})}{\Delta t}+\frac{(UU_x)^{n+1}+(UU_x)^n}{2}=0. \quad (7)$$

The non-linear term in Eq. (7) is approximated by applying Taylor series given in [12] as

$$(UU_x)^{n+1}_m\approx U^n_m(U_x)^{n+1}_m+U^{n+1}_m(U_x)^n_m-U^n_m(U_x)^n_m. \quad (8)$$

At the nth time step we denote U_m, U'_m, U''_m *and* U'''_m at the knots x_m by the following expressions

$$L_{m1}=\delta^n_{m-3}+120\delta^n_{m-2}+1191\delta^n_{m-1}+2416\delta^n_m+1191\delta^n_{m+1}+120\delta^n_{m+2}+\delta^n_{m+3},$$

$$L_{m2} = \frac{7}{h}(-\delta_{m-3}^n - 56\delta_{m-2}^n - 245\delta_{m-1}^n + 245\delta_{m+1}^n + 56\delta_{m+2}^n + \delta_{m+3}^n),$$

$$L_{m3} = \frac{42}{h^2}(\delta_{m-3}^n + 24\delta_{m-2}^n + 15\delta_{m-1}^n - 80\delta_m^n + 15\delta_{m+1}^n + 24\delta_{m+2}^n + \delta_{m+3}^n),$$

$$L_{m4} = \frac{210}{h^3}(-\delta_{m-3}^n - 8\delta_{m-2}^n + 19\delta_{m-1}^n - 19\delta_{m+1}^n + 8\delta_{m+2}^n + \delta_{m+3}^n). \qquad (9)$$

Using the knots $x_m, m = 0,1,2, \dots N$ as the collocation points, the following recurrence relation at the point x_m is obtained

using Eqs. (6)-(9)

$$Z_1\delta_{m-3}^{n+1} + Z_2\delta_{m-2}^{n+1} + Z_3\delta_{m-1}^{n+1} + Z_4\delta_m^{n+1} + Z_5\delta_{m+1}^{n+1} + Z_6\delta_{m+2}^{n+1} + Z_7\delta_{m+3}^{n+1} = 2h^2(L_1 - \mu L_2), \qquad (10)$$

where

$$Z_1 = L_{m0} - 7h\Delta t L_{m1} - 84\mu,$$

$$Z_2 = 120L_{m0} - 392h\Delta t L_{m1} - 2016\mu,$$

$$Z_3 = 1191L_{m0} - 1715h\Delta t L_{m1} - 1260\mu,$$

$$Z_4 = 2416L_{m0} + 6720\mu,$$

$$Z_5 = 1191L_{m0} + 1715h\Delta t L_{m1} - 1260\mu,$$

$$Z_6 = 120L_{m0} + 392h\Delta t L_{m1} - 2016\mu$$

$$Z_7 = L_{m0} + 7h\Delta t L_{m1} - 84\mu,$$

$$L_{m0} = h^2(2 + \Delta t L_{m2}), \qquad (11)$$

where m=0,1,2….N.

The Eq. (10) relates parameters at adjacent time levels. From the above general scheme as stated in Eq. (10) and using the values of m=0,1,2,….N, a septa diagonal matrix is produced containing N+1 equations in N+7 unknowns in the form of δ_i, i=-3,-2 -1…N+3. In order to obtained a unique solution, we eliminate the parameters $\{\delta_{-3}^{n+1}, \delta_{-2}^{n+1}, \delta_{-1}^{n+1}, \delta_{N+1}^{n+1}, \delta_{N+2}^{n+1}, \delta_{N+3}^{n+1}\}$ from Eq. (10). The values of these parameters are obtained from Eq. (9) and the collocation boundary conditions as given below

$$\delta_{-1} = -\frac{40}{27}\delta_0 + \frac{14}{9}\delta_1 + \frac{8}{9}\delta_2 + \frac{1}{27}\delta_3,$$

$$\delta_{-2} = -\frac{220}{27}\delta_0 - \frac{55}{18}\delta_1 - \frac{35}{9}\delta_2 - \frac{11}{54}\delta_3,$$

$$\delta_{-3} = -\frac{280}{3}\delta_0 + 35\delta_1 + 56\delta_2 + \frac{10}{3}\delta_3,$$

$$\delta_{N+1} = \frac{1}{27}\delta_{N-3} + \frac{8}{9}\delta_{N-2} + \frac{14}{9}\delta_{N-1} - \frac{40}{27}\delta_N,$$

$$\delta_{N+2} = -\frac{11}{54}\delta_{N-3} - \frac{35}{9}\delta_{N-2} - \frac{55}{18}\delta_{N-1} + \frac{220}{27}\delta_N,$$

$$\delta_{N+3} = \frac{10}{3}\delta_{N-3} + 56\delta_{N-2} + 35\delta_{N-1} - \frac{280}{3}\delta_N, \qquad (12)$$

By eliminating the parameters from Eq. (10) given in Eq. (12) a linear system of (N+1) equations in (N+1) unknowns parameters δ_i, i=0,1,2,3,…..N is obtained, which is solved by a septa diagonal solver for $\delta_i^n, n = (1,2,3 \dots.)$. Finally the approximate solutions U(x,t) are obtained from Eq. (10). The initial parameters δ_m^0, are determined by using the initial and boundary conditions with the help of the following expressions

$$U'(x_0,0) = u'(x_0,0) = \frac{7}{h}(-\delta_{-3}^0 - 56\delta_{-2}^0 - 245\delta_{-1}^0 + 245\delta_1^0 + 56\delta_2^0 + \delta_3^0) = 0,$$

$$U''(x_0,0) = u'''(x_0,0) = \frac{42}{h^2}(\delta_{-3}^0 + 24\delta_{-2}^0 + 15\delta_{-1}^0 - 80\delta_0^0 + 15\delta_1^0 + 24\delta_2^0 + \delta_3^0) = 0,$$

$$U'''(x_0,0) = u'''(x_0,0) = \frac{210}{h^3}(-\delta_{-3}^0 - 8\delta_{-2}^0 + 19\delta_{-1}^0 - 19\delta_1^0 + 8\delta_2^0 + \delta_3^0) = 0,$$

$$U(x_m,0) = u(x_m,0) = \delta_{m-3}^0 + 120\delta_{m-2}^0 + 1191\delta_{m-1}^0 + 2416\delta_m^0 + 1191\delta_{m+1}^0 + 120\delta_{m+2}^0 + \delta_{m+3}^0 = f(x_m), m = 0, 1, 2, \dots N,$$

$$U'''(x_N,0) = u'''(x_N,0) = \frac{210}{h^3}(-\delta_{N-3}^0 - 8\delta_{N-2}^0 + 19\delta_{N-1}^0 - 19\delta_{N+1}^0 + 8\delta_{N+2}^0 + \delta_{N+3}^0) = 0,$$

$$U''(x_N,0) = u''(x_N,0) = \frac{42}{h^2}(\delta_{N-3}^0 + 24\delta_{N-2}^0 + 15\delta_{N-1}^0 - 80\delta_N^0 + 15\delta_{N+1}^0 + 24\delta_{N+2}^0 + \delta_{N+3}^0) = 0,$$

$$U'(x_N.0) = u'(x_{N,}0) = \frac{7}{h}(-\delta_{N-3}^0 - 56\delta_{N-2}^0 - 24\delta_{N-1}^0 + 245\delta_{N+1}^0 + 56\delta_{N+2}^0 + \delta_{N+3}^0) = 0. \qquad (13)$$

Eq. (13), consists of $(N+1)\times(N+1)$ system of equations which can also be solved by a septa-diagonal solver.

3. Stability Analysis

The non-linear term UU_x in the scheme is linear zed by equating U as a constant k, using the Von-Neumann [11] stability method .The linear zed from of the proposed scheme is given as

$$k_1\delta_{m-3}^{n+1} + k_2\delta_{m-2}^{n+1} + k_3\delta_{m-1}^{n+1} + k_4\delta_m^{n+1} + k_5\delta_{m+1}^{n+1} + k_6\delta_{m+2}^{n+1} + k_7\delta_{m+3}^{n+1} = k_8\delta_{m-3}^n + k_9\delta_{m-2}^n + k_{10}\delta_{m-1}^n + k_{11}\delta_m^n + k_{12}\delta_{m+1}^n + k_{13}\delta_{m+2}^n + k_{14}\delta_{m+3}^n, \quad (14)$$

where

$k_1 = 2h^2 - 7hk\Delta t - 8\mu,$

$k_2 = 240h^2 - 392hk\Delta t - 2016\mu,$

$k_3 = 2382h^2 - 1715hk\Delta t - 1260\mu,$

$k_4 = 4832h^2 + 6720\mu,$

$k_5 = 2382h^2 + 1715hk\Delta t - 1260\mu,$

$k_6 = 240h^2 + 392hk\Delta t - 2016\mu,$

$k_7 = 2h^2 + 7hk\Delta t - 84\mu,$
$k_8 = 2h^2 + 7hk\Delta t - 84\mu,$
$k_9 = 240h^2 + 392hk\Delta t - 2016\mu,$
$k_{10} = 2382h^2 + 1715hk\Delta t - 1260\mu,$
$k_{11} = 4832h^2 + 6720\mu,$
$k_{12} = 2382h^2 - 1715hk\Delta t - 1260\mu,$
$k_{13} = 240h^2 - 392hk\Delta t - 2016\mu,$
$k_{14} = 2h^2 - 7hk\Delta t - 84\mu.$

Substitution of $\delta_m^n = \xi^n \exp(i\mu mh)$ in the general scheme (14), where $i = \sqrt{-1}$ leads to

$\xi((k_1 \exp(-3i\mu h) + k_2 exp(-2i\mu h) + k_3 \exp(-i\mu h) + k_4 + k_5 \exp(i\mu h) + k_6 \exp(2i\mu h) + k_7 \exp(3i\mu h) = k_8 \exp(-3i\mu h) + k_9 exp(-2i\mu h) + k_{10} \exp(-i\mu h) + k_{11} + k_{12} \exp(i\mu h) + k_{13} \exp(2i\mu h) + k_{14} \exp(3i\mu h)).$ (15)

Simplifying Eq. (15) we get

$\xi = \frac{A-iB}{A+iB}$, where

A= $(4h^2 - 168\mu)\cos(3\mu h) + (480h^2 - 4032\mu)\cos(2\mu h) + (4764h^2 - 5220\mu)\cos(\mu h) + (4832h^2 + 6720\mu)$,

B= $(14hk\Delta t)\sin(3\mu h) + (784hk\Delta t)\sin(2\mu h) + (3430hk\Delta t)\sin(\mu h) + (4832h^2 + 6720\mu)$, so that $|\xi|^2 = \frac{A^2+B^2}{A^2+B^2} = 1$, hence $\xi = 1$, shows that the scheme for EW equation is unconditionally stable.

4. The numerical tests and problems

The numerical method proposed in the previous section is tested for single solitary wave and interaction of two solitary waves. The accuracy of the method is measured using the following error norms

$L_\infty = \max_i |u_i - U_i|,\ L_2 = \sqrt{h\sum_{i=0}^{N}|u_i - U_i|^2}$

where u and U denote the exact and approximate solutions respectively. The analytical solution of EW equation given in the literature [18] is written as

$$u(x,t) = 3c\ \operatorname{sech}^2\big(k(x - x_0 - vt)\big), \quad (16)$$

where $k = \frac{1}{\sqrt{4\mu}}$ measures width of the wave pulse, v=c is the wave velocity and x_0 is an arbitrary constant. The initial condition is given by

$$u(x,t) = 3c\ \operatorname{sech}^2\big(k(x - x_0)\big), \quad (17)$$

The conservative properties of the EW equation related to mass, momentum and energy given in [13] are determined by assessing the following three invariants,

$$C_1 = \int_a^b U dx, \qquad C_2 = \int_a^b (U^2 + \mu(U_x)^2)dx$$

$$C_3 = \int_a^b U^3. \quad (18)$$

4.1 Motion of single solitary wave

Problem 1. We use the parameters $\mu = 1$, $x_0 = 10, c = 0.1$ *and* 0.03, h=0.03 ,0.15 and time step $\Delta t = 0.05$, run up time t=80 so that the solitary waves have amplitudes 0.3 and 0.09. The initial conditions are extracted from the exact solution whereas the following boundary conditions are used

$$u_x(0,t) = u_{xx}(0,t) = u_{xxx}(0,t) = 0,$$
$$u_x(30,t) = u_{xx}(30,t) = u_{xxx}(30,t) = 0,$$

The results are compared with [3,7,21]. L_2, L_∞ and the three invariants C_1, C_2 *and* C_3 are recorded in Table 1 and Table 2.

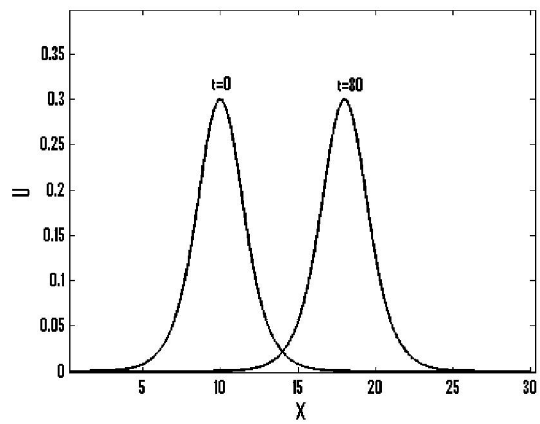

Fig. 1. Solitary wave profile for amplitude 0.3 corresponding to problem 1.

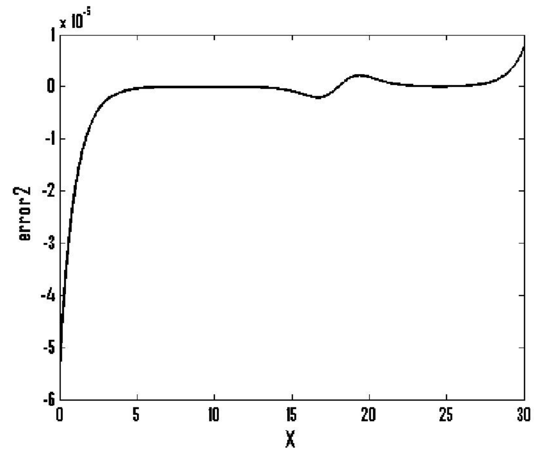

Fig2. Error graph for c=0.1 corresponding to problem 1.

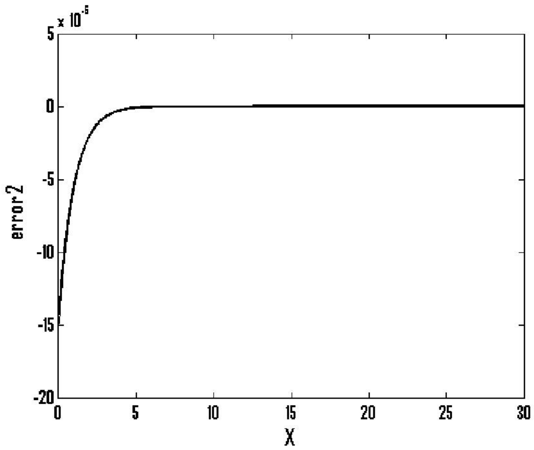

Fig3. Error graph for c = 0.3 corresponding to problem 1.

Table 1. Invariants and norms for the single solitary wave

T	$L_2 \times 10^3$	$L_\infty \times 10^3$	C_1	C_2	C_3
0	0.0	0.0	1.19995	0.28799	0.05760
10	0.02468	0.03443	1.20002	0.28800	0.05760
20	0.03377	0.04710	1.20004	0.28800	0.05760
30	0.0313	0.05176	1.20005	0.28800	0.5760
40	0.03837	0.05347	1.20005	0.28800	0.05760
50	0.3886	0.05411	1.20005	0.28800	0.05760
60	0.38907	0.05434	1.20005	0.28800	0.5760
70	0.03922	0.05443	1.20005	0.28800	0.05760
80	0.03962	0.05446	1.20004	0.28800	0.05760
80[3]	0.024697	0.016425	1.23387	0.029915	0.06097
80[7]	3.849	2.646	1.1910	0.2855	0.05582
80[21]	7.444	4.373	1.1964	0.2858	0.0569

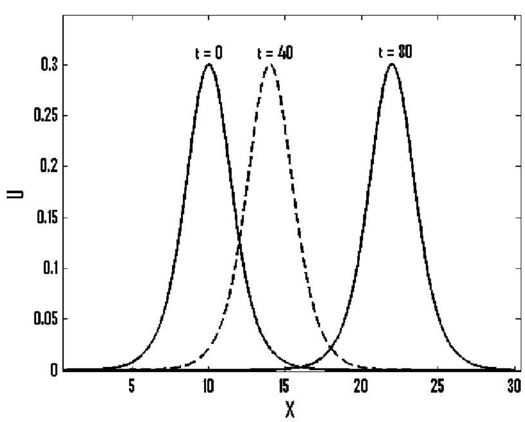

Fig. 4. Solitary wave profile for amplitude 0.3

The exact values of these invariants are given in reference [18] as

$$C_1 = \frac{6c}{k} = 1.2, C_2 = \frac{12c^2}{k} + \frac{48kc^2\mu}{5} = 0.288$$

and $C_3 = \frac{144c^3}{5k} = 0.0576\,.$ (19)

When c = 0.03, then the analytical values Given in the reference [18] are

$$C_1 = 0.36, C_2 = 0.02592 \; and \; C_3 = 0.00156. \quad (20)$$

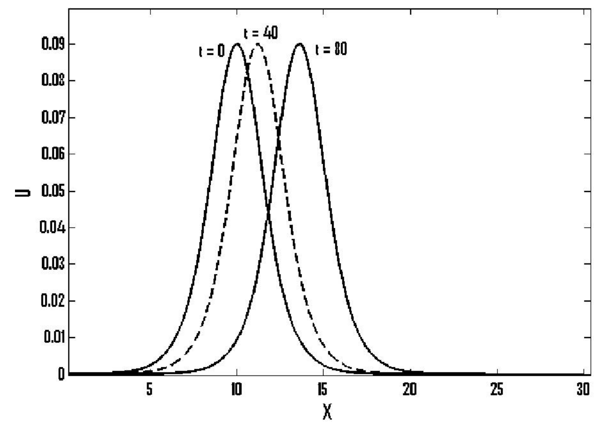

Fig. 5 Solitary wave profile for amplitude 0.09 corresponding to problem 1.

Table 2. Invariants and norms for the single solitary wave

T	$L_2 \times 10^3$	$L_\infty \times 10^3$	C_1	C_2	C_3
0	0.00034	0.00048	0.35998	0.02591	0.00155
10	0.00330	0.00424	0.35999	0.02592	0.00156
20	0.00528	0.00737	0.35999	0.02592	0.00156
30	0.00695	0.00969	0.36000	0.02592	0.00156
40	0.00818	0.01142	0.36000	0.02592	0.00156
50	0.00909	0.01269	0.36000	0.02592	0.00156
60	0.00978	0.01364	0.36001	0.02592	0.00156
70	0.01028	0.01434	0.36001	0.02592	0.00156
80	0.01064	0.01485	0.36001	0.02592	0.00156
80[21]	0.22	0.16	0.3593	0.0259	0.00155
80[3]	0.002683	0.001836	0.36665	0.02658	0.00162

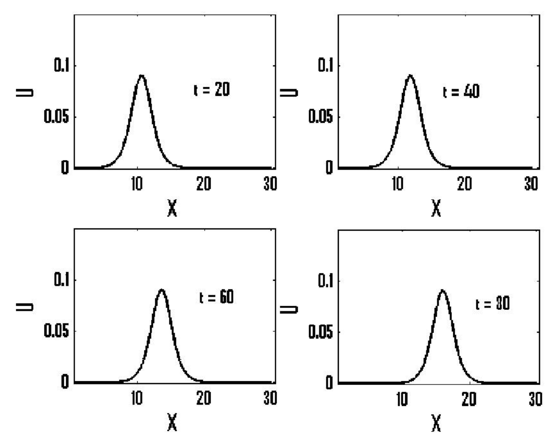

Fig. 6. Motion of single solitary wave at Selected times corresponding to problem 1.

It is clear from Table 1 that the error norms of the present method are smaller than those of [3,7,21]. The values of the three invariants and two error norms obtained by the present method are shown in Table 1,2 for different amplitudes c=0.1, 0.03. The analytical values of the three invariants are given in Eqs. (19)-(20).

4.2 Interaction of two solitary waves

Consider Eq. (1) along with collocation boundary conditions

$$U_x(0,t) = U_{xx}(0,t) = U_{xxx}(0,t) = 0,$$
$$U_x(80,t) = U_{xx}(80,t) = U_{xxx}(80,t) = 0,$$

and the initial condition

$$U(x,0) = 3A_1 \sec h^2\big(k_1(x - x_1 - A_1)\big) + 3A_2 \sec h^2\big(k_2(x - x_2 - A_2)\big). \quad (19)$$

which represents interaction of two solitary waves, one with amplitude $3A_1$ and other with amplitude $3A_2$ placed initially at $x = x_1$ and $x = x_2$.

Problem 2. For interaction of two solitary waves, using the parameters $\mu = 1$, $x_1 = 10$, $x_2 = 25$, $k_1 = k_2 = 0.5$, $A_1 = 1.5$, $A_2 = 0.75$
$c = 0.1$, h=0.1, $\Delta t = 0.1$, and the runup time t=30. The values of the three invariants obtained by the present method are recorded in Table 3. They are compared with some earlier methods [15,18] in the literature. The results of the present method are in a good agreement. The analytical values given in [18] are

$$C_1 = 12(A_1 + A_2) = 27,$$
$$C_2 = 28.8(A_1^2 + A_2^2) = 81,$$
$$C_3 = 57.6(A_1^3 + A_2^3) = 218.7.$$

Table 3. Values of three invariants at different time.

T	C_1	C_2	C_3
0	26.99999	81.00000	218.70300
1	27.00011	81.00044	218.70289
5	27.00019	81.00034	218.70213
10	27.00019	80.99414	218.66204
15	27.00019	80.94087	218.32369
20	27.00019	80.99234	218.65317
25	27.00019	81.00013	218.70157
30	27.00019	81.00045	218.70312
25[15]	27.124800	81.220630	218.698300
30[18]	26.99973	80.99778	218.69094

Table 4. Values of the three invariants at various times.

T	C_1	C_2	C_3
0.0	-2.400000	97.916015	-58.519209
1.0	-2.399999	97.916019	-58.519207
4.0	-2.399999	97.905400	-58.485293
6.0	-2.399999	97.889200	-58.378440
8.0	-2.399999	97.907313	-58.384056
10.0	-2.399999	97.8511039	-58.397048

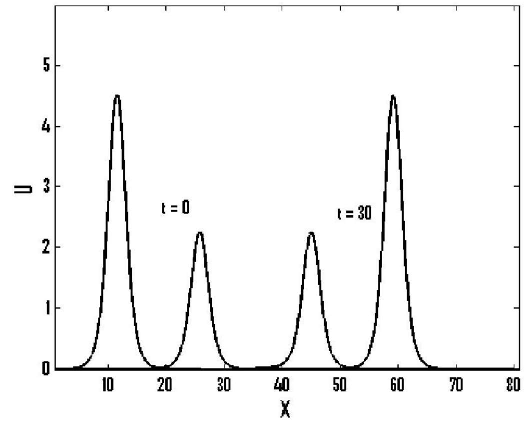

Fig. 7. Interaction of two solitary waves at t = 0, t = 30 corresponding to problem 2.

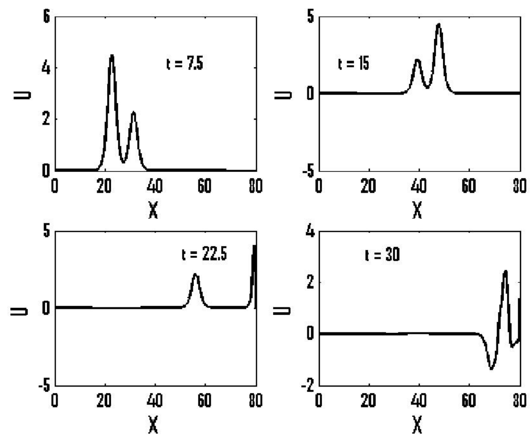

Fig.8 Interaction of two solitary waves at selected times corresponding to problem 2

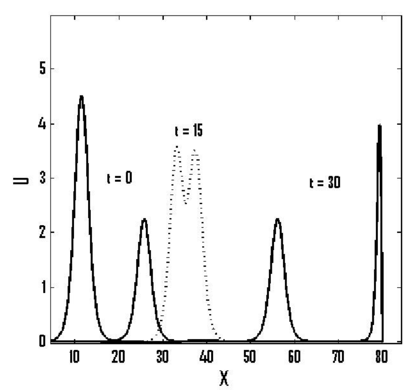

Fig. 9. Interaction of two solitary waves at t = 0, t = 15 and t = 30 corresponding to problem 2.

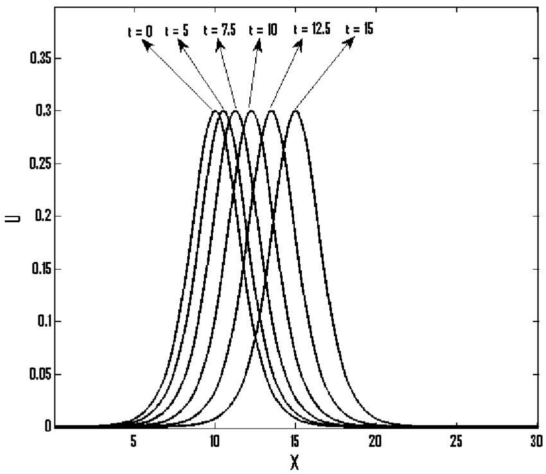

Fig. 10. Motion of single solitary wave at Selected times corresponding to problem 1.

4.3 Interaction of solitons

For this simulation we use Gardner [24], which has simulated the interaction of two waves (a positive and a negative) for the EW equation. The collision was confirmed given in references [25-28]. The initial condition is using as

$u(x,0) = u_1 + u_2,$ where

$u_i = 3A_i\,\mathrm{sech}(k(x_i - x_0 - A_i))^2,\ for\ i = 1,2.$

Problem 3. We use the parameters $a = 0, b = 80, \mu = 1, x_1 = 23, x_2 = 38, k_1 = k_2 = 0.5, A_1 = 1.2, A_2 = -1.4, c = 0.1, h = 0.1, \Delta t = 0.1$ *and run up time* $t = 10$. The values of the three invariants obtained by the present method are recorded in Table 4 for two solitary waves. The analytical values of the three invariants given in the reference [29] as listed below

$$C_1 = 12(A_1 + A_2) = -2.4,$$
$$C_2 = 28.8(A_1^2 + A_2^2) = 97.9,$$
$$C_3 = 57.6(A_1^3 + A_2^3) = -58.5216.$$

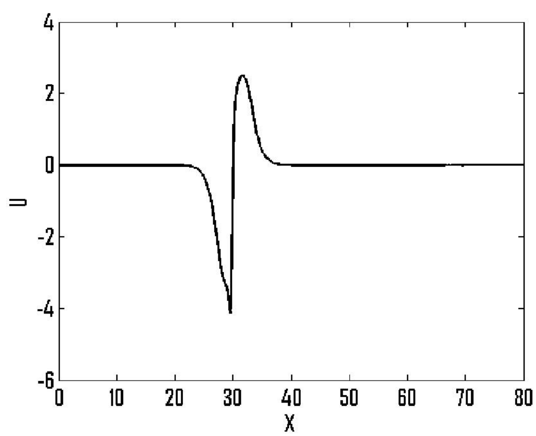

Fig11. Interaction of two opposite solitons corresponding to problem 3.

5. Conclusion

We studied motion of a single solitary wave, interaction of two solitary waves and interaction of two solitons using septic B-spline collocation method .Three tests problems are given and the results are compared with some earlier work from literature. In this work, the performance and accuracy of the septic B-spline collocation method was demonstrated by evaluating the two error norms L_2 , L_∞ and three invariants on the motion of signal solitary wave and interaction of two solitary waves. The error norms are sufficiently small and the invariants are very close to the analytic values.

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