Comparisons between the Solutions of the Generalized Ito System by Different Methods

Hassan Zedan[1&2], Wafaa Albarakati[1] and Eman El Adrous[1]

[1]Department of Mathematics, Faculty of Science, king Abdualziz University, P.O.Box:80203, Jeddah 21589, Saudi Arabia. [2]Department of Mathematics, Faculty of Science, Kafr El-Sheikh University, Egypt
hassanzedan2003@yahoo.com

Abstract: The main objective of this paper is to find the solutions of the generalized Ito system by using the following three different methods, Sine-cosine method, the homotopy Perturbation method and the differential transformation method. Moreover, we will make some comparisons between the solutions in those three methods.

[Hassan Zedan, Wafaa Albarakati and Eman El Adrous. **Comparisons between the Solutions of the Generalized Ito System by Different Methods**. *Life Sci J* 2013;10(1):1512-523] (ISSN:1097-8135). http://www.lifesciencesite.com. 223

Keyword: Solutions, Generalized Ito System.

1. Introduction

In the last two decades with the rapid development of nonlinear science, there has appeared ever-increasing interest of scientists and engineers in the analytical techniques for nonlinear problems. The widely applied techniques are perturbation methods. But, like other nonlinear analytical techniques, perturbation methods have their own limitations. All but few [17] perturbation methods are based on such an assumption that a small parameter must exist in an equation. This so-called small parameter assumption greatly restricts applications of perturbation techniques. As is well known, an overwhelming majority of nonlinear problems, especially those having strong nonlinearity have no small parameters at all.

To eliminate the small parameter assumption, in 1997**, Liu [17],** proposes a new perturbation technique, where an artificial parameter is embedded in an equation at its appropriate place, and the embedding parameter is used as a "small parameter". Unfortunately, there is an uncertainty about an appropriate artificial parameter, and often enough the approximations obtained by such method will not be uniform, so that its applicability range is severely limited. Just recently, in order to be freed from the limitation of "small parameter" assumption, Liao [18-19] proposes a new technique which, based on homotopy in topology, does not require small parameters in equations, using the interesting property of homotopy, he transforms a nonlinear problem into an infinite number of linear problems without using the perturbation techniques. In this paper a novel method is successfully handled from an entirely different point of view. Namely, homotopy perturbation method. By the simple property of homotopy, the problem is converted into a special perturbation problem with the small embedding parameter, which is considered as a small parameter, so the method is caught the name of the homotopy perturbation method. The proposed method can take full advantage of the traditional perturbation methods and Liao's homotopy method.

Now, we introduce the exact solutions for the generalized Ito system by Sine-cosine method,

$$\begin{aligned} &U_t = V_x, \\ &V_t = -2V_{xxx} - 6(UV)_x + aWW_x + bPW_x + cWP_x + dPP_x + fW_x + gP_x, \\ &W_t = W_{xxx} + 3UW_x, \\ &P_t = P_{xxx} + 3UP_x. \end{aligned} \tag{1}$$

2. Sine-cosine method

We consider the following generalized Ito system:

$$u_t = v_x, \tag{1}$$

$$v_t = -2v_{xxx} - 6(uv)_x + aww_x + bpw_x + cwp_x + dpp_x + fw_x + gp_x, \tag{2}$$

$$w_t = w_{xxx} + 3uw_x, \tag{3}$$

$$p_t = p_{xxx} + 3up_x. \tag{4}$$

Now,we can define ,

$$\xi = x - \gamma t \tag{5}$$

Take into consideration (5) , therefore Eqs (1-4) gives:

$$-\gamma u' - v' = 0, \tag{6a}$$

$$-\gamma v' + 2v''' + 6(uv)' - aww' - bpw' - cwp' - dpp' - fw' - gp' = 0, \tag{6b}$$

$$-\gamma w' - w''' - 3uw' = 0, \tag{6c}$$

$$-\gamma p' - p''' - 3up' = 0. \tag{6d}$$

Suppose that the general solution of Eqs. (6a-6d) take the forms:

$$u = \lambda_1 \mathrm{Sin}[\mu\xi]^{\beta_1},$$
$$v = \lambda_2 \mathrm{Sin}[\mu\xi]^{\beta_2},$$
$$w = \lambda_3 \mathrm{Sin}[\mu\xi]^{\beta_3},$$

$$p = \lambda_4 \mathrm{Sin}[\mu\xi]^{\beta_4}. \qquad (7)$$

From (7) and (6a-6d), after some calculation we have the system

$$-\gamma\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_1}\beta_1\lambda_1 - \mu \mathrm{Sin}[\mu\xi]^{-1+\beta_2}\beta_2\lambda_2 = 0 \qquad (8)$$

$$\begin{aligned}
&4\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_2}\beta_2\lambda_2 - \gamma\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_2}\beta_2\lambda_2 - 6\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_2}\beta_2^2\lambda_2 \\
&+2\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_2}\beta_2^3\lambda_2 - 2\mu^3 \mathrm{Sin}[\mu\xi]^{-1+\beta_2}\beta_2^3\lambda_2 + 6\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_1+\beta_2}\beta_1\lambda_1\lambda_2 \\
&+6\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_1+\beta_2}\beta_2\lambda_1\lambda_2 - f\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_3}\beta_3\lambda_3 - a\mu \mathrm{Sin}[\mu\xi]^{-1+2\beta_3}\beta_3\lambda_3^2 \\
&-g\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_4}\beta_4\lambda_4 - b\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_3+\beta_4}\beta_3\lambda_3\lambda_4 - c\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_3+\beta_4}\beta_4\lambda_3\lambda_4 \\
&=0
\end{aligned} \qquad (9)$$

$$\begin{aligned}
&-2\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_3}\beta_3\lambda_3 - \gamma\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_3}\beta_3\lambda_3 + 3\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_3}\beta_3^2\lambda_3 \\
&-\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_3}\beta_3^3\lambda_3 + \mu^3 \mathrm{Sin}[\mu\xi]^{-1+\beta_3}\beta_3^3\lambda_3 - 3\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_1+\beta_3}\beta_3\lambda_1\lambda_3 \\
&= 0,
\end{aligned} \qquad (10)$$

$$\begin{aligned}
&-2\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_4}\beta_4\lambda_4 - \gamma\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_4}\beta_4\lambda_4 + 3\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_4}\beta_4^2\lambda_4 \\
&-\mu^3 \mathrm{Sin}[\mu\xi]^{-3+\beta_4}\beta_4^3\lambda_4 + \mu^3 \mathrm{Sin}[\mu\xi]^{-1+\beta_4}\beta_4^3\lambda_4 - 3\mu \mathrm{Sin}[\mu\xi]^{-1+\beta_1+\beta_4}\beta_4\lambda_1\lambda_4 \\
&= 0.
\end{aligned} \qquad (11)$$

From (8), (9), (10) and (11) and balance the terms of sine function and equating the exponent of each pair of the sine function we have:

$$\beta_i(\beta_i - 1)(\beta_i - 2) \neq 0, \ \beta_i = -2, \ i = 1,2,3,4 \qquad (12)$$

Substituting (12) into (8), (9), (10) and (11), equating the coefficient of $\mathrm{Sin}[\mu\xi]^i$ to be zero we have:

$$\begin{aligned}
&2\gamma\mu\lambda_1 + 2\mu\lambda_2 = 0, \\
&24\mu^3\lambda_3 + 6\mu\lambda_1\lambda_3 = 0, \\
&2\gamma\mu\lambda_3 - 8\mu^3\lambda_3 = 0, \\
&24\mu^3\lambda_4 + 6\mu\lambda_1\lambda_4 = 0, \\
&2\gamma\mu\lambda_4 - 8\mu^3\lambda_4 = 0 \\
&2\gamma\mu\lambda_2 + 16\mu^3\lambda_2 + 2f\mu\lambda_3 + 2g\mu\lambda_4 = 0, \\
&-48\mu^3\lambda_2 - 24\mu\lambda_1\lambda_2 + 2a\mu\lambda_3^2 + 2b\mu\lambda_3\lambda_4 + 2c\mu\lambda_3\lambda_4 + 2d\mu\lambda_4^2 = 0
\end{aligned} \qquad (13)$$

Solving (13) we obtain that

$$\begin{aligned}
&\mu = -\frac{\sqrt{\gamma}}{2}, \\
&\lambda_1 = -\gamma, \\
&\lambda_2 = \gamma^2, \\
&\lambda_3 = \frac{-3\gamma^3 - \frac{3bfg\gamma^3}{2(df^2 - bfg - cfg + ag^2)} - \frac{3cfg\gamma^3}{2(df^2 - bfg - cfg + ag^2)}}{f} + \frac{\frac{3ag^2\gamma^3}{df^2 - bfg - cfg + ag^2}}{f} + \frac{-\frac{g\sqrt{(-3bf - 3cf + 6ag)^2\gamma^6 + 4(df^2 - bfg - cfg + ag^2)\gamma^3(-6f^2 - 9a\gamma^3)}}{2(df^2 - bfg - cfg + ag^2)}}{f}, \\
&\lambda_4 = \frac{\sqrt{(-3bf - 3cf + 6ag)^2\gamma^6 + 4(df^2 - bfg - cfg + ag^2)\gamma^3(-6f^2 - 9a\gamma^3)}}{2(df^2 - bfg - cfg + ag^2)} - \frac{(-3bf - 3cf + 6ag)\gamma^3}{2(df^2 - bfg - cfg + ag^2)},
\end{aligned} \qquad (14)$$

Consequently, we can get the value of u, v, w and p as follow :

$$u(x,t) = -\gamma \mathrm{Csc}\left[\frac{1}{2}\sqrt{\gamma}(x - t\gamma)\right]^2,$$

$$v(x,t) = \gamma^2 \mathrm{Csc}\left[\frac{1}{2}\sqrt{\gamma}(x - t\gamma)\right]^2,$$

$$\begin{aligned}
w(x,t) = \frac{1}{f}\Bigg(&-3\gamma^3 - \frac{3bfg\gamma^3}{2(df^2 - bfg - cfg + ag^2)} - \frac{3cfg\gamma^3}{2(df^2 - bfg - cfg + ag^2)} + \frac{3ag^2\gamma^3}{df^2 - bfg - cfg + ag^2} \\
&- \frac{g\sqrt{(-3bf - 3cf + 6ag)^2\gamma^6 + 4(df^2 - bfg - cfg + ag^2)\gamma^3(-6f^2 - 9a\gamma^3)}}{2(df^2 - bfg - cfg + ag^2)}\Bigg) \\
&\times \mathrm{Csc}\left[\frac{1}{2}\sqrt{\gamma}(x - t\gamma)\right]^2,
\end{aligned}$$

$$p(x,t) = \Bigg(\frac{(3bf + 3cf - 6ag)\gamma^3}{2(df^2 - bfg - cfg + ag^2)}$$

$$+\frac{\sqrt{(-3bf-3cf+6ag)^2\gamma^6+4(df^2-bfg-cfg+ag^2)\gamma^3(-6f^2-9a\gamma^3)}}{2(df^2-bfg-cfg+ag^2)}\Bigg)$$

$$\times \operatorname{Csc}\left[\frac{1}{2}\sqrt{\gamma}(x-t\gamma)\right]^2. \quad (15)$$

3 Homotopy Perturbation Method:

Consider the system of the form (1) and the exact solution of system (1) is

$$u(x,t)=\frac{8-\beta}{3}-4\tanh(x-t\beta)^2,$$
$$v(x,t)=-\beta\left(\frac{8-\beta}{3}-4\tanh(x-t\beta)^2\right),$$
$$w(x,t)=b_0+\frac{4\sqrt{6c}\sqrt{\beta}\tanh(x-t\beta)^2}{\sqrt{a}\sqrt{bc-ad}},$$
$$h(x,t)=\frac{1}{6}\left(-\frac{6cf}{bc-ad}+\frac{6ag}{bc-ad}+\frac{16\sqrt{6a\beta}}{\sqrt{bc-ad}}-\frac{5\sqrt{6a}\beta^{\frac{3}{2}}}{\sqrt{bc-ad}}\right)-\frac{4\sqrt{6a\beta}\tanh(x-t\beta)^2}{\sqrt{bc-ad}} \quad (16)$$

Where a,b,c,d,f,g,β,b_0 are arbitrary constants such that $bc\neq ad, a\neq 0, \beta\neq 8$.

At $b=7, a=6, c=1, f=1, d=1, g=1, \beta=\frac{1}{4}, b_0=4$, hence system (1) gives:

$$u_t-v_x=0,$$
$$v_t+2v_{xxx}+6(uv)_x-6ww_x-7hw_x-wh_x-hh_x-w_x-h_x=0,$$
$$w_t-w_{xxx}-3uw_x=0,$$
$$h_t-h_{xxx}-3uh_x=0, \quad (17)$$

and the initial data is:

$$u(x,0)=\frac{31}{12}-4\tanh(x)^2,$$
$$v(x,0)=-\frac{31}{48}+\tanh(x)^2,$$
$$w(x,0)=4+2\tanh(x)^2,$$
$$h(x,0)=\frac{99}{8}-12\tanh(x)^2. \quad (18)$$

To solve system (17) by means of HPM, we choose the initial approximations

$$u_0=\frac{31}{12}-4\tanh(x)^2,$$
$$v_0=-\frac{31}{48}+\tanh(x)^2,$$
$$w_0=4+2\tanh(x)^2,$$
$$h_0=\frac{99}{8}-12\operatorname{Tanh}(x)^2, \quad (19)$$

and construct the following homotopy

$$(1-p)\cdot\left(\frac{\partial}{\partial t}U(x,t)-\frac{\partial}{\partial t}u_0(x,t)\right)+p\cdot\left(\frac{\partial}{\partial t}U(x,t)-\frac{\partial}{\partial x}V(x,t)\right)=0,$$

$$(1-p)\cdot\left(\frac{\partial}{\partial t}V(x,t)-\frac{\partial}{\partial t}v_0(x,t)\right)+p\cdot\left(\begin{array}{c}\frac{\partial}{\partial t}V(x,t)+2\cdot\frac{\partial^3}{\partial x^3}V(x,t)+6\cdot\frac{\partial}{\partial x}(u(x,t)\cdot v(x,t))-6\cdot w(x,t)\cdot\frac{\partial}{\partial x}w(x,t)\\ -7\cdot h(x,t)\cdot\frac{\partial}{\partial x}w(x,t)-w(x,t)\cdot\frac{\partial}{\partial x}h(x,t)-h(x,t)\cdot\frac{\partial}{\partial x}h(x,t)-\frac{\partial}{\partial x}w(x,t)\\ -\frac{\partial}{\partial x}h(x,t)\end{array}\right)=0,$$

$$(1-p)\cdot\left(\frac{\partial}{\partial t}W(x,t)-\frac{\partial}{\partial t}w_0(x,t)\right)+p\cdot\left(\frac{\partial}{\partial t}w(x,t)-\frac{\partial^3}{\partial x^3}w(x,t)-3\cdot u(x,t)\cdot\frac{\partial}{\partial x}w(x,t)\right)=0,$$

$$(1-p)\cdot\left(\frac{\partial}{\partial t}h(x,t)-\frac{\partial}{\partial t}h_0(x,t)\right)+p\cdot\left(\frac{\partial}{\partial t}h(x,t)-\frac{\partial^3}{\partial x^3}h(x,t)-3\cdot u(x,t)\cdot\frac{\partial}{\partial x}h(x,t)\right)=0 \quad (20)$$

Suppose the solution of system (17) has the form

$$u(x,t)=U_0(x,t)+pU_1(x,t)+p^2\cdot U_2(x,t)+p^3\cdot U_3(x,t)+p^4\cdot U_4(x,t)+\cdots$$
$$v(x,t)=V_0(x,t)+p\cdot V_1(x,t)+p^2\cdot V_2(x,t)+p^3\cdot V_3(x,t)+p^4\cdot V_4(x,t)+\cdots,$$

$$
\begin{aligned}
w(x,t) &= W_0(x,t) + p \cdot W_1(x,t) \\
&\quad + p^2 \cdot W_2(x,t) + p^3 \\
&\quad \cdot W_3(x,t) + p^4 \\
&\quad \cdot W_4(x,t) + \cdots
\end{aligned}
$$

$$
\begin{aligned}
&h(x,t) \\
&= H_0(x,t) + p \cdot H_1(x,t) + p^2 \\
&\cdot H_2(x,t) + p^3 \cdot H_3(x,t) + p^4 \\
&\cdot H_4(x,t) \\
&+ \cdots
\end{aligned}
\tag{21}
$$

Substitute (21) into (17) and then Separate all coefficients of powers of p we get :

$$
p^0: \quad \frac{\partial}{\partial t}U_0(x,t) - \left(\frac{\partial}{\partial t}u_0(x,t)\right) = 0
$$

$$
\frac{\partial}{\partial t}V_0(x,t) - \left(\frac{\partial}{\partial t}v_0(x,t)\right) = 0
$$

$$
\frac{\partial}{\partial t}W_0(x,t) - \left(\frac{\partial}{\partial t}w_0(x,t)\right) = 0
$$

$$
\frac{\partial}{\partial t}H_0(x,t) - \left(\frac{\partial}{\partial t}h_0(x,t)\right) = 0
$$

$$
p^1: \quad \frac{\partial}{\partial t}U_1(x,t) + \frac{\partial}{\partial t}u_0(x,t) - \left(\frac{\partial}{\partial x}V_0(x,t)\right) = 0
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}V_1(x,t) + \frac{\partial}{\partial t}v_0(x,t) \\
&+ 2\left(\frac{\partial^3}{\partial x^3}V_0(x,t)\right) - \left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&- \left(\frac{\partial}{\partial x}H_0(x,t)\right) \\
&+ 6\left(\frac{\partial}{\partial x}U_0(x,t)\right)V_0(x,t) \\
&+ 6\,U_0(x,t)\left(\frac{\partial}{\partial x}V_0(x,t)\right) \\
&- 6\,W_0(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&- 7\,H_0(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&- W_0(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) \\
&- H_0(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}W_1(x,t) + \frac{\partial}{\partial t}w_0(x,t) \\
&- \left(\frac{\partial^3}{\partial x^3}W_0(x,t)\right) \\
&- 3\,U_0(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}H_1(x,t) + \frac{\partial}{\partial t}h_0(x,t) \\
&- \left(\frac{\partial^3}{\partial x^3}H_0(x,t)\right) \\
&- 3\,U_0(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) = 0
\end{aligned}
$$

$$
p^2: \quad \frac{\partial}{\partial t}U_2(x,t) - \left(\frac{\partial}{\partial x}V_1(x,t)\right) = 0,
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}V_2(x,t) \\
&+ 2\left(\frac{\partial^3}{\partial x^3}V_1(x,t)\right) - \left(\frac{\partial}{\partial x}W_1(x,t)\right) \\
&- \left(\frac{\partial}{\partial x}H_1(x,t)\right) \\
&- 7\,H_1(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&+ 6\,U_0(x,t)\left(\frac{\partial}{\partial x}V_1(x,t)\right) \\
&- 6\,W_1(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&+ 6\left(\frac{\partial}{\partial x}U_0(x,t)\right)V_1(x,t) \\
&+ 6\left(\frac{\partial}{\partial x}U_1(x,t)\right)V_0(x,t) \\
&+ 6\,U_1(x,t)\left(\frac{\partial}{\partial x}V_0(x,t)\right) \\
&- 6\,W_0(x,t)\left(\frac{\partial}{\partial x}W_1(x,t)\right) \\
&- 7\,H_0(x,t)\left(\frac{\partial}{\partial x}W_1(x,t)\right) \\
&- W_0(x,t)\left(\frac{\partial}{\partial x}H_1(x,t)\right) \\
&- W_1(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) \\
&- H_0(x,t)\left(\frac{\partial}{\partial x}H_1(x,t)\right) \\
&- H_1(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) = 0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}W_2(x,t) \\
&- \left(\frac{\partial^3}{\partial x^3}W_1(x,t)\right) \\
&- 3\,U_1(x,t)\left(\frac{\partial}{\partial x}W_0(x,t)\right) \\
&- 3\,U_0(x,t)\left(\frac{\partial}{\partial x}W_1(x,t)\right) = 0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}H_2(x,t) - \left(\frac{\partial^3}{\partial x^3}H_1(x,t)\right) \\
&- 3\,U_1(x,t)\left(\frac{\partial}{\partial x}H_0(x,t)\right) \\
&- 3\,U_0(x,t)\left(\frac{\partial}{\partial x}H_1(x,t)\right) = 0,
\end{aligned}
$$

…………...

$$
p^n: \quad \frac{\partial}{\partial t}U_n(x,t) - \left(\frac{\partial}{\partial x}V_{n-1}(x,t)\right) = 0,
$$

$$
\begin{aligned}
&\frac{\partial}{\partial t}V_n(x,t)\\
&+2\left(\frac{\partial^3}{\partial x^3}V_{n-1}(x,t)\right)\\
&+6\sum_{i=1}^{n-1}U_i(x,t)\left(\frac{\partial}{\partial x}V_{n-i-1}(x,t)\right)\\
&+6\sum_{i=1}^{n-1}\left(\frac{\partial}{\partial x}U_i(x,t)\right)V_{n-i-1}(x,t)\\
&-6\sum_{i=1}^{n-1}W_i(x,t)\left(\frac{\partial}{\partial x}W_{n-i-1}(x,t)\right)\\
&-7\sum_{i=1}^{n-1}U_i(x,t)\left(\frac{\partial}{\partial x}W_{n-i-1}(x,t)\right)\\
&-\sum_{i=1}^{n-1}W_i(x,t)\left(\frac{\partial}{\partial x}H_{n-i-1}(x,t)\right)\\
&-\sum_{i=1}^{n-1}H_i(x,t)\left(\frac{\partial}{\partial x}H_{n-i-1}(x,t)\right)\\
&-\left(\frac{\partial}{\partial x}W_{n-1}(x,t)\right)\\
&\left(\frac{\partial}{\partial x}H_{n-1}(x,t)\right)=0,\\
&\frac{\partial}{\partial t}W_n(x,t)\\
&-\left(\frac{\partial^3}{\partial x^3}W_{n-1}(x,t)\right)\\
&-3\sum_{i=1}^{n-1}U_i(x,t)\left(\frac{\partial}{\partial x}W_{n-i-1}(x,t)\right)\\
&=0,\\
&\frac{\partial}{\partial t}H_n(x,t)\\
&-\left(\frac{\partial^3}{\partial x^3}H_{n-1}(x,t)\right)\\
&-3\sum_{i=1}^{n-1}U_i(x,t)\left(\frac{\partial}{\partial x}H_{n-i-1}(x,t)\right)\\
&=0\\
&=0,
\end{aligned}
\qquad (22)
$$

Solving system (22) we have:

$$U_0=\frac{31}{12}-4\tanh(x)^2,$$

$$V_0=-\frac{31}{48}+\tanh(x)^2,$$

$$W_0=4+2\tanh(x)^2,$$

$$H_0=\frac{99}{8}-12\tanh(x)^2$$

$$U_1=\frac{2t\sinh(x)}{\cosh(x)^3},$$

$$V_1=-\frac{1}{2}\frac{t\sinh(x)}{\cosh(x)^3},$$

$$W_1=-\frac{t\sinh(x)}{\cosh(x)^3},$$

$$H_1=\frac{6t\sinh(x)}{\cosh(x)^3},$$

$$U_2=\frac{1}{4}\frac{t^2(2\cosh(x)^2-3)}{\cosh(x)^4},$$

$$V_2=-\frac{1}{16}\frac{t^2(2\cosh(x)^2-3)}{\cosh(x)^4},$$

$$W_2=-\frac{1}{8}\frac{t^2(2\cosh(x)^2-3)}{\cosh(x)^4},$$

$$H_2=\frac{3}{4}\frac{t^2(2\cosh(x)^2-3)}{\cosh(x)^4}$$

$$U_3=\frac{1}{12}\frac{t^3\sinh(x)(\cosh(x)^2-3)}{\cosh(x)^5},$$

$$V_3=-\frac{1}{48}\frac{t^3\sinh(x)(\cosh(x)^2-3)}{\cosh(x)^5},$$

$$W_3=-\frac{1}{24}\frac{t^3\sinh(x)(\cosh(x)^2-3)}{\cosh(x)^5},$$

$$H_3=\frac{1}{4}\frac{t^3\sinh(x)(\cosh(x)^2-3)}{\cosh(x)^5}$$

$$U_4=\frac{1}{192}\frac{t^4(2\cosh(x)^4-15\cosh(x)^2+15)}{\cosh(x)^6},$$

$$V_4=-\frac{1}{768}\frac{t^4(2\cosh(x)^4-15\cosh(x)^2+15)}{\cosh(x)^6},$$

$$W_4=-\frac{1}{384}\frac{t^4(2\cosh(x)^4-15\cosh(x)^2+15)}{\cosh(x)^6},$$

$$H_4=\frac{1}{64}\frac{t^4(2\cosh(x)^4-15\cosh(x)^2+15)}{\cosh(x)^6},$$

$$U_5=\frac{1}{1920}\frac{t^5\sinh(x)(2\cosh(x)^4-30\cosh(x)^2+45)}{\cosh(x)^7},$$

$$V_5=-\frac{1}{7680}\frac{t^5\sinh(x)(2\cosh(x)^4-30\cosh(x)^2+45)}{\cosh(x)^7},$$

$$W_5=-\frac{1}{3840}\frac{t^5\sinh(x)(2\cosh(x)^4-30\cosh(x)^2+45)}{\cosh(x)^7},$$

$$H_5=\frac{1}{640}\frac{t^5\sinh(x)(2\cosh(x)^4-30\cosh(x)^2+45)}{\cosh(x)^7},$$

$$U_6=\frac{1}{46080}\frac{t^6(-126\cosh(x)^4+420\cosh(x)^2+4\cosh(x)^6-315)}{\cosh(x)^8},$$

$$V_6=-\frac{1}{184320}\frac{t^6(-126\cosh(x)^4+420\cosh(x)^2+4\cosh(x)^6-315)}{\cosh(x)^8},$$

$$W_6=-\frac{1}{92160}\frac{t^6(-126\cosh(x)^4+420\cosh(x)^2+4\cosh(x)^6-315)}{\cosh(x)^8},$$

$$H_6 = \frac{1}{15360}\frac{t^6\left(-126\cosh(x)^4+420\cosh(x)^2+4\cosh(x)^6-315\right)}{\cosh(x)^8},$$

$$U_{12} = \frac{t^{12}\left(390390\cosh(x)^8+4\cosh(x)^{12}-3603600\cosh(x)^6-8190\cosh(x)^{10}+11351340\cosh(x)^4-14189175\cosh(x)^2+6081075\right)}{1961990553600\cosh(x)^{14}}$$

$$V_{12} = -\frac{t^{12}\left(390390\cosh(x)^8+4\cosh(x)^{12}-3603600\cosh(x)^6-8190\cosh(x)^{10}+11351340\cosh(x)^4-14189175\cosh(x)^2+6081075\right)}{7847962214400\cosh(x)^{14}},$$

$$W_{12} = -\frac{t^{12}\left(390390\cosh(x)^8+4\cosh(x)^{12}-3603600\cosh(x)^6-8190\cosh(x)^{10}+11351340\cosh(x)^4-14189175\cosh(x)^2+6081075\right)}{3923981107200\cosh(x)^{14}},$$

$$H_{12} = \frac{t^{12}\left(390390\cosh(x)^8+4\cosh(x)^{12}-3603600\cosh(x)^6-8190\cosh(x)^{10}+11351340\cosh(x)^4-14189175\cosh(x)^2+6081075\right)}{653996851200\cosh(x)^{14}}$$

The approximate solution of (17) can be obtained by setting $p = 1$ and get

$$u(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + \cdots$$

$$v(x,t) = V_0(x,t) + V_1(x,t) + V_2(x,t) + V_3(x,t) + \cdots$$

$$w(x,t) = W_0(x,t) + W_1(x,t) + W_2(x,t) + W_3(x,t) + \cdots$$

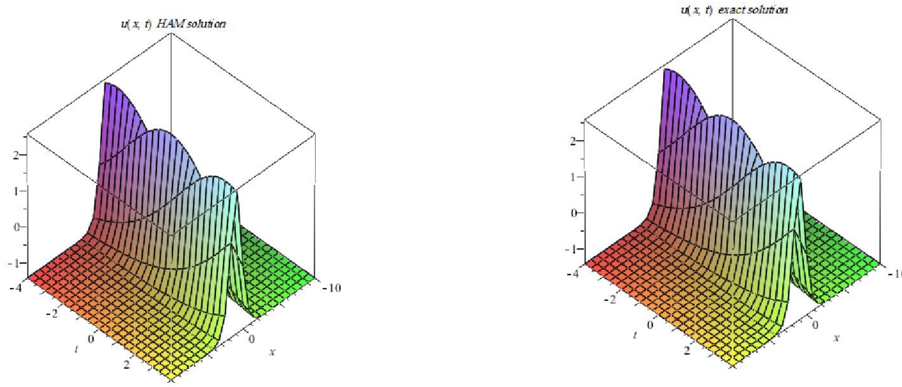


Fig. 1 exact solution and HPM solution for u(x ,t) at (-10 ≤ x ≤10 , -4 ≤ t ≤ 4)

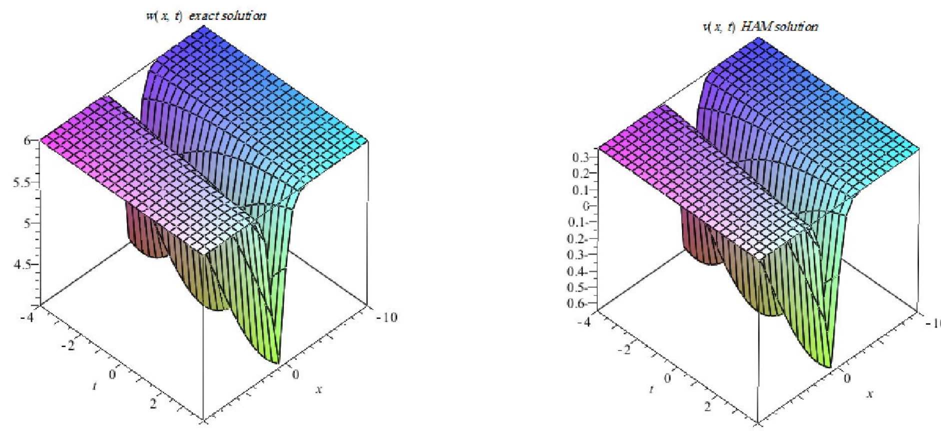


Fig 2 exact solution and HPM solution for v(x ,t) at (-10 ≤ x ≤10 , -4 ≤ t ≤ 4

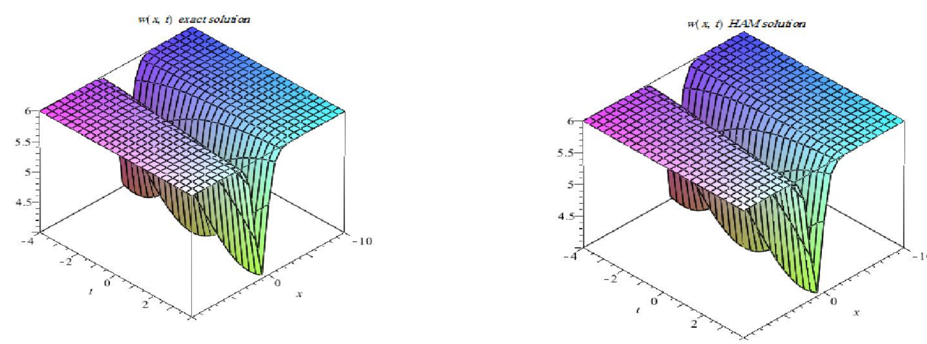


Fig. 3 exact solution and HPM solution for w(x ,t) at (-10 ≤ x ≤10 , -4 ≤ t ≤ 4)

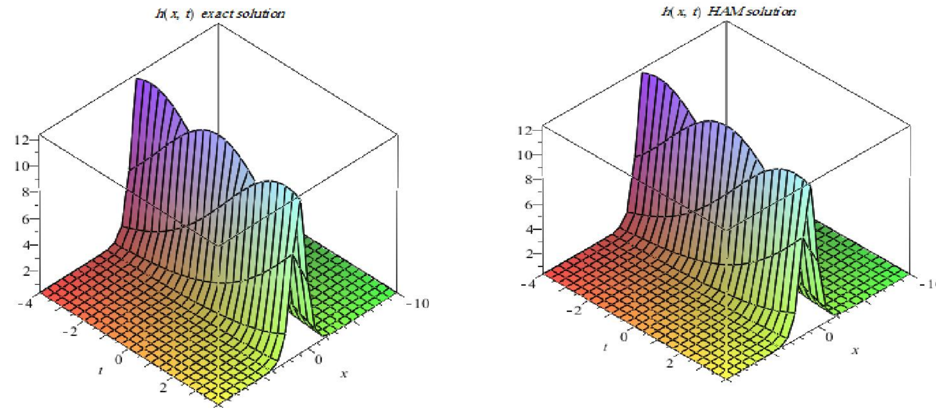


Fig. 4 exact solution and HPM solution for h(x ,t) at ($-10 \le x \le 10$, $-4 \le t \le 4$)

Table 1. Absolute error between exact solution and HPM solution

t , $x = 20$	$\lvert u_{exact} - u_{HPM} \rvert$	$\lvert v_{exact} - v_{HPM} \rvert$	$\lvert w_{exact} - w_{HPM} \rvert$	$\lvert h_{exact} - h_{HPM} \rvert$
0.1	4.8×10^{-46}	1.8×10^{-46}	2.3×10^{-46}	1.4×10^{-45}
0.2	7.8×10^{-42}	1.9×10^{-42}	3.9×10^{-42}	2.3×10^{-41}
0.3	2.9×10^{-39}	5.7×10^{-40}	1.1×10^{-39}	6.8×10^{-39}
0.4	1.3×10^{-37}	3.2×10^{-38}	6.4×10^{-38}	3.8×10^{-37}
0.5	3.0×10^{-36}	7.3×10^{-37}	1.4×10^{-36}	8.8×10^{-36}
0.6	3.8×10^{-35}	9.5×10^{-36}	1.9×10^{-35}	1.1×10^{-34}
0.7	3.3×10^{-34}	8.2×10^{-35}	1.6×10^{-34}	9.9×10^{-34}
0.8	2.1×10^{-33}	5.3×10^{-34}	1.1×10^{-33}	6.4×10^{-33}
0.9	1.1×10^{-32}	2.8×10^{-32}	5.6×10^{-33}	3.4×10^{-32}
1.0	4.9×10^{-32}	1.2×10^{-32}	2.4×10^{-32}	1.4×10^{-31}

4 Differential Transformation Method

Consider the system and the transformations between the original function and transformed functions in table 1.

$$u_t = v_y \quad (1)$$

$$v_t = -2v_{xxx} - 6(uv)_x + awu_x + bpv_x + cwp_x + dvp_x + fw_x + gp_x. \quad (2)$$

$$w_t = w_{xxx} + 3uw_x. \quad (3)$$

$$p_t = p_{xxx} + 3up_x. \quad (4)$$

We take

$b = 7, a = 6, c = 1, f = 1, d = 1, g = 1, \beta = \frac{1}{4}, b_0 = 4$

Eqs.(5)-(8) gives:

The initial data is:

$$u(x,0) = \frac{31}{12} - 4\mathrm{Tanh}[x]^2, \quad (13)$$

$$v(x,0) = \frac{31}{48} - \mathrm{Tanh}[x]^2, \quad (14)$$

$$w(x,0) = 4 + 2\mathrm{Tanh}[x]^2, \quad (15)$$

$$p(x,0) = \frac{99}{8} - 12\mathrm{Tanh}[x]^2. \quad (16)$$

Using table (1) and taking the differential transformation of (1) and (4) , we obtain

$$(h+1)U[k,h+1] = (k+1)V[k+1,h]. \quad (17)$$

$$(h+1)V[k,h+1] = -2(k+1)(k+2)(k+3)V[k+3,h] - 6\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)U(r,h-s)V(k-r+1,s) - 6\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)V(r,h-s)U(k-r+1,s)$$

$$
\begin{aligned}
&+6\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)W(r,h-s)W(k-r+1,s)\\
&+7\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)P(r,h-s)W(k-r+1,s)\\
&+\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)W(r,h-s)P(k-r+1,s)\\
&+\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)P(r,h-s)P(k-r+1,s)\\
&+(k+1)W[k+1,h]+(k+1)P[k+1,h] \qquad (18)
\end{aligned}
$$

Table 2. Taking into consideration solution (16) of Eqs. (1) - (4) and without loss the generality

original function	transformed function
$1-\ W(x,t)=u(x,t)\pm v(x,t)$	$W(k,h)=U(k,h)\pm V(k,h)$
$2-\ W(x,t)=\lambda u(x,t)$	$W(k,h)=\lambda U(k,h),\lambda$ is constant.
$3-\ W(x,t)=\dfrac{\partial u(x,t)}{\partial x}$	$W(k,h)=(k+1)U(k+1,h)$
$4-\ W(x,t)=\dfrac{\partial u(x,t)}{\partial y}$	$W(k,h)=(h+1)U(k,h+1)$
$5-W(x,t)=\dfrac{\partial^{r+s}u(x,t)}{\partial x^{r}\partial y^{s}}$	$W(k,h)=(k+1)(k+2)\ldots(k+r)\times(h+1)(h+2)\ldots(h+s)\times U(k+r,h+s)$
$6-W(x,t)=u(x,y)v(x,t)$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{k}U(r,h-s)V(k-r,s)$
$7-W(x,t)=\dfrac{\partial u(x,t)}{\partial x}\dfrac{\partial v(x,t)}{\partial x}$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)U(r+1,h-s)V(k-r+1,s)$
$8-W(x,t)=u(x,t)\dfrac{\partial u(x,t)}{\partial x}$	$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)U(r,h-s)U(k-r+1,s)$

$$
\begin{aligned}
(h+1)W[k,h+1]&=(k+1)(k+2)(k+3)W[k+3,h]\\
&+3\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)U(r,h-s)W(k-r+1,s) \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
(h+1)P[k,h+1]&=(k+1)(k+2)(k+3)W[k+3,h]\\
&+3\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)U(r,h-s)P(k-r+1,s) \qquad (20)
\end{aligned}
$$

From initial data (13) and (16) we have :

$U[0,0]=\frac{31}{12}, U[1,0]=0, U[2,0]=4, U[3,0]=0, U[4,0]=\frac{8}{3},$

$U[5,0]=0, U[6,0]=-\frac{68}{45}, U[7,0]=0, U[8,0]=\frac{248}{315}, U[9,0]=0,$

$U[14,0]=-\frac{3718276}{42567525}, U[15,0]=0, U[16,0]=\frac{25618328}{630512075}, U[17,0]=0,$

$U[18,0]=-\frac{1775444648}{97692469875}, U[19,0]=0, U[20,0]=\frac{75553864336}{9280784638125},\ldots$

$V[0,0]=-\frac{31}{48}, V[1,0]=0, V[2,0]=1, V[3,0]=0, V[4,0]=-\frac{2}{3},$

$V[5,0]=0, V[6,0]=\frac{17}{45}, V[7,0]=0, V[8,0]=\frac{62}{315}, V[9,0]=0,$

$V[10,0]=\frac{1382}{14175}, V[11,0]=0, V[12,0]=-\frac{21844}{467775}, V[13,0]=0,$

$V[14,0]=\frac{929569}{42567525}, V[15,0]=0, V[16,0]=-\frac{6404582}{638512875}, V[17,0]=0,$

$V[18,0]=\frac{443861162}{97692469875}, V[19,0]=0, V[20,0]=-\frac{18883466084}{9280784638125},\ldots$

$W[0,0]=4, W[1,0]=0, W[2,0]=2, W[3,0]=0, W[4,0]=-\frac{4}{3},$

$$W[5,0]=0, W[6,0]=\frac{34}{45}, W[7,0]=0, W[8,0]=-\frac{124}{315}, W[9,0]=0,$$

$$W[10,0]=\frac{2764}{14175}, W[11,0]=0, W[12,0]=-\frac{43688}{467775}, W[13,0]=0,$$

$$W[14,0]=\frac{1059130}{42567525}, W[15,0]=0, W[16,0]=-\frac{12809164}{638512875}, W[17,0]=0,$$

$$W[18,0]=\frac{887722324}{97692469875}, W[19,0]=0, W[20,0]=-\frac{37776932168}{9280784638125},\ldots$$

$$P[0,0]=\frac{99}{8}, P[1,0]=0, P[2,0]=-12, P[3,0]=0, P[4,0]=8,$$

$$P[5,0]=0, P[6,0]=-\frac{68}{15}, P[7,0]=0, P[8,0]=\frac{248}{105}, P[9,0]=0,$$

$$P[10,0]=-\frac{5528}{4725}, P[11,0]=0, P[12,0]=\frac{87376}{155925}, P[13,0]=0,$$

$$P[14,0]=-\frac{3718276}{14189175}, P[15,0]=0, P[16,0]=\frac{25618328}{212837625}, P[17,0]=0,$$

$$P[18,0]=-\frac{1775444646}{32564156625}, P[19,0]=0, P[20,0]=\frac{7555386436}{3093594875375},\ldots \quad (21)$$

Substituting from (21) into (17) - (20) and by recursive method we have:

$$U[0,1]=0, V[0,1]=0, W[0,1]=0, P[0,1]=0,$$

$$U[1,1]=2, V[1,1]=-\frac{1}{2}, W[1,1]=-1, P[1,1]=6,$$

$$U[2,1]=0, V[2,1]=0, W[2,1]=0, P[2,1]=0, P[4,1]=0,$$

$$U[3,1]=-\frac{8}{3}, V[3,1]=\frac{2}{3}, W[3,1]=\frac{4}{3}, P[3,1]=-8,$$

$$U[4,1]=0, V[4,1]=0, W[4,1]=0, P[4,1]=0,$$

$$U[5,1]=\frac{34}{15}, V[5,1]=-\frac{17}{30}, W[5,1]=-\frac{17}{15}, P[5,1]=\frac{34}{5},$$

$$U[6,1]=0, V[6,1]=0, W[6,1]=0, P[6,1]=0,$$

$$U[7,1]=-\frac{496}{315}, V[7,1]=\frac{124}{315}, W[7,1]=\frac{248}{315}, P[7,1]=-\frac{496}{105},$$

$$U[8,1]=0, V[8,1]=0, W[8,1]=0, P[8,1]=0,$$

$$U[9,1]=\frac{2764}{2835}, V[9,1]=-\frac{691}{2835}, W[9,1]=\frac{1382}{2835}, P[9,1]=\frac{2764}{945},$$

$$U[10,1]=0, V[10,1]=0, W[10,1]=0, P[10,1]=0,$$

$$U[4,4]=\frac{31}{144}, V[4,4]=-\frac{31}{576}, W[4,4]=-\frac{31}{288}, P[4,4]=\frac{31}{48},$$

$$U[5,4]=0, V[5,4]=0, W[5,4]=0, P[5,4]=0,$$

$$U[5,5]=\frac{691}{7200}, V[5,5]=-\frac{691}{28800}, W[5,5]=-\frac{691}{14400}, P[5,5]=\frac{691}{2400},\ldots \quad (22)$$

Substituting all U(k, h) , V(k, h), W(k, h) and P(k, h) into ,

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U[h,k]\,x^{k}t^{h},\quad v(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}V[h,k]\,x^{k}t^{h},$$

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W[h,k]\,x^{k}t^{h},\quad p(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}P[h,k]\,x^{k}t^{h},$$

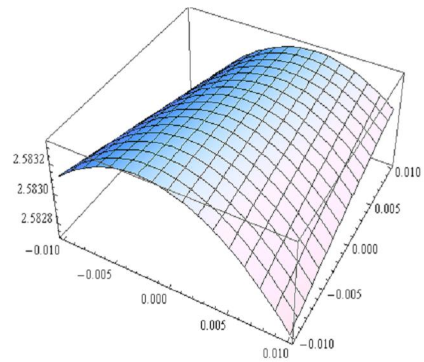
respectively, we have:

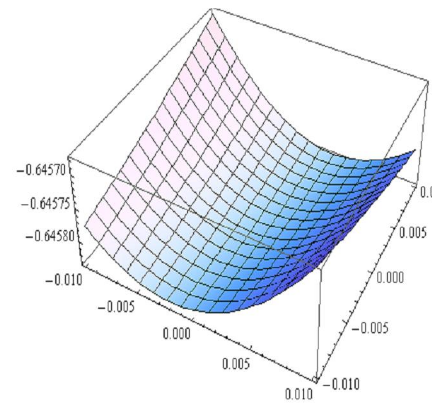
$$\begin{aligned}u(x,t)&=\frac{31}{12}-4x^{2}+\frac{8x^{4}}{3}+t\left(2x-\frac{8x^{3}}{3}+\frac{34x^{5}}{15}\right)\\&+t^{2}\left(-\frac{1}{1}+x^{2}-\frac{17x^{4}}{12}\right)\\&+t^{3}\left(-\frac{x}{6}+\frac{17x^{3}}{36}-\frac{31x^{5}}{45}\right)\\&+t^{4}\left(\frac{1}{96}-\frac{17x^{2}}{192}+\frac{31x^{4}}{144}\right)\\&+t^{5}\left(\frac{17x}{1920}-\frac{31x^{3}}{720}+\frac{691x^{5}}{7200}\right)\\&+\cdots,\end{aligned}\quad (23)$$

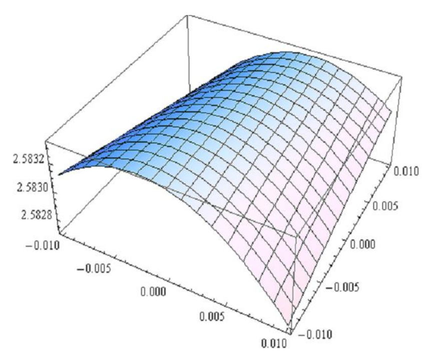
$$\begin{aligned}v(x,t)&=-\frac{31}{48}+x^{2}-\frac{2x^{4}}{3}+t\left(-\frac{x}{2}+\frac{2x^{3}}{3}-\frac{17x^{5}}{30}\right)\\&+t^{2}\left(\frac{1}{16}-\frac{x^{2}}{4}+\frac{17x^{4}}{48}\right)\\&+t^{3}\left(\frac{x}{24}-\frac{17x^{3}}{144}+\frac{31x^{5}}{180}\right)\\&+t^{4}\left(-\frac{1}{384}+\frac{17x^{2}}{768}-\frac{31x^{4}}{576}\right)\\&+t^{5}\left(-\frac{17x}{7680}+\frac{31x^{3}}{2880}-\frac{691x^{5}}{28800}\right)\\&+\cdots,\end{aligned}\quad (24)$$

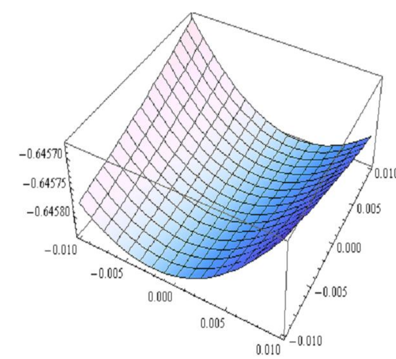
$$\begin{aligned}w(x,t)&=-4+2x^{2}-\frac{4x^{4}}{3}+t\left(-x+\frac{4x^{3}}{3}-\frac{17x^{5}}{15}\right)\\&+t^{2}\left(\frac{1}{8}-\frac{x^{2}}{2}+\frac{17x^{4}}{24}\right)\\&+t^{3}\left(\frac{x}{12}-\frac{17x^{3}}{72}+\frac{31x^{5}}{90}\right)\\&+t^{4}\left(-\frac{1}{192}+\frac{17x^{2}}{384}-\frac{31x^{4}}{288}\right)\\&+t^{5}\left(-\frac{17x}{3840}+\frac{31x^{3}}{1440}-\frac{691x^{5}}{14400}\right)\\&+\cdots,\end{aligned}\quad (25)$$

$$
\begin{aligned}
&p(x,t)\\
&-\frac{99}{8}-12x^2+8x^4+t\left(6x-8x^3+\frac{34x^5}{5}\right)\\
&+t^2\left(-\frac{3}{4}+3x^2-\frac{17x^4}{4}\right)\\
&+t^3\left(-\frac{x}{2}+\frac{17x^3}{12}-\frac{31x^5}{15}\right)\\
&+t^4\left(\frac{1}{32}-\frac{17x^2}{64}+\frac{31x^4}{48}\right)\\
&+t^5\left(\frac{17x}{640}-\frac{31x^3}{240}+\frac{691x^5}{2400}\right)\\
&+\cdots \qquad (26)
\end{aligned}
$$





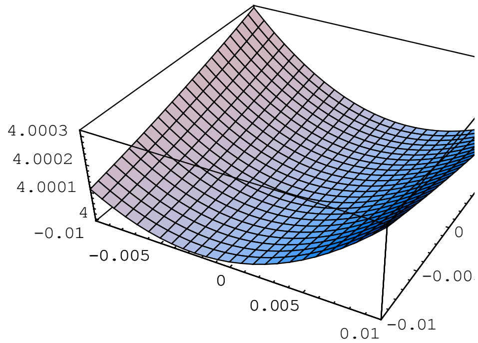


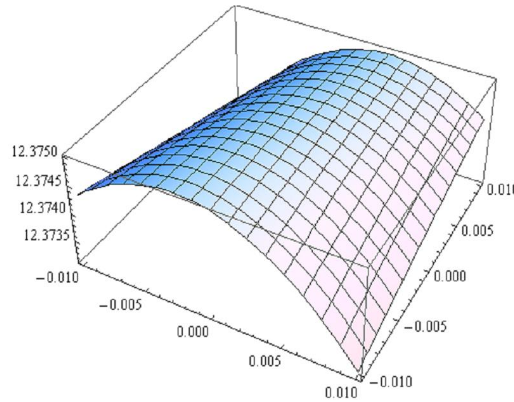


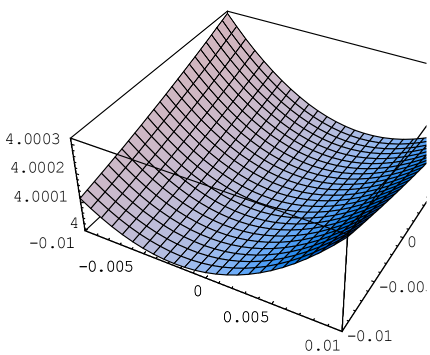
$u=\frac{31}{12}-4\mathrm{Tanh}[x-\frac{t}{4}]^2$

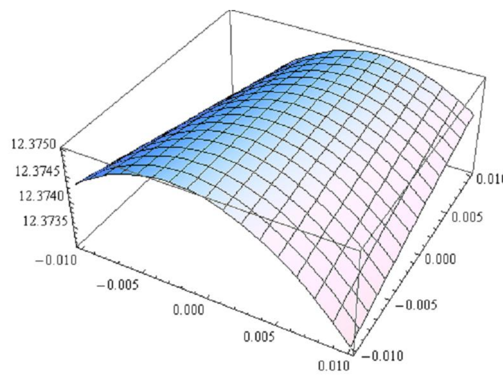
$v(x,t)=\frac{1}{4}(-\frac{31}{12}+4\mathrm{Tanh}[x-\frac{t}{4}]^2$

$$
u(x,t)=\frac{31}{12}-\frac{t^2}{4}+\frac{t^4}{96}+2tx-\frac{t^3x}{6}+\frac{17t^5x}{1920}-4x^2+t^2x^2-\frac{17t^4x^2}{192}-\frac{8tx^3}{3}+\frac{17t^3x^3}{36}-\frac{31t^5x^3}{720}+\frac{8x^4}{3}-\frac{17t^2x^4}{12}+\frac{31t^4x^4}{144}+\frac{34tx^5}{15}-\frac{31t^3x^5}{45}+\frac{691t^5x^5}{7200}
$$









$$w(x,t) = 4 + 2\mathrm{Tanh}[x - \tfrac{t}{4}]^2 \qquad p(x,t) = \frac{99}{8} - 12\mathrm{Tanh}[\tfrac{t}{4} - x]^2$$

$$w(x,t) = 4 + \frac{t^2}{8} - \frac{t^4}{192} - tx + \frac{t^3x}{12} - \frac{17t^5x}{3840} + 2x^2 - \frac{t^2x^2}{2} + \frac{17t^4x^2}{384} + \frac{4tx^3}{3} - \frac{17t^3x^3}{72} + \frac{31t^5x^3}{1440} - \frac{4x^4}{3} + \frac{17t^2x^4}{24} - \frac{31t^4x^4}{288} - \frac{17tx^5}{15} + \frac{31t^3x^5}{90} - \frac{691t^5x^5}{14400}$$

$$p(x,t) = \frac{99}{8} - \frac{3t^2}{4} + \frac{t^4}{32} + 6tx - \frac{t^3x}{2} + \frac{17t^5x}{640} - 12x^2 + 3t^2x^2 - \frac{17t^4x^2}{64} - 8tx^3 + \frac{17t^3x^3}{12} - \frac{31t^5x^3}{240} + 8x^4 - \frac{17t^2x^4}{4} + \frac{31t^4x^4}{48} + \frac{34tx^5}{5} - \frac{31t^3x^5}{15} + \frac{691t^5x^5}{2400}$$

Table 1. exact ,numerical solution and error of u(x,t).

t	U exact	U_{DTM}	Error
0.01	2.5831083417705645	2.5831083417720760	1.51168×10^{-12}
0.02	2.5832333349999765	2.5832333350015110	1.53433×10^{-12}
0.03	2.5833083334375000	2.5833083334392786	1.77858×10^{-12}
0.04	2.5833333333333335	2.5833333333363520	3.01847×10^{-12}
0.05	2.5833083334375000	2.5833083334447666	7.26663×10^{-12}
0.06	2.5832333349999765	2.5832333350186800	1.87033×10^{-11}
0.07	2.5831083417705645	2.5831083418154352	4.48708×10^{-11}
0.08	2.5829333599984890	2.5829333600966233	9.81344×10^{-11}
0.09	2.5827083984317363	2.5827083986291446	1.97408×10^{-10}
0.10	2.5824334683161230	2.5824334686862720	3.70149×10^{-10}

Table 2. exact ,numerical solution and error of v(x,t).

t	V_{exact}	V_{DTM}	Error
0.01	-0.6457770854426411	-0.6457770854430190	3.7792×10^{-13}
0.02	-0.6458083337499941	-0.6458083337503777	3.83582×10^{-13}
0.03	-0.6458270833593750	-0.6458270833598196	4.44644×10^{-13}
0.04	-0.6458333333333334	-0.6458333333340880	7.54619×10^{-13}
0.05	-0.6458270833593750	-0.6458270833611917	1.81666×10^{-12}
0.06	-0.6458083337499941	-0.6458083337546700	4.67582×10^{-12}

0.07	-0.6457770854426411	-0.6457770854538588	1.12177×10^{-11}
0.08	-0.6457333399996222	-0.6457333400241558	2.45336×10^{-11}
0.09	-0.6456770996079341	-0.6456770996572861	4.93521×10^{-11}
0.10	-0.6456083670790308	-0.6456083671715680	9.25372×10^{-11}

Table 3. exact ,numerical solution and error of w(x,t).

t	W_{exact}	W_{DTM}	Error
0.01	4.0001124957813845	4.0001124957806290	7.5584×10^{-13}
0.02	4.0000499991666790	4.0000499991659115	7.67386×10^{-13}
0.03	4.0000124999479170	4.0000124999470280	8.89067×10^{-13}
0.04	4.0000000000000000	3.9999999999984905	1.50946×10^{-12}
0.05	4.0000124999479170	4.0000124999442830	3.63354×10^{-12}
0.06	4.0000499991666790	4.0000499991573270	9.35163×10^{-12}
0.07	4.0001124957813840	4.0001124957589500	2.24345×10^{-11}
0.08	4.0001999866674220	4.0001999866183560	4.90656×10^{-11}
0.09	4.0003124674507990	4.0003124673520950	9.87042×10^{-11}
0.10	4.0004499325086050	4.0004499323235310	1.85074×10^{-10}

Table 4. exact ,numerical solution and error of p(x,t).

t	P_{exact}	P_{DTM}	Error
0.01	12.374325025311693	12.374325025316228	4.53504×10^{-12}
0.02	12.374700004999928	12.374700005004533	4.60432×10^{-12}
0.03	12.374925000312500	12.374925000317836	5.33618×10^{-12}
0.04	12.375000000000000	12.375000000009056	9.05587×10^{-12}
0.05	12.374925000312500	12.374925000334300	2.18012×10^{-11}
0.06	12.374700004999928	12.374700005056038	5.61098×10^{-11}
0.07	12.374325025311693	12.374325025446305	1.34612×10^{-10}
0.08	12.373800079995467	12.373800080289870	2.94403×10^{-10}
0.09	12.373125195295207	12.373125195887434	5.92227×10^{-10}
0.10	12.372300404948370	12.372300406058816	1.11045×10^{-9}

Acknowledgement:

This paper was funded by the Deanship of Scientific Research (DSR), King Abdulaziz university, Jeddah, under grant No ( ).

The authors therefore acknowledge with thans DSR technical and financial support.

Corresponding Author:
Hassan Zedan[1&2]
[1]Department of Mathematics, Faculty of Science, king Abdualziz University, P.O.Box:80203, Jeddah 21589, Saudi Arabia.

[2]Department of Mathematics, Faculty of Science, Kafr El-Sheikh University, Egypt
hassanzedan2003@yahoo.com

References

1. Whitham G.B., Linear and Nonlinear waves, Wiley, New York, 1999.
2. Kichenassamy S., Olver, Existence and nonexistence of solitary wave solutions to higher-order model evolution equations, SIAM J. Math. Anal. 23 (5) (1992) 1141–1166.
3. D. Baldwin, U. Goktas, W. Hereman, L. Hong, R.S. Martino, J.C. Miller, Symbolic computation of exact solutions in hyperbolic and elliptic functions for nonlinear PDEs, J. Symbolic Comput. 37 (2004) 669–705.
4. Hassan A. Zedan, Symmetry analysis of an integrable Ito coupled systems, Computers and Mathematics with Applications 60 (2010) 3088–3097. and the Pad e technique, Computers and Mathematics with Applications 54 (2007) 940–954.
5. Hassan A. Zedan , Applications of the New Compound Riccati Equations Rational Expansion Method and Fan's Subequation Method for the Davey Stewartson Equations Hindawi Publishing Corporation Boundary Value Problems Volume 2010, 23 pages.
6. Hassan A. Zedan "Exact solutions for the generalized KdV equation by Using Backlund transformations" Journal of the Franklin Institute 348;(2011)1751-1768.
7. Liao, S. J., An approximate solution technique not depending on small parameters: a special example, Int. J. Non-Linear Mechanics, 1995, 30(3):371-380.
8. He, J. H., Approximate solution of nonlinear differential equations with convolution product nonlinearities, Computer Methods in Applied Mechanics and Engineering, 1998, 167(1-2):69-73.
9. Nayfeh, A. H., Problems in Perturbation, John Wiley & Sons, 1985.
10. J.K. Zhou, Differential Transformation and its Application for Electrical Circuits, Huazhong University Press, Wuhan, China, 1986 (in Chinese).
11. C.K. Chen, S.H. Ho, Solving partial differential equations by two dimensional differential transform, Appl. Math. Comput. 106 (1999) 171–179.
12. M.J. Jang, C.L. Chen, Y.C. Liu, Two-dimensional differential transform for partial differential equations, Appl. Math. Comput. 121 (2001) 261–270.

1/20/2013