A new integrated mathematical model for optimizing facility location and network design policies with facility disruptions

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Abstract: Considering the reliability in modeling of facility location problems is one of the most effective ways to hedge against failures of system from time to time. In reality, the combined facility location network design problem with respect to system reliability has a number of applications in industries and services. In this paper, a mixed integer non linear programming formulation is developed to model the combined facility location network design problem incorporating the option of facility hardening to hedge against the risk of facility disruptions. The new mathematical formulation first considers different costs including facility location, link construction, and transportation costs; it particularly takes the facility hardening cost of the system into account. Then, the proposed model is linearized by suitable techniques and, at the follow; a practical numerical example is presented in detail to illustrate the application of the proposed mathematical model. The results demonstrate the capability of the model.
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1. Introduction

In general, the facility location problems seeks optimal facility locations to minimize the one-time investment for facility constructions and the long-run transportation costs for serving spatially distributed customers. This topic has been studied extensively; see (Daskin 1995; Drezner and Hamacher 2004; Farahani and Hekmatfar 2009) for comprehensive surveys. As a significant point in modeling of facility location problems, proposing an efficient mathematical model which displays a more realistic description of the problem can give more practical solutions and also considerably reduce the related costs. Two significant topics that help reach such intention are network design and system reliability. The importance of these topics in modeling of facility location problems will be explained further.

In most of the typical facility location problems, it is assumed that the link arcs among potential facility locations in the considered network are predetermined. On the other hands, in network design, the basic problem is to optimally construct a network that enables some kind of flow, and possibly that satisfies additional constraints. The nodes are given and the network is constructed from a set of potential edges, or links and the flow involved is that between clients and facilities. As is suggested by the name, facility location–network design problem (FLNDP) combines facility location and network design topics. Facility location deals with optimally locating facilities. There are two main parties involved in any facility location problem: the facilities themselves and the clients of the facilities. Typically we want the facilities to be close to the clients, which can be defined in several ways, such as minimizing total travel cost. In FLNDP, the objective may be met using the means of both facility location and network design: by building both facilities and links (Cocking and Reinelt 2009).

In the literature review, FLNDP was first considered by Daskin et al. in 1993 (Daskin, Hurter et al. 1993). They presented some preliminary results which showed the effect of network design topic in mathematical modeling of facility location problems and their optimal solution. Later, Melkote (Melkote 1996) in his doctoral thesis researched three models for the FLNDP including UFLNDP, the capacitated facility location-network design problem (CFLNDP), and the maximum covering location-network design problem (MCLNDP). The results of the thesis were published in (Melkote and Daskin 2001a; Melkote and Daskin 2001b). Drezner and Wesolowsky (Drezner and Wesolowsky 2003) proposed a new network design problem with potential links, each of which could be either constructed at a given cost or not. Moreover, each constructed link could be constructed as either a one-way or two-way link. They developed four basic problems subject to two objective functions; finally, they solved the problems by a descent algorithm, a simulated annealing, a tabu search, and a genetic algorithm as main solution procedures. In another doctoral thesis, Cocking (Cocking 2008; Cocking and Reinelt 2009) expanded some efficient approaches to solve the static budget constrained (FLND) problem. Some useful algorithms were developed to find good upper bounds and good lower bounds on the optimal solution. Simple greedy heuristics, a local search heuristic, and meta-heuristics including simulated annealing (SA) and variable neighborhood search (VNS), as well as a custom heuristic based on the problem-specific structure of FLND were the main heuristics and metaheuristics that were proposed in Cocking's doctoral thesis. Besides, a branch-and-cut algorithm using heuristic solutions as upper bounds, and cutting planes to improve the lower bound of the problem were developed. The method reduced the number of nodes which were needed to approach optimality. Recently, Bigotte et al. (Bigotte, Krass et al. 2010) have proposed a mixed-integer optimization model for integrated urban hierarchy and transportation network planning. The model simultaneously determines which urban centers and which network links should be transferred to a new level of hierarchy in order to improve availability of all groups of facilities. Jabalameli and Mortezaei (JabalAmeli and Mortezaei 2011) proposed an extension of the CFLNDP in which the maximum amount of demands can be carried by a link is limited. They presented a bi-objective mixed integer programming formulation of the problem and developed a hybrid algorithm to solve the resulted problem. Contreras and Fernandez (Contreras and Fernández 2011) reviewed the relevant modeling aspects, alternative formulations and several algorithmic strategies for the FLNDP. In fact, they studied general network design problems in which, design decisions to locate facilities and to select links on an underlying network are combined with operational allocation and routing decisions to satisfy demands. Contreras et al. (Contreras, Fernández et al. 2012) presented a combined FLNDP to minimize the maximum customer-facility travel time. They developed and compared two mixed integer programming formulations by generalizing the model of the classical P-center problem so that the models simultaneously consider the location of facilities and the design of its underlying network.

As another significant subject, traditional models generally assume that the facilities, once built, remain operational forever. In reality, however, facility operations may be disrupted from time to time due to reasons such as natural disasters, power outages, operational accidents, labor actions or terrorist attacks. The failure of a facility will force its customers to either travel longer distances to obtain service from another facility, or give up service and incur a penalty. Both way, system operation cost increases and customer satisfaction deteriorates. The adverse effect may be further exacerbated if multiple facilities fail simultaneously(Zhang, Berman et al. 2009). So it can be easily said that system reliability is another important topic that can affect facility location and allocation. The importance of the reliability of system is recognized when a set of facilities has been constructed, but one or some of facilities occasionally become unavailable in situations such as inclement weather, labor actions, sabotage, or changes in ownership. There are different types of such catastrophic plight, many of which caused facilities to shut down including a series of mail-based anthrax attacks in the United States in 2001-2002 (Snyder 2003; Snyder and Ülker 2005; Snyder and Daskin 2005; Berman, Krass et al. 2007) and SARS outbreak in Toronto, Canada, in the summer of 2003 (Berman, Krass et al. 2007). It is observed that when a facility failure occurs, customers may have to be reassigned from their original facilities to the other available facilities, a condition that surely requires higher transportation or other special costs. Therefore, study of facility location problems with respect to the reliability of system and also network design can practically ameliorate solving of the mentioned problems in industries and services by obtaining more accurate and efficient solutions.

In the related traditional literature, Snyder and Daskin in 2003 were the first to propose an implicit formulation of the stochastic P-median and fixed charge problems based on the level assignments, in which the candidate sites are affected by random disruptions with equal probability (Snyder 2003; Snyder and Ülker 2005; Snyder and Daskin 2005). Shen et al. (Shen 2009) and Berman et al. (Berman, Krass et al. 2007) relaxed the assumption of uniform failure probabilities, formulated the stochastic fixed-charged facility location problem as a nonlinear mixed integer program, and expanded several heuristic solution algorithms. Berman et al. (Berman, Krass et al. 2007) concentrated on an asymptotic property of the problem and verified that the solution to the stochastic P-median problem coincides with the deterministic problem as the failure probabilities approach zero. They also presented some efficient heuristics with bounds on the worst-case performance. Lim et al. (Lim, Bassamboo et al. 2009) suggested a reliability continuum approximation (CA) approach for facility location problems with uniform customer density. For simplification, a specific form of failure-proof facility was supposed to exist; a customer was always reassigned to a failure-proof facility after its nearest regular facility failed, regardless of other regular facilities. With respect to the huge investment for facility location and network design, the attention to the failures of system based on facility disruptions in facility locating and network design has been increased recently (Qi and Shen 2007; Qi, Shen et al. 2010). Hanley and Church (O'Hanley and Church 2011) developed a new facility location–interdiction covering model for finding a robust alignment of facilities that has a suitable efficiency in the worst situations of facility loss. They formulated the problem as two mathematical models. At the first model, all possible interdiction patterns are considered and a standard MIP formulation is proposed. In the second model, the optimal interdiction pattern is implicitly defined in terms of the chosen facility location layout and more compact bi-level programming formulation is developed. Peng et al. (Peng, Snyder et al. 2011) studied the effect of considering of reliability topic on logistic networks design with facility disruptions and illustrated that applying a reliable network design are often possible with negligible increases in total location and allocation costs depends on decision makers opinion. They considered the commodity production/delivery system without respect to open/close decisions on the arcs of supply chain system and by applying the *p*-robustness criterion (which bounds the cost in disruption scenarios), they simultaneously minimize the nominal cost (the cost when no disruptions occur) and reduce the disruption risk. Recently, Liberatore et al. (Liberatore, Scaparra et al. 2012) introduced the problem of optimizing fortification plans in median distribution systems in the face of disruptions that involve large areas. They developed an effective exact solution algorithm to solve it optimally. Also, they showed empirically that ignoring correlation effects in a system can lead to suboptimal protection plans that result in an unnecessary increase in the system cost when disruptions take place. Moreover, Aryanezhad et al. (Aryanezhad 2011) studied a supply chain design problem with an unreliable supplier and random demand. Due to imperfect performance of the supplier, the quantity of the product received from the supplier may be less than the quantity ordered by distribution centers. A nonlinear integer programming model has been presented that minimizes the expected total costs including costs of location, inventory, transportation, and lost sales. The presented model simultaneously determines which customers are served, where distribution centers are located and how distribution centers are assigned to the customers. In order to solve the model, a heuristic approach based on genetic algorithm has been proposed. Computational results for different data sets have revealed that the proposed solution approach is quiet effective.

The literature review shows that there is an unfulfilled research in facility locating with respect to more realistic factors such as network design and system reliability to manage the practical facility location problems. In fact, considering simultaneously network design and reliability of system in facility location problems is relatively rare. However, there are numerous practical instances of facility location problems with such constitution that can lead to a more realistic and practical mathematical modeling of the problem. One can refer to locating health care service centers, locating gas compressor stations, and designing water tubing networks as the most obvious and practical examples. As a result, proposing a new mathematical model formulation, which can obtain optimum facility location and link constructing under some special conditions such as predetermined maximum failure cost, can lead decision makers to more accurate solutions for the considered problem. In other words, the proposed model provides an enough effective and confidant approach to be applied in industries and services.

In this paper, we study the facility location-network design problem: how to design a reliable facility location with respect to network design in the presence of random facility disruptions with the option of hardening selected facilities. We consider a reliable facility location-network design problem (RFLNDP) incorporating two types of facilities, one that is unreliable and another that is reliable (which is not subject to disruption, but is more expensive). The motivation of this research is to consider simultaneously two practical factors (network design and reliability of facilities) to develop the mathematical modeling of facility location problems, which has not been considered until now based on the authors' best knowledge. The main contributions that differentiate this paper from the existing ones in the related literature can be summarized as follows:

- ✓ By formulating and solving the RFLNDP, we study the relationship between the facility and network decisions (e.g., facility locations, network design, hardening investment) and some key factors (e.g., disruption probability and customer demand) in the presence of random disruptions.
- ✓ The option of "facility hardening;" is considered here; hence another set of decisions is made. The notion of facility hardening implies various protection plans ranging from physical facility preservation to exogenous outsourcing agreements.

- ✓ Considering different costs including facility location, link construction, transportation costs, and particularly hardening cost of facilities to improve the system reliability.
- ✓ Introducing a new mathematical optimization model to consider simultaneously facility location and allocation, network design and reliability of facilities as a mixed-integer, nonlinear programming (MINLP) problem. A model that integrates the managerial and strategic decision making such as determining the optimum locating of new (reliable/unreliable) facilities, optimum constructing of the transportation links, and optimum allocating demand nodes to located facilities so that locating, allocating, link constructing, transporting and hardening costs as well as system reliability are optimized. Proposing such mathematical modeling can present a more accurate and integrated description of the problem and eliminate the obstacles of using stochastic optimization models (Snyder and Ülker 2005; Snyder and Daskin 2005; Snyder and Daskin 2006; Snyder and Daskin 2007; Azaron, Brown et al. 2008); besides, some studies have recently emphasized on integrating strategic and tactical decisions to obtain more accurate improvement on considered practical problems (Snyder and Daskin 2006; Salmerón and Apte 2009; Rawls and Turnquist 2010).

The rest of the paper is organized as follows: In section 2, the mathematical model formulation of RFLNDP is developed. In section 3, the linearization of proposed model is presented, and a numerical example that exactly shows the application of the model formulation is demonstrated in section 4. Finally, conclusions and future works are presented in section 5.

2. Problem definition and model formulation

2-1. Definition

In this section, the general structure of mentioned problem is exactly described. Suppose that a set of demand nodes exists in a geographical region and a set of transportation roads as links is defined to construct a transportation network on the mentioned region. It is clearly desired to locate a set of new facilities to harden some of them and to construct new candidate links so that the total operational costs (including location costs, hardening costs, construction costs, and transportation costs) are minimized. One point that should be considered is that all of the facilities are not reliable, but they can be reliable by spending more extra cost for hardening them. If an unreliable facility fails, the demands of nodes, that directly connected to it, must be re-assigned to the nearest reliable facility. But it should be noted that the new re-assigning is not optimal and the demand nodes have to travel more distance to achieve the backup facility, leading to raise transportation costs and sometimes further costs for constructing of link roads. If the increase in the mentioned costs is considered as failure cost (Snyder and Daskin 2006; Snyder and Daskin 2007), then we can have some expressions in our formulation for it. Therefore, the failure costs (in other words, reliability of facilities) in the proposed problem can be evaluated and optimized.

The problem is to determine: (1) the optimum locations of new facilities regarding to network design and system reliability, (2) the facilities that must be paid more extra costs for hardening them, (3) the transportation links that should be constructed in the proposed network, (4) the amount of demands of nodes that should be transported by the transportation links in nominal (normal) and disruption conditions, and, (5) the fraction of every demand that should be supplied by facilities in nominal and disruption conditions.

2-2. Assumptions

The assumptions for RFLNDP can be described as follow:

1. Each node of network represents a demand node.
2. Based on the geographical situation, the facilities are divided to two categories: (I) reliable facilities (II) unreliable facilities.
3. Both reliable and unreliable facilities can be located on each node.
4. The demands of each node are provided by closest (reliable/unreliable) facility in primary assignment.
5. The demands of each node are provided by closest reliable facility in backup assignment.
6. When the closest facility from a certain demand node is a reliable facility, the primary assignment and the backup assignment are identical. In this state, we have a saving cost.
7. At most one new (reliable/unreliable) facility can be located on each node.
8. The infrastructure of the network is planned based on a *customer-to-serve*r system, which means that the demands themselves travel to the relevant facilities in order to be served.
9. All travel costs are symmetric.
10. All network links are directed.
11. Just only one (reliable/unreliable) facility can be located on a demand node.
12. The facilities and links are uncapacitated.
13. It may happen that several facilities simultaneously fail and not be available at a time.

14. In order to simplify the calculation of the total costs and control the complexity of the problem, neither the probability nor the duration of a failure will be considered; In fact, our goal is simply to hamper the cost that results from a failure, regardless of how frequently this cost incurs.

2-3. Notifications

Sets and input parameters:

N : set of nodes in the network

L : set of links in the network

F : number of facilities to open, ($F \geq 2$)

d_i : demand at node $i \in N$

f_i^U : fixed cost of locating an unreliable facility which is subject to failure at node $i \in N$

f_i^R : fixed cost of locating an reliable facility which is subject to failure at node $i \in N$ ($f_i^U \leq f_i^R$)

q_j : probability that an unreliable facility at $i \in N$ will be in the failure (disruption) state

c_{ij}^P : construction cost of link $(i, j) \in L$

t_{ij}^{kP} : unit transportation cost for a primary assignment from demand node $i \in N$ to a facility at $j \in N$

t_{ij}^{kD} : unit transportation cost for a backup assignment from demand node $i \in N$ to a reliable facility at $j \in N$

$t_{ij}^{kS} = t_{ij}^{kD} - t_{ij}^{kP}$: unit savings cost when demand node $i \in N$ is assigned to a reliable facility at $j \in N$ as both the primary and backup facility

Decision variables:

Z_i^U : 1 if an unreliable facility is located at node i, 0 otherwise

Z_i^R : 1 if a reliable facility is located at node i, 0 otherwise

X_{ij}^P : 1 if link (i,j) is constructed for nominal (normal) condition, 0 otherwise

X_{ij}^D : 1 if link (i,j) is constructed for disruption condition, 0 otherwise

X_{ij}^S : 1 if link (i,j) is constructed for both nominal and disruption condition, 0 otherwise

Y_{ij}^{kP} : fraction of demand of node l that flows on link (i,j) in nominal condition

Y_{ij}^{kD} : fraction of demand of node l that flows on link (i,j) in disruption condition

Y_{ij}^{kS} : fraction of demand of node l that flows on link (i,j) in both nominal and disruption condition

Y_{ij}^{iP} : X_{ij} $(i,j) \in L$

W_i^l : fraction of demand of node l that is served by a facility at node $i \in N$

$W_i^i = (Z_i^U + Z_i^R)$ $i \in N$

2-4. Mathematical formulation

Now, we present a new integrated model for the problem. The objective function is included five expressions in table (1) (Equations (1)–(7) are defined for the convenience of formulation):

Table (1): Description of objective function in detail

Row	***Expression in objective function***	***Description***
(1)	$\sum_{i \in N} f_i^U Z_i^U + \sum_{i \in N} f_i^R Z_i^R$	***Lo C***: Cost of reliable/unreliable facilities locating
(2)	$\sum_{(i,j) \in L} c_{ij} (X_{ij}^P + X_{ij}^D)$	***Con C***: Cost of link construction
(3)	$\sum_{(i,j) \in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} Y_{ji}^{lP}$	***Tran C***: Transportation cost of different demands
(4)	$\sum_{i \in N} \left(\begin{array}{c} (Z_i^U + Z_i^R) \\ \left[\sum_{(i,j) \in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} Y_{ji}^{lP} \right] \end{array} \right)$	***DTran C***: Transportation costs of demands that are directly transferred to reliable/unreliable facilities.
(5)	$\sum_{i \in N} \left(\begin{array}{c} (1-q_i)(Z_i^U + Z_i^R) \\ \left[\sum_{(i,j) \in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} Y_{ji}^{lP} \right] \end{array} \right)$	***QTran C***: Transportation costs of demands that are directly transferred to reliable/unreliable facilities. In this state, the primary facility is OK with probability (1-q_i) and is serving to assigned demands

(6)	$\sum_{i\in N}\left(Z_i^R q_i\left[\sum_{(i,j)\in L} t_{ji}^{jD} X_{ji}^{D} + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lD} Y_{ji}^{lD}\right]\right)$	***BTran C*:** Transportation costs of demands that are directly transferred to backup reliable facilities. In this state, the primary facility has disrupted with probability q_i and the related assigned demands are served by backup facility
(7)	$\sum_{i\in N}\left(Z_i^R q_i\left[\sum_{(i,j)\in L} t_{ji}^{jS} X_{ji}^{S} + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lS} Y_{ji}^{lS}\right]\right)$	***STran C*:** Saving transportation cost of demands that are assigned to reliable facilities. In this state, the reliable facilities are known as both facility for primary conditions and facility for disruption conditions

We recast the above discussion in the following integrated mathematical formulation:

$$Minmize \quad Lo\,C + Con\,C + Tran\ C - DTran\ C$$
$$+QTran\ C + BTran\ C - STran\ C \quad =$$

$$\begin{aligned}
&Minmize \sum_{i\in N} f_i^U Z_i^U + \sum_{i\in N} f_i^R Z_i^R + \\
&+\sum_{(i,j)\in L} c_{ij}(X_{ij}^P + X_{ij}^D) + \sum_{(i,j)\in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lP} Y_{ji}^{lP} \\
&-\sum_{i\in N}(Z_i^U + Z_i^R)\left[\sum_{(i,j)\in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lP} Y_{ji}^{lP}\right] \\
&+\sum_{i\in N}\left((1-q_i)(Z_i^U + Z_i^R)\left[\sum_{(i,j)\in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lP} Y_{ji}^{lP}\right]\right) \\
&+\sum_{i\in N} Z_i^R q_i\left[\sum_{(i,j)\in L} t_{ji}^{jD} X_{ji}^D + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lD} Y_{ji}^{lD}\right] \\
&-\sum_{i\in N} Z_i^R q_i\left[\sum_{(i,j)\in L} t_{ji}^{jS} X_{ji}^S + \sum_{(i,j)\in L}\sum_{l\in N} t_{ji}^{lS} Y_{ji}^{lS}\right]
\end{aligned} \tag{8}$$

(RFLNDP) $s.t.$

$$(Z_i^U + Z_i^R) + \sum_{j\in N} X_{ij}^P = 1 \quad \forall i\in N \tag{9}$$

$$X_{li}^P + \sum_{j\in N: j\neq l} Y_{ji}^{lP} = \sum_{j\in N} Y_{ij}^{lP} + W_i^l \qquad \forall i,l\in N: i\neq l,\ \forall (l,i)\in L \tag{10}$$

$$\sum_{j\in N: j\neq l} Y_{ji}^{lP} = \sum_{j\in N} Y_{ij}^{lP} + W_i^l \qquad \forall i,l\in N: i\neq l,\ \forall (l,i)\notin L \tag{11}$$

$$(Z_i^U + Z_i^R) + \sum_{i\in N: i\neq l} W_i^l = 1 \quad \forall l\in N \tag{12}$$

$$W_i^l \leq (Z_i^U + Z_i^R) \quad \forall i,l\in N: i\neq l \tag{13}$$

$$Z_i^U + Z_i^R \leq 1 \quad \forall i\in N \tag{14}$$

$$\sum_{i\in N}(Z_i^U + Z_i^R) = F \tag{15}$$

$$\sum_{i\in N} Z_i^R \geq 1 \tag{16}$$

$$Y_{ji}^{lP} \le X_{ji}^{P} \quad \forall (i,j) \in L \tag{17}$$

$$Y_{ji}^{lD} \le Z_i^R (X_{ji}^P + X_{ji}^D) \quad \forall (i,j) \in L \tag{18}$$

$$Y_{ji}^{lS} \le Y_{ji}^{lD} \quad \forall (i,j) \in L \tag{19}$$

$$Y_{ji}^{lS} \le Y_{ji}^{lP} \quad \forall (i,j) \in L \tag{20}$$

$$X_{ji}^{S} \le X_{ji}^{P} \quad \forall (i,j) \in L \tag{21}$$

$$X_{ji}^{S} \le X_{ji}^{D} \quad \forall (i,j) \in L \tag{22}$$

$$\sum_{i \in N} (Z_i^U + Z_i^R) Y_{ji}^{lP} = \sum_{k \in N} Z_k^R Y_{ji}^{lD} \quad \forall (i,j) \in L, \ j \ne l \tag{23}$$

$$X_{ji}^{P} (Z_i^U + Z_i^R) = \sum_{k \in N} Z_k^R X_{jk}^D \quad \forall (i,j) \in L, \ j \ne l \tag{24}$$

$$Y_{ij}^{lP}, Y_{ij}^{lD}, Y_{ij}^{lS} \ge 0 \quad \forall i,j,l \in N \tag{25}$$

$$W_i^l \ge 0 \quad \forall i,l \in N \tag{26}$$

$$X_{ij}^P, X_{ij}^D, X_{ij}^S \in \{0,1\} \quad \forall (i,j) \in L \tag{27}$$

$$Z_i^U, Z_i^R \in \{0,1\} \quad \forall i \in N \tag{28}$$

The objective function (8) illustrates the summation of five expressions which present in equations (1-7). In general observation, constraints (9-12) consider the rational conditions of the transportation flow between demand nodes and facilities. Specifically, constraints (9) ensure that demand at i is either served by a facility at i or by shipping on some links out of i. Constraints (10) and (11) state conservation of flow for transshipped demand. Constraints (12) impose that the demand of node l must find a destination, whether it is estimated by node l itself $(Z_i^U + Z_i^R)$ or by the other nodes i (W_i^l). Constraints (13) guarantee that the reliable/unreliable facilities are not used if they are not located. Constraints (14) emphasize that in node $i \in N$, we cannot locate both an unreliable and a reliable facility . Constraint (15) restricts the total number of newly located facilities to the predetermined facilities of F. Constraint (16) illustrates that at least one reliable facility must be located. Constraints (17) state that the primary assignment must be to an open (constructed) link. Constraints (18) illustrate that the backup assignment must be to open links which are directly connected to nodes with facilities. Constraints (19) and (20) state that the savings associated with any assignment can only be realized if the demand node is assigned to the same facility as its primary and its backup facility. Also, constraints (21) and (22) state that the savings associated with any construction can only be realized if the demand node is assigned to the same facility as its primary and its backup facility. Constraints (23) illustrate that the demands, directly assigned to each (reliable/unreliable) facility, must directly reassigned to a reliable facility as backup assignment. Constraints (24) guarantee constructing backup links for the demands that are directly assigned to reliable/unreliable facilities in primary assignment. Constraints (25-26) force the flow variables to be non-negative; while, constraints (27-28) enforce the binary restriction on the primary and backup facility location and link decision variables.

As it mentioned, according to the single assignment property, every demand of node is completely assigned to the closest single facility. That is, nothing is gained by “splitting up” a demand and sending parts of it to different facility. Therefore, the fractions of demands, which served a single facility, are integer-valued, while W_i^l and Y_{ij}^l are integral (Melkote 1996).

3. Linearization of Mathematical Model

With respect to the mathematical model structure of RFLNDP, since the variable Z_i^U and Z_i^R are multiplied in the variables (X_{ij}^P, X_{ij}^D, X_{ij}^S) and (Y_{ij}^{lP}, Y_{ij}^{lD}, Y_{ij}^{lS}) in equations (4-7) in the objective function, the proposed model is a mixed-integer non-linear programming (MINLP) model. However, it can be easily linearized by introducing three categories of new binary variables and additional constraints as table (2) (Lee and Dong 2009; Bozorgi-Amiri, Jabalameli et al. 2011).

Table (2): Categories of variables change and linearization

Category	***Description***	***Linearization***
(I)	Variables change	$U_{ij}^{UP} = Z_i^U X_{ij}^P \quad ; \quad V_{ijl}^{UP} = Z_i^U Y_{ij}^{lP}$
	Extra constraints	$U_{ij}^{UP} \le Z_i^U \quad ; \quad V_{ijl}^{UP} \le MZ_i^U$ $U_{ij}^{UP} \le X_{ij}^P \quad ; \quad V_{ijl}^{UP} \le Y_{ij}^{lP}$ $\begin{cases} U_{ij}^{UP} \ge Z_i^U + X_{ij}^P - 1 \;; \\ V_{ijl}^{UP} \ge Y_{ij}^{lP} - M(1 - Z_i^U) \end{cases}$
(II)	Variables change	$U_{ij}^{RP} = Z_i^R X_{ij}^P \quad ; \quad V_{ijl}^{RP} = Z_i^R Y_{ij}^{lP}$
	Extra constraints	$U_{ij}^{RP} \le Z_i^R \quad ; \quad V_{ijl}^{RP} \le MZ_i^R$ $U_{ij}^{RP} \le X_{ij}^P \quad ; \quad V_{ijl}^{RP} \le Y_{ij}^{lP}$ $\begin{cases} U_{ij}^{RP} \ge Z_i^R + X_{ij}^P - 1 \;; \\ V_{ijl}^{RP} \ge Y_{ij}^{lP} - M(1 - Z_i^R) \end{cases}$
(III)	Variables change	$U_{ij}^{RB} = Z_i^R X_{ij}^B \quad ; \quad V_{ijl}^{RB} = Z_i^R Y_{ij}^{lB}$
	Extra constraints	$U_{ij}^{RB} \le Z_i^R \quad ; \quad V_{ijl}^{RB} \le MZ_i^R$ $U_{ij}^{RB} \le X_{ij}^B \quad ; \quad V_{ijl}^{RB} \le Y_{ij}^{lB}$ $\begin{cases} U_{ij}^{RB} \ge Z_i^R + X_{ij}^B - 1 \;; \\ V_{ijl}^{RB} \ge Y_{ij}^{lB} - M(1 - Z_i^R) \end{cases}$
(IV)	Variables change	$U_{ij}^{RS} = Z_i^R X_{ij}^S \quad ; \quad V_{ijl}^{RS} = Z_i^R Y_{ij}^{lS}$
	Extra constraints	$U_{ij}^{RS} \le Z_i^R \quad ; \quad V_{ijl}^{RS} \le MZ_i^R$ $U_{ij}^{RS} \le X_{ij}^S \quad ; \quad V_{ijl}^{RS} \le Y_{ij}^{lS}$ $\begin{cases} U_{ij}^{RS} \ge Z_i^R + X_{ij}^S - 1 \;; \\ V_{ijl}^{RS} \ge Y_{ij}^{lS} - M(1 - Z_i^R) \end{cases}$

Where M is the aforementioned arbitrarily large number. Consequently, the objective function of model (RFLNDP) can be substituted with:

$$
\begin{aligned}
& Minmize \sum_{i \in N} f_i^U Z_i^U + \sum_{i \in N} f_i^R Z_i^R + \\
& + \sum_{(i,j) \in L} c_{ij} (X_{ij}^P + X_{ij}^B) + \sum_{(i,j) \in L} t_{ji}^{jP} X_{ji}^P + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} Y_{ji}^{lP} \\
& - \sum_{i \in N} \left[\sum_{(i,j) \in L} t_{ji}^{jP} \left(U_{ji}^{UP} + U_{ji}^{RP} \right) + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} \left(V_{jil}^{UP} + V_{jil}^{RP} \right) \right] \\
& + \sum_{i \in N} (1 - q_i) \left[\sum_{(i,j) \in L} t_{ji}^{jP} \left(U_{ji}^{UP} + U_{ji}^{RP} \right) + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lP} \left(V_{jil}^{UP} + V_{jil}^{RP} \right) \right] \\
& + \sum_{i \in N} q_i \left[\sum_{(i,j) \in L} t_{ji}^{jB} U_{ji}^{RB} + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lB} V_{jil}^{RB} \right] \\
& - \sum_{i \in N} q_i \left[\sum_{(i,j) \in L} t_{ji}^{jS} U_{ji}^{RS} + \sum_{(i,j) \in L} \sum_{l \in N} t_{ji}^{lS} V_{jil}^{RS} \right]
\end{aligned}
\qquad (29)
$$

Therefore, the final model of RFLNDP converted to mixed integer linear programming (MILP) model easily.

4. Numerical Example

An application of the proposed mathematical model is described in as a numerical example. The example is referenced from literature review (Melkote 1996).

4-1. Definition

A 21 node network with the travel costs scaled up by a factor of ten is supposed in which 38 potential links can be constructed. This network is shown in Figure 1. The demands, which are shown in parentheses beside each node, are normalized so that they sum to 1000. The fixed unreliable facility location costs are taken from a uniform (500, 1500) distribution and normalized so that their mean is \$1000. These costs are assigned to the demand nodes in ascending order, that is, they increase with the amount of demand. Also, the fixed location costs of reliable facility are defined as $f_i^R = 1.5\times f_i^U$. The travel cost t_{ij} of each candidate link may be interpreted as its length. Also, all distances on this network satisfy the triangle inequality. We again assume that we have a unit link construction cost that is distinguished as $c_{ij}=30\times t_{ij}$; in other words, for each link, the unit link construction cost is a fixed coefficient of unit transportation cost.

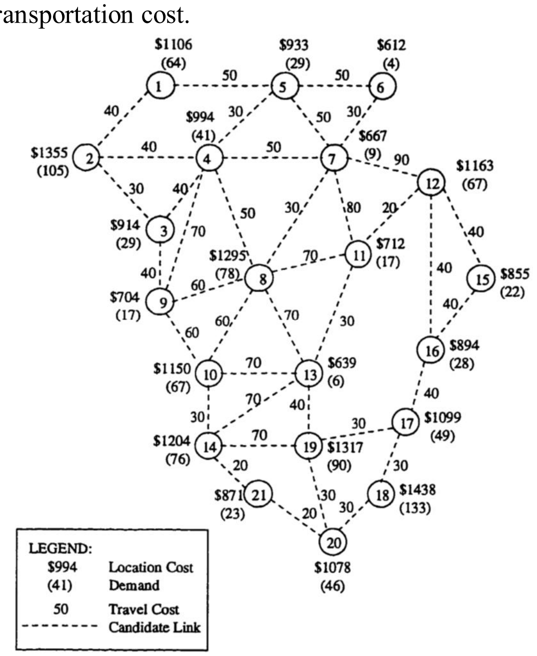

Figure 1: The 21 node network (Melkote 1996)

4-2. Computational results

Due to the above description and the predetermined value of different parameters, the numerical example was coded in GAMS 23.8 and solved by CPLEX solver. The results are presented in Figure 2 which visually illustrates the obtained optimal solution. As Figure 2 illustrates, the value of Z_2^R , Z_{10}^R , Z_{12}^U and Z_{18}^U are determined to 1. This means that the optimum locations for four new facilities are nodes 2 , 10 , 12 and 18. Also the facilities, located in nodes 2 and 10, should be selected for hardening.

In additions, 17 new roads must be constructed between several nodes ($X_{1,2}$ =1; $X_{5,4}$ =1; $X_{3,2}$=1; $X_{4,2}$=1; $X_{6,7}$=1; $X_{7,8}$=1; $X_{9,8}$=1; $X_{8,10}$=1; $X_{13,10}$=1; $X_{14,10}$=1; $X_{19,20}$=1; $X_{21,20}$=1; $X_{20,18}$=1; $X_{17,16}$=1; $X_{16,12}$=1; $X_{11,12}$=1; $X_{15,12}$=1). As well as, 4 new roads should be constructed between several nodes as a potential links for disruption conditions ($X_{8,11}$ =1; $X_{19,14}$ =1; $X_{21,14}$=1, $X_{17,18}$=1). The optimal value of objective function for the numerical example is \$59332.

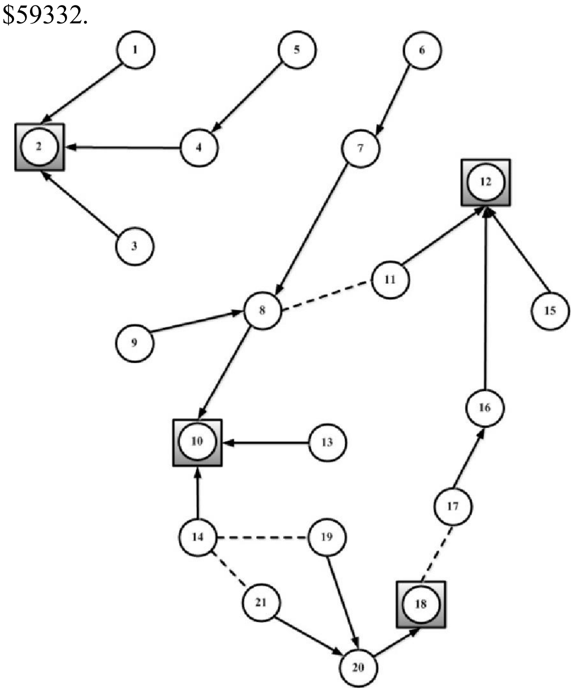

Figure 2: The optimal solution of the numerical example

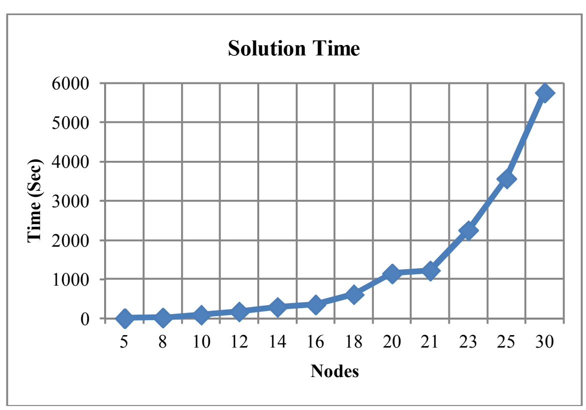

Figure 3: The changing procedure of CPU time for different sizes of problem

As a remarkable point, the CPU time of the model is one of the key factors in applying of the proposed methodology. Different instance problems, that the number of their demand nodes was set from 5 to 30, were generated randomly and solved on a PC with Pentium IV-2.33 GHz and 2 GB RAM DDR 3 under win XP SP3. Figure 3 presents the changing procedure of CPU time according to different increasing values in the number of demand nodes.

As expected, the CPU times increase exponentially. Therefore, the model is not suitable for large scale problems. However, with respect to the several applications of the RFLNDP in different industrial and services environments, developing of a mathematical formulation for the RFLNDP is an auspicious beginning point for starting works on this kind of facility location problem as future studies.

5. Conclusions and future research

In this paper, the combined facility location-network design problem with respect to the system reliability, named reliable facility location-network design problem (RFLNDP), was considered and a mixed integer non linear programming (MINLP) formulation for the mentioned problem was proposed. The basic principal in the proposed formulation is the concept of “facility hardening investment” to hedge against the risk of facility disruptions which implies various protection plans ranging from physical facility protection to exogenous outsourcing contracts.

The proposed model was linearized by the efficient techniques. Also, a practical numerical example was presented in detail to illustrate the application of the proposed mathematical model (RFLNDP). The results illustrate that the proposed model not only can present a more accurate description of RFLNDP but also can propose efficient feasible solutions to use in industries and services.

Our findings raise some appropriate questions for future research. First, the size of the numerical example may be small and if the size of the problem increases, a suitable solution procedure should be proposed to obtain optimal or near optimal solution. We are particularly interested in seeking apposite and efficient heuristics and metaheuristics such as tabu search (TS) and particle swarm optimization (PSO) for the mentioned propose. Second, only a single objective function was studied in this paper; however, considering the RFLNDP as a multi objective problem such as minimizing the operating costs and maximizing the system reliability can have more practical application in industries and services. Finally, we would explore other applications of the proposed model, especially in the fields of integrated facility sitting and supply chain design.

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