Forced Vibration Responses of Functionally Graded Conical Shell under Harmonic Load

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Abstract: An efficient method for studying the forced vibration of functionally graded conical shell is demonstrated. Hamilton's principle with the Rayleigh-Ritz method is utilized to obtain the equation of motion of the FG conical shell. A group of simpler principal vibration modes of the conical shell with two simply supported boundaries are demonstrated. The natural frequencies of FG conical shell can be obtained by solving eigenvalue problem of the equation of motion and the steady responses of forced vibration can also be obtained by solving the equation of motion. The exponential variation of material properties in the thickness direction of the plate is considered. Numerical comparisons with the outcomes in the open literature are done to confirm the present methodology. In addition, the forced vibration responses of a functionally graded conical shell are also computed.
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Introduction

The thought of the construction of functionally graded materials (FGMs) was first initiated in 1984 by a team of Japanese materials researchers Koizumi M. (1993). From 20 years ago, FGMs have tried a remarkable raise in terms of investigation and development programs. Worldwide dispersion and division of the outcomes through publications, international symposiums and exchange programs attests to this increasing development. They have many acquired requests in rocket engine pieces, space plan body, nuclear reactor components, first wall of fusion reactor, engine components, turbine blades, hip implant and other engineering and technological applications. A detailed discussion on their design, processing and applications can be found in Refs. (Obata Y et al., 1993; Wetherhold RC et al., 1996; Suresh S et al., 1998; Miyamoto Y et al., 1999; Kieback B et al., 2003) Commonly, FGMs are constructed from a mixture of metals and ceramics and are more characterized by a continuous and smooth change of the mechanical properties from one surface to another. In addition, a mixture of ceramic and metal with a continuously changing volume fraction can be easily fabricated. (see, for instance, (Miyamoto Y et al.,1999; Reddy JN et al.,1998; Liew KM et al.,2002)) FGMs now have been considered as one of the most encouraging candidates for coming smart composites in numerous engineering fields such as fast computers, aerospace, biomedical industry, and environmental sensors. Regarding the obvious significance in practical applications, the vibration behaviors of FGM shell structures have attracted increasing research effort. Conical shells are greatly utilized in many engineering applications. It is essential to do the analysis of vibration of conical shells for the purpose of vibration control, dynamical design and others. To the present time, there are many papers that have been published on the vibration problems of conical shells (Khatri and Asnani, 1995; Goldberg et al., 1960; Chang, 1978; Serpico, 1963). Lam and Li (1999) and Li (2000) studied the frequency characteristics of free vibration of rotating conical shell using the Galerkin method. Fares et al. (2004) inquire into the design and active vibration control of composite laminated conical shells. They utilized the Liapunov–Bellman theory to acquire the controlled deflections of the shells. Liew et al. (2005) analyzed the free vibration of thin conical shells utilizing the element-free kp-Ritz method and discussed the frequency properties under different parameters. Chai et al. (2006) investigated the spatially distributed microscopic control characteristics of distributed actuator patches on a rocket conical shell. Sofiyev et al. (2008) studied the vibration and stability of orthotropic conical shells with non-homogeneous material properties under a hydrostatic pressure. It should be noted that Liew and Lim (Liew et al.,1995; Lim and Liew, 1996, 1995) methodically investigated the free vibration of shallow conical shells using the pb-2 Ritz method. Despite the fact that many studies on the dynamic problems of the conical shells have been published, the forced vibration problems of the functionally graded conical shells have to be comprehensively studied. Through the forced vibration analysis, an efficient way for computing the forced vibration responses can be acquired and further utilized in the vibration control and dynamic designs of the conical shells.

Commonly, the equation of motion of the conical shell is extremely complex and some of the coefficients of the equation of motion are variables (Soedel, 1981). Therefore, it is difficult to analytically solve the equation of motion of the conical shell. In this study, an efficient method for the forced vibration analysis of functionally graded conical shells is presented. The steady responses of the forced vibration of functionally graded conical shells can also be acquired by solving the equation of motion. In addition, the effect of changing (g) gradient index on the force vibration responses is studied.

2. Functionally gradient materials

A thin, isotropic and functionally graded conical shell with constant thickness is assumed. Fig. 1 shows the schematic diagram of the conical shell. The two boundaries of the conical shell are simply supported (S-S). In which for functionally graded materials with two constituent materials Poisson ratio υ is assumed to be constant through the thickness, whereas the variations through the thickness of Young's modulus E(η) and the mass density per unit volume ρ(η). According some investigators (Nie et al., (2007); Zhong et al., (2003); Gu et al., (1997)) assumed the exponential variation of material properties in the thickness direction of the plates, can be written as

$$E(\eta) = E_m e^{g(\frac{\eta}{h}+\frac{1}{2})} \quad (1)$$

$$\rho(\eta) = \rho_m e^{g(\frac{\eta}{h}+\frac{1}{2})} \quad (2)$$

in which η is the thickness coordinate ($\frac{-h}{2} \le \eta \le \frac{h}{2}$), and $g \ge 0$ is the gradient index, and E_m denote the Young's module of the bottom materials, ρ_m denote the mass density per unit volume of the bottom materials.

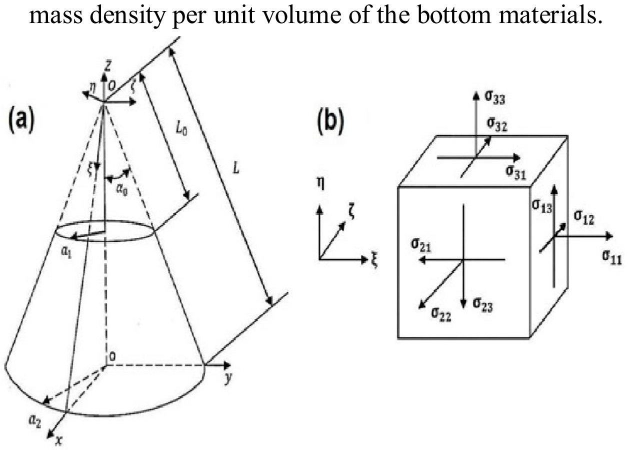


Figure 1. The schematic diagram of a FGM conical shell (a) The geometry and the curvilinear surface and Cartesian Coordinate Systems; (b) the infinitesimal shell element and the corresponding stresses.

3. Equation of motion of FG conical shell

A thin and FG conical shell with constant thickness is assumed. Figure 1 shows the schematic diagram of the conical shell. The two boundaries of the conical shell are simply supported (S-S). The corresponding curvilinear surface coordinates $0-\xi\zeta\eta$ and Cartesian coordinates $O-xyz$ are also shown in Figure 1. The curvilinear surface coordinates are limited to be orthogonal ones which coincide with the lines of principal curvature of the neutral surface. For conical shells, the lines of principal curvature of the neutral surface are the circles (ζ-axis) and parallel meridians (ξ-axis).

For a thin conical shell, plane stress condition is assumed and the constitutive relation is given by

$$\{\sigma\} = [Q]\{e\}, \quad (3)$$

Where $\{\sigma\}$ is the stress vector, $\{e\}$ is the strain vector and $[Q]$ is the reduced stiffness matrix. The stress vector and the strain vector are defined as

$$\{\sigma\}^T = \{\sigma_{11}\ \sigma_{22}\ \sigma_{12}\ \sigma_{23}\}, \quad (4)$$

$$\{e\}^T = \{\varepsilon_{11}\ \varepsilon_{22}\ \varepsilon_{12}\ \varepsilon_{23}\}, \quad (5)$$

Where σ_{11} and σ_{22} are the stresses in ξ and ζ directions, and σ_{12} is the shear stress on the ξζ plane, and σ_{23} is the shear stress on the ζη plane. ε_{11} and ε_{22} are the strains in the ξ and ζ directions, and ε_{12} is the shear strain on the ξζ plane, and ε_{23} is the shear strain on the ζη plane. The reduced stiffness matrix is defined as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 \\ 0 & 0 & Q_{66} & 0 \\ 0 & 0 & 0 & Q_{66} \end{bmatrix} \quad (6)$$

For FGM materials the reduced stiffness Q_{ij} $(i,j = 1, 2 \text{ and } 6)$ are defined as

$$Q_{11} = Q_{22} = \frac{E(\eta)}{(1-\mu^2)}, \quad Q_{12} = \frac{\mu E(\eta)}{(1-\mu^2)}, \quad Q_{66} = \frac{E(\eta)}{2(1+\mu)}, \quad (7)$$

Where $E(\boldsymbol{\eta})$ is the Young's modulus and μ is the Poisson's ratio. According Soedel (1981) and Love's shell theory, the components in the strain vector $\{e\}$ are defined as

$$\varepsilon_{11} = e_1 + \eta k_1, \varepsilon_{22} = e_2 + \eta k_2, \varepsilon_{12} = \gamma + 2\eta\tau, \quad \varepsilon_{23} = e_{23} \quad (8)$$

where e_1, e_2, γ and e_{23} are the references surface strains, and k_1, k_2 and τ are the surface curvatures. These surface strains and curvatures are defined as

$$\{e_1,e_2,\gamma,e_{23}\}=\left\{\frac{\partial u}{\partial \xi},\frac{1}{\xi\, sin\alpha_0}\frac{\partial v}{\partial \zeta}+\frac{u}{\xi}+\frac{w}{\xi\, tan\alpha_0},\frac{1}{\xi\, sin\alpha_0}\frac{\partial u}{\partial \zeta}+\frac{\partial v}{\partial \xi}-\frac{v}{\xi},\frac{v}{\xi\, tan\alpha_0}\right\}, \quad (9)$$

$$\{k_1,k_2,\tau\}=\left\{-\frac{\partial^2 w}{\partial \xi^2},\frac{1}{\xi^2 sin\alpha_0 tan\alpha_0}\frac{\partial v}{\partial \zeta}-\frac{1}{\xi^2 sin^2\alpha_0}\frac{\partial^2 w}{\partial \zeta^2}-\frac{1}{\xi}\frac{\partial w}{\partial \xi},-\frac{1}{\xi \sin\alpha_0}\frac{\partial^2 w}{\partial \xi \partial \zeta}+\frac{1}{2\xi \tan\alpha_0}\frac{\partial v}{\partial \xi}-\frac{1}{\xi^2 \tan\alpha_0}v+\frac{1}{\xi^2 \sin\alpha_0}\frac{\partial w}{\partial \zeta}\right\} \quad (10)$$

For a thin conical shell the force and moment resultants are defined as

$$\{N_\xi,N_\zeta,N_{\xi\zeta},N_{\zeta\eta}\}=\int_{-h/2}^{h/2}\{\sigma_{11},\sigma_{22},\sigma_{12},\sigma_{23}\}\, d_\eta, \quad (11)$$

$$\{M_\xi,M_\zeta,M_{\xi\zeta}\}=\int_{-h/2}^{h/2}\{\sigma_{11},\sigma_{22},\sigma_{12}\}Z\, d_\eta, \quad (12)$$

Substituting Eq. (3), with substitution from Eq. (8), into Eqs. (11) and (12), the constitutive equation is obtained as

$$\{N\}=[S]\{\varepsilon\}, \quad (13)$$

where $\{N\}$ and $\{\varepsilon\}$ are, respectively, defined as

$$\{N\}^T=\{N_\xi\ N_\zeta\ N_{\xi\zeta}\ N_{\zeta\eta}\ M_\xi\ M_\zeta\ M_{\xi\zeta}\}, \quad (14)$$

$$\{\varepsilon\}^T=\{e_1\ e_2\ \gamma\ e_{23}\ k_1\ k_2\ 2\tau\}, \quad (15)$$

and $[S]$ is defined as

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & B_{11} & B_{12} & 0\\ A_{12} & A_{22} & 0 & 0 & B_{12} & B_{22} & 0\\ 0 & 0 & A_{66} & 0 & 0 & 0 & B_{66}\\ 0 & 0 & 0 & A_{66} & 0 & 0 & 0\\ B_{11} & B_{12} & 0 & 0 & D_{11} & D_{12} & 0\\ B_{12} & B_{22} & 0 & 0 & D_{12} & D_{22} & 0\\ 0 & 0 & B_{66} & 0 & 0 & 0 & D_{66} \end{bmatrix} \quad (16)$$

where A_{ij}, B_{ij} and D_{ij} (i, j=1, 2 and 6) are the extensional, coupling and bending stiffnesses defined as

$$\{A_{ij},B_{ij},D_{ij}\}=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{ij}\{1,z,z^2\}\, d_z. \quad (17)$$

Notice that, unlike a homogeneous isotropic conical shell where the coupling stiffnesses B_{ij} do not exist, the B_{ij} are present in the constitutive equation for a functionally graded conical shell. This arises because of material properties asymmetry about the mid-plane, when Q_{ij} are a function of Z for functionally gradient materials. Notice that, unlike a homogeneous isotropic conical shell where the coupling stiffnesses B_{ij} do not exist, the B_{ij} are present in the constitutive equation for a functionally graded conical shell. This arises because of material properties asymmetry about the mid-plane, when Q_{ij} are a function of Z for functionally gradient materials. The strain energy and kinetic energy and virtual work of a conical shell can be written as

$$T=\frac{1}{2}\int_{-h/2}^{h/2}\int_0^{2\pi}\int_{L_0}^{L}\rho(\eta)\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]\xi\, sin\alpha_0\, d_\xi d_\zeta d_\eta, \quad (18)$$

$$U=\frac{1}{2}\int_{-h/2}^{h/2}\int_0^{2\pi}\int_{L_0}^{L}\{\varepsilon\}^T[S]\{\varepsilon\}\xi\, sin\alpha_0\, d_\xi d_\zeta d_\eta \quad (19)$$

$$\delta W=\int_0^{2\pi}\int_{L_0}^{L}(q_1\delta u+q_2\delta v+q_3\delta w)\,\xi\, sin\alpha_0\, d_\xi d_\zeta \quad (20)$$

where q_1, q_2 and q_3 are the distributed load components per unit area along the $\xi,\zeta,$ and η directions and are assumed to act on the neutral surface of the shell. The units of q_1, q_2 and q_3 are $[N/m^2]$. For simply supported conical shell, the boundary conditions at both ends can be written as

$$v(l_0,\zeta,t)=v(l,\zeta,t)=0,$$
$$w(l_0,\zeta,t)=w(l,\zeta,t)=0$$
$$N_{11}(l_0,\zeta,t)=N_{11}(l,\zeta,t)=0,$$
$$M_{11}(l_0,\zeta,t)=M_{11}(l_0,\zeta,t)=0, \quad (21)$$

In the Rayleigh–Ritz method, the shape of deformation of the continuous system is approximated using a series of trial shape functions that must satisfy the geometric boundary conditions. The displacements can be written as

$$u(\xi,\zeta,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}U_{ij}(\xi,\zeta)p_{ij}(t)=U^T(\xi,\zeta)P(t),$$
$$v(\xi,\zeta,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}V_{ij}(\xi,\zeta)\, r_{ij}(t)=V^T(\xi,\zeta)r(t),$$
$$w(\xi,\zeta,t)=\sum_{i=1}^{m}\sum_{j=1}^{n}W_{ij}(\xi,\zeta)s_{ij}(t)=W^T(\xi,\zeta)s(t), \quad (22)$$

Where U, V and W are the trial shape functions or the principal vibration modes, and p, r, s are the generalized coordinates or modal coordinates. It is necessary to demonstrate the formulations of the principle mode shapes U, V and W in Eq. (22). A number of vibration mode shapes of conical shells have been utilized. For instance, Lam and Li (1999) and Li (2000) utilized a kind of vibration modes of conical shells that are similar to those of cylindrical shells. The main mode shapes of conical shells with simply supported boundaries can be declared as

$$U_{ij}(\xi,\zeta)=cos\left[\frac{i\pi(\xi-l_0)}{l-l_0}\right]cos(j\zeta),$$
$$V_{ij}(\xi,\zeta)=sin\left[\frac{i\pi(\xi-l_0)}{l-l_0}\right]sin(j\zeta),$$
$$W_{ij}(\xi,\zeta)=sin\left[\frac{i\pi(\xi-l_0)}{l-l_0}\right]cos(j\zeta),$$
$$i=1,2,\dots,m\,;j=1,2,\dots,n, \quad (23)$$

where i and j denote the wave numbers in the meridional and circumferential directions.

Hamilton's principle is written by

$$\int_{t_1}^{t_2} \delta(T-U)dt + \int_{t_1}^{t_2} \delta W dt = 0, \quad (24)$$

where T the kinetic energy, U strain energy and W work (Khatri and Asnani, 1995; Mecitog˘lu, 1996) , t_1 and t_2 are the integration time limits, $\delta(0)$ denotes the first variation. Then the kinetic energy, strain energy and work are expressed in terms of the generalized coordinates and displacement shape functions. Substituting Eq. (22) into Eq. (18), the kinetic energy is written by

$$T = \frac{1}{2}\frac{dp^T}{dt}M_1\frac{dp}{dt} + \frac{1}{2}\frac{dr^T}{dt}M_2\frac{dr}{dt} + \frac{1}{2}\frac{ds^T}{dt}M_3\frac{ds}{dt} \quad (25)$$

where M_1, M_2 and M_3 are the modal mass matrices of the functionally graded conical shell and they are listed in Appendix A. Substituting Eq. (22) into (19), the strain energy is written by

$$U = \frac{1}{2}p^T K_1 p + \frac{1}{2}p^T K_2 r + \frac{1}{2}p^T K_3 s + \frac{1}{2}r^T K_2^T p + \frac{1}{2}r^T K_4 r + \frac{1}{2}r^T K_5 s + \frac{1}{2}s^T K_3^T p + \frac{1}{2}s^T K_5^T r + \frac{1}{2}s^T K_6 S, \quad (26)$$

where $K_1, K_2, \dots, K_6$ are the modal stiffness matrices which are also presented in Appendix A. Substituting Eq. (22) into Eq. (20), the virtual work is expressed as

$$\delta W = q_1 F_{q1}\delta p + q_2 F_{q2}\delta r + q_3 F_{q3}\delta s, \quad (27)$$

where F_{q1}, F_{q2} and F_{q3} are the forcing matrices which are also given in Appendix A. Substituting Eqs. (25), (26) and (27) into Eq. (24) and fulfilling the variation operation in terms of p, r and s. They can be acquired as

$$M_t\frac{d^2X}{dt^2} + K_t X = Q, \quad (28)$$

where M_t the generalized mass matrix, K_t the stiffness matrix , Q the forcing matrix and written by

$$M_t = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}, \quad K_t = \begin{bmatrix} K_1 & K_2 & K_3 \\ K_2^T & K_4 & K_5 \\ K_3^T & K_5^T & K_6 \end{bmatrix}$$

$$Q = [F_{q1}q_1 \quad F_{q2}q_2 \quad F_{q3}q_3]^T,$$
$$X = [p^T \quad r^T \quad s^T]^T, \quad (29)$$

a solution of Eq. (28) is in the form

$$X(t) = X_0 e^{\lambda t}, \quad (30)$$

where $\boldsymbol{\lambda}$ is the characteristic values or the eigenvalue and X_0 is the eigenvector. Substituting Eq. (30) into the homogeneous differential equation of Eq.(28) leads to the following standard eigenvalue problem:

$$(M_t\lambda^2 + K_t)X_0 = 0, \quad (31)$$

The imaginary parts of the eigenvalues are the natural frequencies of the functionally graded conical shell. The distributed loads are assumed to be harmonic and deviate from symmetry. Since this is never exactly true for engineering applications to assume that the load does where ω is the frequency of the dynamic loads and q_{10}, q_{20} and q_{30} are the amplitudes. The steady state not deviate from symmetry. The distributed loads are written by

$$q_1(\xi,\zeta,t) = q_{10}(\zeta)\sin\omega t, q_2(\xi,\zeta,t) = q_{20}(\zeta)\sin\omega t$$
$$q_3(\xi,\zeta,t) = q_{30}(\zeta)\sin\omega t, \quad (32)$$

solution of Eq. (28), according to the application of the external dynamic loads, can be expressed as

$$X_i(t) = A_i\sin\omega t, \quad i = 1,2,\dots,3mn, \quad (33)$$

where A_i is the amplitude that it should be determined. Substituting Eq. (33) into Eq. (28) yields

$$(K_t - \omega^2 M_t)A = Q_0, \quad (34)$$

Where $A = [A_1, A_2, \dots, A_{3mn}]^T$ and $Q_0 = [F_{q1}q_{10}, F_{q2}q_{20}, F_{q3}q_{30}]^T$. The amplitudes A_i of $X_i(t)$can be found by solving Eq. (34). Then, the steady state responses of the functionally graded conical shell can be found.

4. Results and discussions

4.1. Validation of the present method

So that to confirm the present methodology, the results are compared with the open literature in Table 1. In the numerical calculations, the non-dimensional frequency parameter is defined as (Lam and Li, 1999; Liew et al., 2005; Irie et al., 1984).

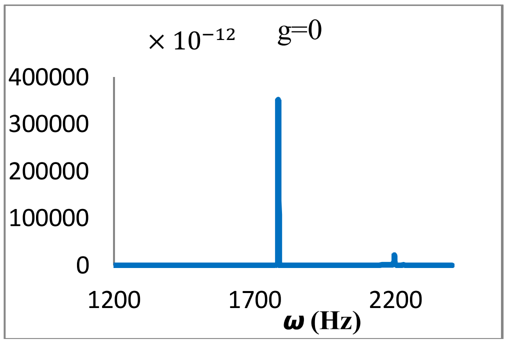
$$f = \omega_0\alpha_2\sqrt{\frac{\rho_m(1-\mu^2)}{E_m}}, \quad (35)$$

Table 1. Comparisons of frequency parameter f for the conical shell with S-S boundaries (m = 1, Metal).

	n	Irie et al (1984)	Lam and Li (1999)	Present
$\alpha_0 = 30^0$	2	0.7910	0.8420	0.84307
	3	0.7284	0.7376	0.74163
	4	0.6352	0.6362	0.64194
	5	0.5531	0.5528	0.55902
	6	0.4949	0.4950	0.50079
	7	0.4653	0.4661	0.47079
	8	0.4654	0.4660	0.46921
	9	0.4892	0.4916	0.49318
$\alpha_0 = 45^0$	2	0.6879	0.7655	0.76424
	3	0.6973	0.7212	0.72108
	4	0.6664	0.6739	0.67467
	5	0.6304	0.6323	0.63364
	6	0.6032	0.6035	0.60492
	7	0.5918	0.5921	0.59311
	8	0.5992	0.6001	0.60045
	9	0.6257	0.6273	0.62691
$\alpha_0 = 60^0$	2	0.5772	0.6348	0.63423
	3	0.6001	0.6238	0.62361
	4	0.6054	0.6145	0.61459
	5	0.6077	0.6111	0.61128
	6	0.6159	0.6171	0.61721
	7	0.6343	0.6350	0.63479
	8	0.6650	0.6660	0.66525
	9	0.7084	0.7101	0.70873

Figure 2. The amplitude of the displacements in the ξ, ζ and η directions at the position $(l_0 + \frac{s}{2}, \frac{\pi}{4})$ of the middle surface of functionally graded conical shell varying with the frequency ω of the dynamical load. (a) In the $\boldsymbol{\xi}$ direction;(b)in the $\boldsymbol{\zeta}$ direction;(c)in the $\boldsymbol{\eta}$ direction

(a) The Amplitudes of the Displacement u (m) for g=0, 1, 2



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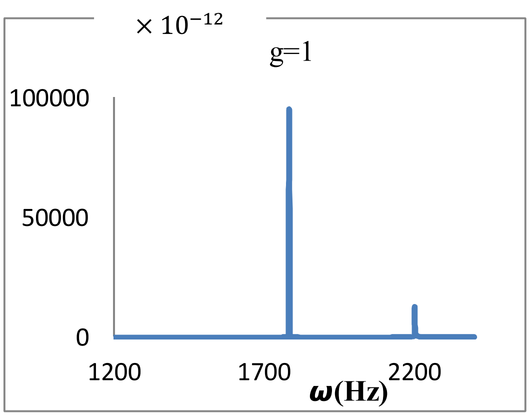
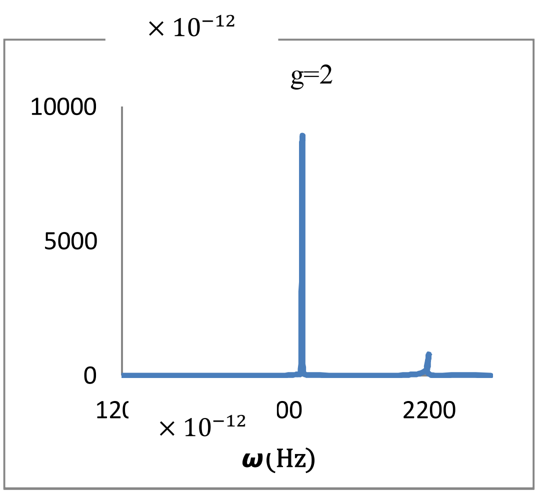
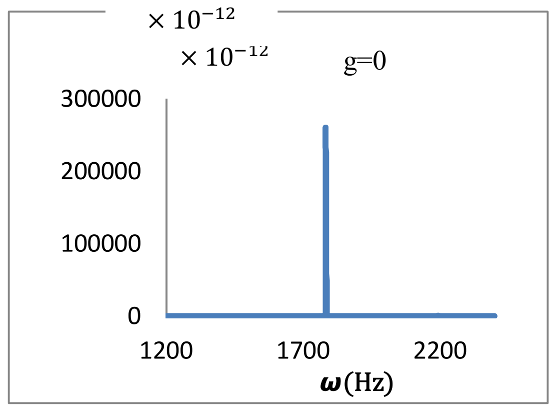
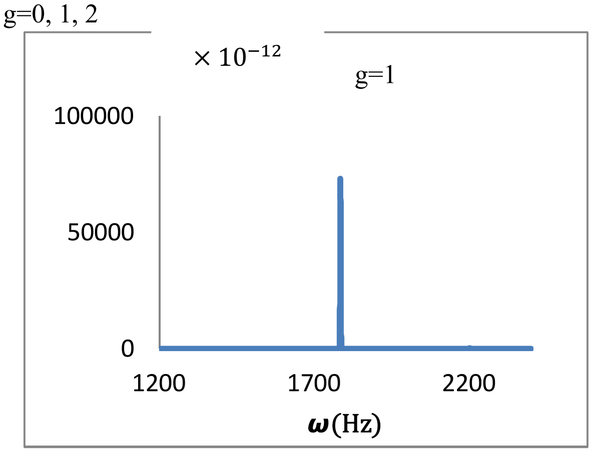


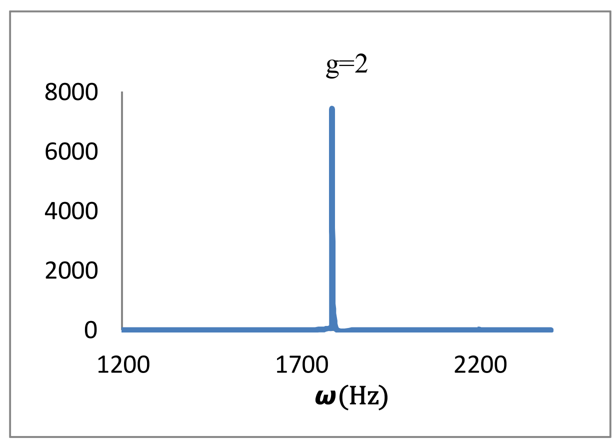


(b) The Amplitudes of the Displacement v (m) for g=0, 1, 2

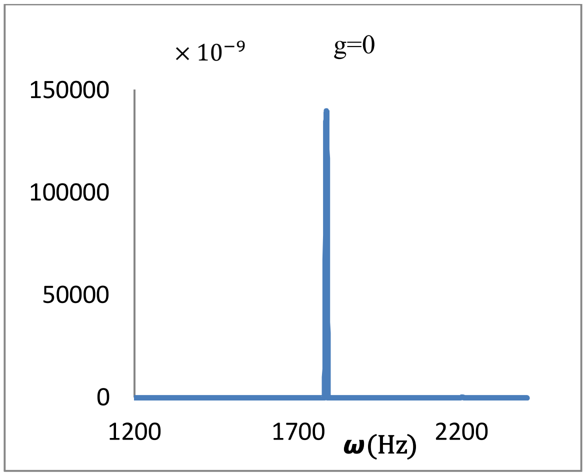


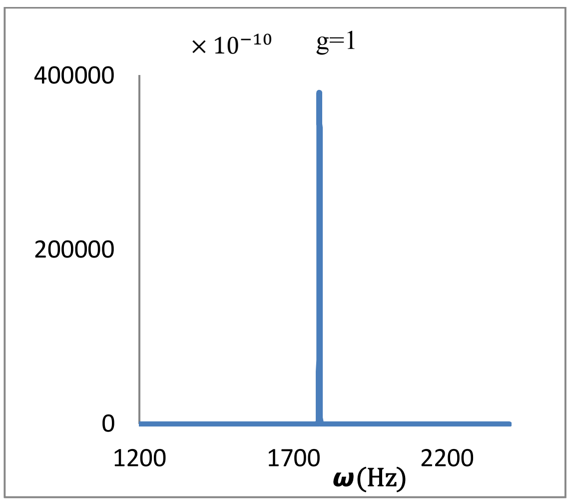
(C) The Amplitudes of the Displacement v (m) for g=0, 1, 2

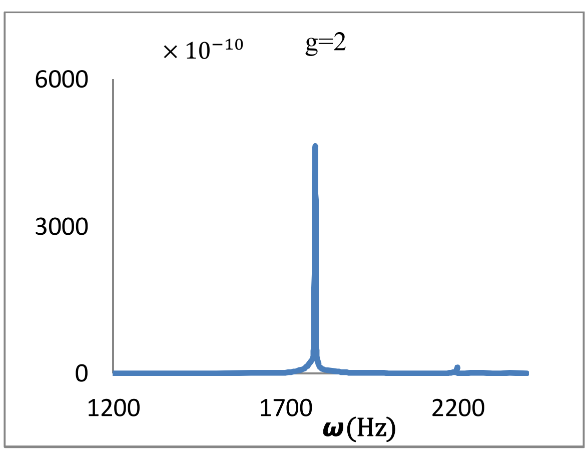




where ω_0 is the natural frequency of the conical shell in radians per second.







The constituent material properties of the functionally graded conical shell is $E_m = 70$ GPa, $\rho_m = 2710\ \mathrm{kg/m^3}$ and $\mu = \mu_C = 0.3$ for the material Al. The variation through the thickness of Young's modulus $E(\eta)$ and mass density per unit volume $\rho(\eta)$ are the same as Eqs. (1) and (2). The structural parameters are h =0.004 m, $h/a_2 = 0.01$, $(l - l_0)\sin\alpha_0 / a_2$ =0.25. For the case of the meridional wave number $m = 1$ and semi-vertex cone angles $\alpha_0 = 30^0, 45^0, 60^0$ the frequency parameters computed by Eq. (35) are listed in Table 1. Also the corresponding outcomes by Lam and Li (1999) and Irie et al. (1984) are listed in Table 1. It is clear from (Table 1) that the frequency parameters acquired by the Rayleigh-Ritz method are in good agreement with those in the open literature, which confirms the validity of the present analytical method. In addition, the principal mode shapes declared by Eq. (23) can be utilized for the conical shells with two simply supported boundaries. The forced vibration responses of functionally graded conical shell with two simply supported boundaries are computed. The structural parameters of the functionally graded conical shell sample are identical to those utilized in Section 3.1. The semi-vertex cone angle is $\alpha_0 = 30^0$. The radii at the two ends are $a_1 = 0.3$ m and $a_2 = 0.4$ m. l_0= 0.6 m and l= 0.8 m are the position coordinates of the conical shell in the curvilinear surface coordinates O-$\xi\zeta\eta$. Thus the length of the conical shell is $s = l - l_0$ = 0.2 m. q_{10}, q_{20} and q_{30} are the amplitudes of the external dynamic loads that all of them equal to 10 Pa. In the computation, m and n in Eq. (23) are all set to be 1. The amplitudes of the displacements in the ξ, ζ and η directions of the middle surface of functionally graded conical shell at position $(l_0 + \frac{s}{2}, \frac{\pi}{4})$ varying with the frequency ω (Hz) of external dynamic loads and the gradient index g are shown in Figure 2. There are some peak values of the displacements in Figure 2 that relate to the resonant responses of the functionally graded conical shell under the external dynamic loads. In addition, it is clear that the displacements in the frequency-response curves are reduced when (g) gradient index increase from 0 to 2.

5. Conclusions

The purpose of the investigation described in this paper is to determine the forced vibration responses of functionally graded conical shell under harmonic load. it is clear that the displacements in the frequency-response curves are reduced when g increase from 0 to 2. Hamilton's principle with the Rayleigh–Ritz method is utilized to obtain the equation of motion of the functionally graded conical shell.

The eigenvalue problem of the equation of motion is solved to obtain the natural frequencies of the functionally graded conical shell. The equation of motion is solved to obtain the steady responses of forced vibration. To validate the present method, numerical comparisons with the results in the open literature are done that show very good agreement. In addition, the forced vibration responses of functionally graded conical shell varying with the external dynamical loads and gradient index are calculated. This method can be utilized for other kinds of boundary conditions of the conical shell.

Appendix A. The expressions of the modal mass, modal stiffness and forcing matrices in Eqs. (29) are given by

$$M_1 = \sin\alpha_0 \int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2} UU^T\rho(\eta)\xi d\eta d\xi d\zeta$$

$$M_2 = \sin\alpha_0 \int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2} VV^T\rho(\eta)\xi d\eta d\xi d\zeta$$

$$M_3 = \sin\alpha_0 \int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2} WW^T\rho(\eta)\xi d\eta d\xi d\zeta$$

$$K_1 = \frac{\sin\alpha_0}{1-\mu^2}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \xi}\frac{\partial U^T}{\partial \xi}\xi + UU^T\frac{1}{\xi} + \mu\frac{\partial U}{\partial \xi}U^T\right.$$
$$\left. +\mu U\frac{\partial U^T}{\partial \xi}\right)E(\eta)d\eta d\xi d\zeta$$
$$+\frac{1}{\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\frac{\partial U}{\partial \zeta}\frac{\partial U^T}{\partial \zeta}\frac{1}{\xi}G(\eta)d\eta d\xi d\zeta$$

$$K_2 = \frac{1}{1-\mu^2}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(U\frac{\partial V^T}{\partial \zeta}\frac{1}{\xi}\right.$$
$$\left. + \mu\frac{\partial U}{\partial \xi}\frac{\partial V^T}{\partial \zeta}\right)E(\eta)d\eta d\xi d\zeta$$
$$+\frac{1}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \zeta}\frac{\partial V^T}{\partial \xi}\right.$$
$$\left. - \frac{\partial U}{\partial \zeta}V^T\frac{1}{\xi}\right)E(\eta)d\eta d\xi d\zeta$$
$$+\frac{\mu}{(1-\mu^2)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \xi}\right)\left(\frac{\partial V^T}{\partial \zeta}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$+\frac{1}{(1-\mu^2)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(U)\left(\frac{\partial V^T}{\partial \zeta}\right)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$+\frac{1}{2(1+\mu)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \zeta}\right)\left(\frac{\partial V^T}{\partial \xi}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{2}{2(1+\mu)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \zeta}\right)(V^T)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$

$$K_3 = \frac{\cos\alpha_0}{1-\mu^2}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(UW^T\frac{1}{\xi}\right.$$
$$\left. + \mu\frac{\partial U}{\partial \xi}W^T\right)E(\eta)d\eta d\xi d\zeta$$
$$-\frac{\sin\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \xi}\right)\left(\frac{\partial^2 W^T}{\partial \xi^2}\right)E(\eta)\eta\xi d\eta d\xi d\zeta$$
$$-\frac{\mu\sin\alpha_0}{(1-\mu^2)\sin^2\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \xi}\right)\left(\frac{\partial^2 W^T}{\partial \zeta^2}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{\mu\sin\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \xi}\right)\left(\frac{\partial W^T}{\partial \xi}\right)E(\eta)\eta d\eta d\xi d\zeta$$
$$-\frac{1}{(1-\mu^2)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(U)\left(\frac{\partial^2 W^T}{\partial \zeta^2}\right)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$-\frac{\sin\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(U)\left(\frac{\partial W^T}{\partial \xi}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{\mu\sin\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(U)\left(\frac{\partial^2 W^T}{\partial \xi^2}\right)E(\eta)\eta d\eta d\xi d\zeta$$
$$-\frac{2}{2(1+\mu)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \zeta}\right)\left(\frac{\partial^2 W^T}{\partial \xi\partial \zeta}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$+\frac{2}{2(1+\mu)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial U}{\partial \zeta}\right)\left(\frac{\partial W^T}{\partial \zeta}\right)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$

$$K_4 = \frac{1}{(1-\mu^2)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\frac{\partial V^T}{\partial \zeta}\frac{1}{\xi}\right.$$
$$\left. +\frac{\eta^2}{\tan^2\alpha_0}\frac{\partial V}{\partial \zeta}\frac{\partial V^T}{\partial \zeta}\frac{1}{\xi^3}\right)E(\eta)d\eta d\xi d\zeta$$

$$+\sin\alpha_0\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\frac{\partial V^T}{\partial \xi}\xi + VV^T\frac{1}{\xi}\right.$$
$$\left. -\frac{\partial V}{\partial \xi}V^T - V\frac{\partial V^T}{\partial \xi}\right)G(\eta)d\eta d\xi d\zeta + \frac{\sin\alpha_0}{\tan^2\alpha_0}\times$$
$$\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(VV^T\frac{1}{\xi} + \eta^2\left(\frac{\partial V}{\partial \xi}\frac{\partial V^T}{\partial \xi}\frac{1}{\xi} + 4VV^T\frac{1}{\xi^3} - 2\frac{\partial V}{\partial \xi}V^T\frac{1}{\xi^2} - 2V\frac{\partial V^T}{\partial \xi}\frac{1}{\xi^2}\right)\right)$$
$$G(\eta)d\eta d\xi d\zeta +$$
$$\frac{2}{(1-\mu^2)\sin\alpha_0\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\right)\left(\frac{\partial V^T}{\partial \zeta}\right)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$+\frac{2\cos\alpha_0}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\right)\left(\frac{\partial V^T}{\partial \xi}\right)E(\eta)\eta d\eta d\xi d\zeta$$
$$-\frac{3\cos\alpha_0}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\right)(V^T)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{3\cos\alpha_0}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(V)\left(\frac{\partial V^T}{\partial \xi}\right)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$+\frac{4\cos\alpha_0}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(V)(V^T)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$

$$K_5 = \frac{1}{(1-\mu^2)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\frac{\partial V}{\partial \zeta}\frac{W^T}{\xi}E(\eta)d\eta d\xi d\zeta$$
$$-\frac{1}{(1-\mu^2)\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\frac{\partial W^T}{\partial \xi}\frac{1}{\xi^2} + \mu\frac{\partial V}{\partial \zeta}\frac{\partial^2 W^T}{\partial \xi^2}\frac{1}{\xi} +\right.$$
$$\left.\frac{1}{\sin^2\alpha_0}\frac{\partial V}{\partial \zeta}\frac{\partial^2 W^T}{\partial \zeta^2}\frac{1}{\xi^3}\right)E(\eta)\eta^2 d\eta d\xi d\zeta +$$
$$\frac{2}{\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\frac{\partial W^T}{\partial \zeta}\frac{1}{\xi^2} - \frac{\partial V}{\partial \xi}\frac{\partial^2 W^T}{\partial \xi\partial \zeta}\frac{1}{\xi} + \frac{2V}{\xi^2}\frac{\partial^2 W^T}{\partial \xi\partial \zeta} -\right.$$
$$\left.\frac{2V}{\xi^3}\frac{\partial W^T}{\partial \zeta}\right)G(\eta)\eta^2 d\eta d\xi d\zeta + \frac{1}{(1-\mu^2)\tan^2\alpha_0}\times$$

$$\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\right)(W^T)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$-\frac{1}{(1-\mu^2)\sin^2\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\right)(\frac{\partial^2 W^T}{\partial \zeta^2})\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$-\frac{1}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\right)(\frac{\partial W^T}{\partial \xi})\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{\mu}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \zeta}\right)(\frac{\partial^2 W^T}{\partial \xi^2})E(\eta)\eta d\eta d\xi d\zeta$$
$$-\frac{2}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\right)(\frac{\partial^2 W^T}{\partial \xi\partial \zeta})E(\eta)\eta d\eta d\xi d\zeta$$

$$+\frac{2}{2(1+\mu)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\left(\frac{\partial V}{\partial \xi}\right)(\frac{\partial W^T}{\partial \zeta})\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$+\frac{2}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(V)(\frac{\partial^2 W^T}{\partial \xi\partial \zeta})\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{2}{2(1+\mu)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(V)(\frac{\partial W^T}{\partial \zeta})\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$K_6=\frac{\sin\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{\partial^2 W}{\partial \xi^2}\frac{\partial^2 W^T}{\partial \xi^2}\xi+\frac{\partial W}{\partial \xi}\frac{\partial W^T}{\partial \xi}\frac{1}{\xi}+$$
$$\mu\frac{\partial^2 W}{\partial \xi^2}\frac{\partial W^T}{\partial \xi}+\mu\frac{\partial W}{\partial \xi}\frac{\partial^2 W^T}{\partial \xi^2})E(\eta)\eta^2 d\eta d\xi d\zeta$$
$$+\frac{1}{(1-\mu^2)\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{1}{\sin^2\alpha_0}\frac{\partial^2 W}{\partial \zeta^2}\frac{\partial^2 W^T}{\partial \zeta^2}\frac{1}{\xi^3}$$
$$+\mu\frac{\partial^2 W}{\partial \xi^2}\frac{\partial^2 W^T}{\partial \zeta^2}\frac{1}{\xi}+\mu\frac{\partial^2 W}{\partial \zeta^2}\frac{\partial^2 W^T}{\partial \xi^2}\frac{1}{\xi}+\frac{\partial^2 W}{\partial \zeta^2}\frac{\partial W^T}{\partial \xi}\frac{1}{\xi^2}+\frac{\partial W}{\partial \xi}\frac{\partial^2 W^T}{\partial \zeta^2}\frac{1}{\xi^2})$$
$$E(\eta)\eta^2 d\eta d\xi d\zeta$$
$$+\frac{\sin\alpha_0}{(1-\mu^2)\tan^2\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}\frac{WW^T}{\xi}E(\eta)d\eta d\xi d\zeta+$$
$$\frac{4}{\sin\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{\partial^2 W}{\partial \xi\partial \zeta}\frac{\partial^2 W^T}{\partial \xi\partial \zeta}\xi+\frac{\partial W}{\partial \zeta}\frac{\partial W^T}{\partial \zeta}\frac{1}{\xi}-\frac{\partial^2 W}{\partial \xi\partial \zeta}\frac{\partial W^T}{\partial \zeta}$$
$$-\frac{\partial W}{\partial \zeta}\frac{\partial^2 W^T}{\partial \xi\partial \zeta})\frac{G(\eta)}{\xi^2}\eta^2 d\eta d\xi d\zeta-\frac{1}{(1-\mu^2)\sin\alpha_0\tan\alpha_0}\times$$
$$\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{\partial^2 W}{\partial \zeta^2})(W^T)\frac{E(\eta)\eta}{\xi^2}d\eta d\xi d\zeta$$
$$-\frac{\cos\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{\partial W}{\partial \xi})(W^T)\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{\mu\cos\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(\frac{\partial^2 W}{\partial \xi^2})(W^T)E(\eta)\eta d\eta d\xi d\zeta$$
$$-\frac{1}{(1-\mu^2)\sin\alpha_0\tan\alpha_0}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(W)(\frac{\partial^2 W^T}{\partial \zeta^2})\frac{E(\eta)\eta}{\xi^2}$$
$$d\eta d\xi d\zeta$$
$$-\frac{\cos\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(W)(\frac{\partial W^T}{\partial \xi})\frac{E(\eta)\eta}{\xi}d\eta d\xi d\zeta$$
$$-\frac{\mu\cos\alpha_0}{(1-\mu^2)}\int_0^{2\pi}\int_{l_0}^{1}\int_{-h/2}^{h/2}(W)(\frac{\partial^2 W^T}{\partial \xi^2})E(\eta)\eta d\eta d\xi d\zeta \quad (36)$$

$$F_{q_1}=\sin\alpha_0\int_0^{2\pi}\int_{l_0}^{1}U^T\xi d\xi d\zeta,$$
$$F_{q_2}=\sin\alpha_0\int_0^{2\pi}\int_{l_0}^{1}V^T\xi d\xi d\zeta,$$
$$F_{q3}=\sin\alpha_0\int_0^{2\pi}\int_{l_0}^{1}W^T\xi d\xi d\zeta,$$

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