Impact of Directional Variograms in Fuzzy Type-II Punctual Kriging based Image Restoration

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Abstract: In this paper, we perform an experimental study to investigate directional variograms in punctual kriging and consequently its effect on image restoration. We employ punctual kriging in conjunction with fuzzy logic type-II and fuzzy smoothing based approaches to remove white Gaussian noise from corrupted images. Images degraded with Gaussian white noise are restored by first utilizing fuzzy logic for selecting pixels that needs kriging. Type-II fuzzy set has been used to generate fuzzy map for detection of noisy pixels. Further, local neighborhood information is used to ensure noise free pixels in 3x3 window to estimate the noisy data. The concept of directional semivariance based punctual kriging is then used to estimate the intensity of a noisy pixel. Image restoration performance based comparison has been made against adaptive Weiner filter and existing fuzzy non-directional kriging approaches. Experimental results on various images and different image quality measures show that directional semivariance may provide information how image data is distributed in different directions but could not play an effective role in punctual kriging based image restoration as compared to non-directional punctual kriging estimation.


Keywords: Image restoration; Type-II fuzzy logic; punctual kriging; directional variograms; adaptive spatial filtering

1. Introduction

Image restoration has become a widely investigated field of image processing, dealing with the reconstruction of images by removing noise and blur from degraded images and making them suitable for human perception. In spite of the advances made by recent methods, it is still a challenging task as these methods have yet to achieve a desirable level of applicability in many realistic scenarios. More-over with the ever increasing production of digital contents such as images and videos taken with low resolution cameras and in poor conditions, the importance of image restoration has significantly increased. Images can become corrupted during any of the acquisition, pre-processing, compression, transmission, storage and/or reproduction phases of processing (Gonzalez and Woods, 2002). One of the main aspects in devising intelligent image restoration techniques is noise suppression with keeping edges intact. Further, noise smoothing and edge enhancement are generally considered as contradictory attributes. Since smoothing a region might destroy an edge while sharpening edges might lead to amplification of unnecessary noise (Tsai et al., 2009). In this paper, we present an analysis of directional variograms in punctual kriging, effect of direction based semivariance on matrix \( A \) and vector \( b \) in punctual kriging to estimate unbiased weights as given in equation (8), and consequently its performance in image restoration. In the sequel, we present a brief review of spatial filtering technique, based on punctual kriging and fuzzy logic control, to remove noise while efficiently preserving the image details and edge information.

Punctual kriging, named after its developer, D. G. Krige (Krige, 1992) is heavily used in mining and geostatistics based applications. It is an interpolation technique that gives an optimal linear estimate of an unknown parameter at a sampling point in terms of its known values at the surrounding sampling points (Voloshynovskiy et al., 2005). The estimation involves calculation of the semi-variances and modeling of semi-varigrams from the sampled data. Besides this, kriging has been applied in many other fields as well.

In the field of image processing, Pham and Wagner (Pham and Wagner, 1999, and Pham and Wagner, 2000) reported the first use of kriging along with fuzzy sets to enhance images corrupted by Gaussian noise. They model soft-thresholding by fuzzy sets. In their approach, the pixel intensity in the processed image is a weighted sum of the original (noisy) and the estimated value through kriging. They have evaluated their results qualitatively in comparison with adaptive Wiener filter (AWF). However, their study does not provide any quantitative performance analysis of their proposed technique (Mirza et al., 2007, and Asmatullah, 2007). Further, they apply kriging to all pixels in the degraded image. Considering 3x3 neighborhood,
The inverse of a kriging matrix of size $9 \times 9$ is required, that can make the filtering process computationally expensive. Furthermore, due to zero diagonal, matrix inverse may not always be possible. In addition to this, filter weights also suffer from occurrence of negative values, which leads to an overall poor performance of the filter. Mirza et al. (2007) have highlighted these issues and proposed a spatially adaptive fuzzy kriging (SAFK) approach based on punctual kriging, fuzzy logic and fuzzy smoothing. In their scheme, first they employ fuzzy logic based on homogeneity and DAMdistance to generate a fuzzy decision map for the fate of a pixel whether it needs to be kriged or not. In the next step, they apply punctual kriging to estimate the selected noisy pixels. To tackle the matrix inversion failure problem, they have used fuzzy smoothing value as an estimate for the pixel under consideration. In their scheme, they renormalize the positive weights on occurrence of negative weights in punctual kriging. In addition to this, Chaudhry et al. (2007) have proposed a hybrid image restoration technique using fuzzy punctual kriging and genetic programming. In their technique, they have considered image restoration as an optimization problem. In (Chaudhry et al. 2007), they have also analyzed effect of neighborhood size on negative weights in punctual kriging based image restoration.

In most of the image restoration methods, all pixels in local neighborhood are considered to estimate a pixel under consideration without observing whether all of the pixels other than central pixel are noise free or not. Technically speaking, noisy pixels used to estimate a corrupted pixel in local window can’t offer optimal/near optimal estimate. This is because of noisy pixel has intensity level away from its actual intensity value depending upon the strength of noise in the degraded image. Even AWF, Pham & Wagner fuzzy kriging (PWFK) and SAFK approaches employ the same scenario to estimate the pixel under consideration without taking into account this important fact. So, this certainly needs special attention how to replace intensity values at noisy locations before its use in estimation. In this experimental study, we also take into account this fact, employ fuzzy inference system considering fuzzy type-II and exploit noise free pixels in local neighborhood window in conjunction with punctual kriging for image restoration. We have employed type-II fuzzy sets with improved fuzzy rules for decision making about the pixel’s value. This factor not only improves the results but also gives optimal/near optimal estimates and avoids devastating edges. We call this technique spatially adaptive type-II fuzzy kriging (SAFK-II). Employing fuzzy type-II with improved member functions, fuzzy rules to generate fuzzy decision map, and taking into account vital information about the pixels (noisy or noise free) to estimate the pixel under consideration in local neighborhood window differentiate from the work presented in (Mirza et al., 2007).

Although, the strength of kriging is its ability to account for directional trends of the data at hand, it is possible to analyze the variance with respect to distance only and disregard how pixels in an image are geographically oriented. Whereas, by considering the fact of geographic orientation of the data analysis of the variance with respect to distance as well as direction must be carried out. In addition to this, separating noise and original signal from a single input image is under constrained; in theory it is very difficult to recover the original signal (Ce et al., 2008). This paper is aimed to make an analysis of direction based semivariance estimation and its utilization in punctual kriging to restore noisy images. The results have been compared by considering different scenarios for directional variance estimation by utilizing the technique based on fuzzy inference system type-II, punctual kriging and fuzzy smoothing (Directional SAFK-II) with PWFK, SAFK and SAFK-II. This paper makes the following contributions:

- Comparative analysis of the effect of directional variograms on reduction of negative weights, matrix inversion failure and the consequent improvement in image restoration.

- Edges in an image should be preserved during the denoising process.

- No artifacts should be visible in the denoised image.

- We believe noise free pixels significantly contribute to estimate the noisy pixel whereas; noisy pixels other than the pixel under consideration may enhance the noise effect because of their corruptness. Further, we exploit available local information in the neighborhood window to estimate a noisy pixel utilizing type-II fuzzy decider, punctual kriging and fuzzy averaging.

Rest of the paper is structured as follows. Section 2 introduces punctual kriging and variograms. It also presents some of the most commonly used image quality measures along with the variogram based quality measure. Section 3 explains the SAFK-II technique based on punctual kriging and fuzzy approach of adaptive learning. Experimental results and discussion is presented in section 4. Our findings including directions for future work are given in section 5.

2. Related Theory
2.1. Punctual Kriging and Variogram
In literature, it has been proved that punctual kriging results best linear unbiased estimate of an unknown point on a surface (El-Sheimy et al., 1995). This estimate is the weighted sum of the known neighboring values around the unknown point. The weights are calculated by minimizing the variance of estimation-error. Kriging uses a variogram model (a concept from geostatistics) to fit the experimental data. Based on the variogram model chosen, known values are assigned optimal weights to calculate the unknown value. Whereas, in an image denoising scenario pixel intensity is available at all points including corrupted ones. So, from the experimental variogram we can directly find kriging weights to estimate the noisy pixels without employing any theoretical variogram model. For more details on this topic, we refer the readers to (Mirza et al., 2007, and Asmatullah, 2007). Fig. 1 illustrates the neighborhood of a typical pixel within a digital image, identifying the neighboring pixels at different lags from the central pixel. It also identifies the pixels at different lags and in various directions. Variogram presents the semivariance deviation with respect to distance from a point. Similarly, directional variogram provides variation in data with respect to distance and as well as direction. Semivariance provides a measure of spatial dependence between samples.

Semivariance \([4]\) of the samples at lag ‘d’ can be calculated from eqn. (1).

\[
\gamma(d) = \frac{1}{2} \text{Var}(z_{i+d} - z_i)
\]  

(1)

Different distance metrics can be used to identify a group of neighboring samples having the same lag. In the present investigations, however, we have considered the Euclidean metric as the distance measure. The experimental semivariogram is obtained directly by using the sample values from the experimental data.

For a given lag ‘d’, it is calculated from the available data as:

\[
\gamma(d) = \frac{1}{2N(d)} \sum_{i=1}^{N(d)} (z_i - z_{i+d})^2
\]  

(2)

The above expression for experimental semi-variogram depends upon the spatial configuration of the available image data. One has to consider different cases, as to whether the data is aligned or not and whether it is regularly spaced along the alignments. However in the present case of

![Image of Figure 1. Neighborhood of a pixel and identification of pixels at various lags](http://www.lifesciencesite.com)
digital images, the data is aligned and regularly spaced, which makes the estimation of the semi-variogram easy. In this current research work, we analyze the effect of directional variograms in contrast with simple variogram in punctual kriging based image restoration.

Punctual kriging is a linear combination of the neighboring sample values, as given by eqn. (3).

\[
\hat{z} = \sum_i w_i z_i
\]  
(3)

Where, \( w_i \) are the weights and \( z_i \) are the neighboring values of \( Z \). It is an unbiased estimator if the weights add up to 1. This additional constraint on weights is given by:

\[
\sum_i w_i = 1
\]  
(4)

Statistical variance is measure of how different the estimated value is from its neighboring sample values. It can be found using the eqn. (5).

\[
Var(e) = Var(z - \hat{z})
\]

(5)

A number of such linear unbiased estimators are available, but we find the best one in the sense that it has the smallest estimation variance. Thus, the cost function is defined as:

\[
\phi(w, \lambda) = Var(e) - 2\lambda \left( \sum_i w_i - 1 \right)
\]

(6)

where \( \lambda \) is the Lagrange multiplier. Differentiating the cost function \( \phi(w, \lambda) \) with respect to \( w_i \) and \( \lambda \) and setting the differential equal to zero and rearranging the system of equations, these can be written in matrix form as:

\[
\begin{pmatrix}
\gamma(d_{11}) & \gamma(d_{12}) & \ldots & \gamma(d_{1n}) & 1 & w_1 \\
\gamma(d_{21}) & \gamma(d_{22}) & \ldots & \gamma(d_{2n}) & 1 & w_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\gamma(d_{n1}) & \gamma(d_{n2}) & \ldots & \gamma(d_{nn}) & 1 & w_n \\
1 & 1 & 1 & 0 & \lambda & 1
\end{pmatrix} = 0
\]

(7)

or in matrix-vector notations

\[
Aw = b
\]

(8)

Matrix \( A \) is symmetric and has zero diagonal elements. The elements of the matrix are taken from the semi-variogram (defined in eqn. (1)) for the current point. Solving eqn. (8) gives us the optimal kriging weights \( \{w_1, w_2, \ldots, w_n\} \) for estimating the unknown value \( \hat{z} \) using its neighbors. However, if \( A \) is a singular matrix then punctual kriging fails to estimate pixel intensity. In this paper, we consider direction based semi-variance to find matrix \( A \). Further, we also find the semi-variance vector \( b \) in a similar fashion.

2.2. Image Quality Measures

Besides mean square error (MSE), peak signal-to-noise ratio (PSNR), weighted peak signal-to-noise ratio (wPSNR) and structure similarity index measure (SSIM) [9], other image quality measure in terms of the experimental variograms of the original and degraded images is also used called variance mean square error (VMSE) (Chaudhry et al., 2007). Statistical meaning of VMSE is to measure the mean squared error of the variogram of the estimated and the original images.

3. SAFK-II Approach

The occurrence of singular matrix in kriging is inherently unpredictable as it depends on the variogram for a pixel in the degraded image. The variogram itself depends on neighboring values of a pixel. Such scenarios should be taken care of separately by replacing the processed pixel with a value given by fuzzy ‘averaging’ or ‘median’ filter, which ever makes the error variance ‘small’. Further, it is observed that for some of the selected pixels, the punctual kriging procedure results in negative weights. To handle this problem, approximation has been used to reinitialize the weights i.e. negative weights have been set to zero and positive weights have been renormalized in a similar fashion as described in (Mirza et al., 2007).

Fig. 2 shows the basic architecture of direction based SAFK-II approach. Firstly, a decision map is generated for pixels that needs kriging or not through fuzzy decider. Further, it is ensured that all pixels in local window around the pixel in question are noise free. In case of any noisy pixel found other than the central pixel in local window is replaced by average of its neighbors lying at one pixel apart. Then selected pixels are estimated by applying punctual kriging. The pixels that are not selected for kriging by the fuzzy decider are processed using the robust fuzzy weighted filter. We employ directional variograms in SAFK-II to observe its effect in punctual kriging based image restoration and compare results with AWF, PWFK, SAFK and SAFK-II.
3.1. Details of Different Stages in SAFK-II Methodology

In SAFK-II like SAFK (Mirza et al., 2007), all pixels are not blindly kriged. Rather, based on the homogeneity and deviation of its local neighborhood, a pixel is selected for kriging by a fuzzy logic rule-based system called Fuzzy decider. The inputs to the Fuzzy decider are a measure of homogeneity and DAMdistance which is based on the mean and deviation of the $3 \times 3$ window around the current pixel. The degree of homogeneity is estimated by eqn. (11) as proposed by Tizhoosh (2000). The numerator in eqn. (11) is the difference of the maximum and minimum gray values in the region comprising of the $3 \times 3$ window around a pixel where, the denominator is the difference of the maximum and minimum gray values in the whole image.

$$\mu_{H} = \frac{g_{max}^{local} - g_{min}^{local}}{g_{max}^{global} - g_{min}^{global}}$$  \hspace{1cm} (9)

The DAMdistance in the rules is simply the difference between the gray value of the current pixel and the mean gray value of its neighbors. The fuzzy decider is a basic Mamdani type-II fuzzy logic system consisting of the following rules which differs from Mirza et al. (2007).

3.2. Type-II Fuzzy Sets

If (regionHomogeneity is HomoMed) and (DAMdistance is Acceptable)  
then (decision is KrigingNo)

If (regionHomogeneity is HomoMed) and (DAMdistance is High)  
then (decision is KrigingYes)

If (regionHomogeneity is HomoMed) and (DAMdistance is VeryHigh)

The concept of type-II fuzzy logic is actually an extension of type-I fuzzy sets. Type-II fuzzy sets are capable to handle more uncertainties in spite of the fact that they are difficult to use and understand than type-I fuzzy sets (Mitchell, 2005). A type-II fuzzy set has been used to enhance the fuzzification process for better decision making. Steps involved in a typical fuzzy inference system (FIS) are depicted in Fig. 3.
then (decision is KrigingYes)
If (regionHomogeneity is HomoLow) and (DAMdistance is Acceptable)
then (decision is KrigingNo)
If (regionHomogeneity is HomoLow) and (DAMdistance is High)
then (decision is KrigingYes)
If (regionHomogeneity is HomoLow) and (DAMdistance is VeryHigh)
then (decision is KrigingYes)
If (regionHomogeneity is HomoLow) and (DAMdistance is Acceptable)
then (decision is KrigingNo)
If (regionHomogeneity is HomoHigh) and (DAMdistance is Acceptable)
then (decision is KrigingNo)
If (regionHomogeneity is HomoHigh) and (DAMdistance is High)
then (decision is KrigingNo)
If (regionHomogeneity is HomoHigh) and (DAMdistance is VeryHigh)
then (decision is KrigingYes)

Fig. 4 shows the graphical representation of type-II fuzzy membership functions. Fuzzy decision maps generated by fuzzy type-I &II are described below.

3.3. Generation of Fuzzy Decision Map through Fuzzy-II

In the first stage, noisy image is presented to the fuzzy decider that generates a binary image called the fuzzy decision map. This decision map is generated through type-II fuzzy set and is provided to the punctual kriging based estimation stage, where the decision of whether to estimate or not is enforced. This helps to reduce the computational time. Effectiveness of fuzzy decision map generated through fuzzy type-I and type-II has been compared for image restoration. Fuzzy decision maps generated through fuzzy type-I and type-II for cameraman image are shown in Fig. 5. It can be observed from Fig. 5 that fuzzy map generated through type-II avoids selecting edge pixels for estimation whereas type-I selects more for estimation including edge pixels and results in edge smoothing as compared to type-II fuzzy.

3.4. Employing Punctual Kriging for Estimation

In the second stage, an attempt is made to find a kriging estimate for pixels selected by the fuzzy decider. If the attempt fails, the original pixel value is taken as the processed pixel value. Attempt to find a kriging estimate was found to fail due to two broad reasons: singular kriging matrix and negative-weights. The pixels that were rejected for kriging by the fuzzy decider are processed using the robust fuzzy weighted filter. After this stage, the processed image contains two types of values based on the decision map: kriging estimate and the fuzzy smoothed values. There is a third category of noisy pixels for which kriging could not give optimum weights i.e. the weights either do not sum up to 1 or the sum of square of weights is not less than or equal to 1. Negative weights in this case are set to zero and the positive weights are normalized to sum to 1.
Figure 5. Result from (a) noisy cameraman image (b) Fuzzy type-I and (c) Fuzzy type-II decision map for Cameraman image degraded with variance 0.02.

3.5. Fuzzy Smoothing of Pixels Not Selected for Kriging

In the third stage, the unselected pixels by the fuzzy decider are processed using the robust fuzzy weighted filter. After the second stage, the processed image contains two types of values based on the decision map: kriging estimate and original values (unselected pixels). In this stage, a fuzzy smoothing is applied on the unselected pixels.

4. Results and Discussions

4.1. Variograms of the Original and Degraded Images

The experimental semi-variograms (omni-directional semi-variograms) of three different types of images (Boat, Blood cells and Lena) have been computed and shown in Fig. 6. The shapes of the variograms for all three images near lag zero are continuous. This shows that the pixel values do not change abruptly at lags near zero. However, for Lena and Boat images, fluctuations start appearing for lags greater than 10. This shows that after a lag of 10 pixels, we enter into a new region.

Further, in case of Blood cells image, the fluctuations appear after a lag of 20 pixels. The variograms show sharp changes for larger lags.

If geographic orientation is important then a directional semi-variogram should be calculated which may provide an estimate more accurately. When two or more directions are analyzed an experimental semi-variogram will be generated for each direction. In this experimental study, we have considered four main directions (E-W, N-S, SW-NE and SE-NW) to calculate directional semi-variances and its effect on punctual kriging based image restoration. Semi-variogram experimentation can uncover fundamental information about the data, for instance does the data vary in more than one direction. Directional semi-variograms of different images are shown in Fig. 7. Further, it can be revealed that whether the image data is isotropic (varies the same in all directions) or anisotropic (data varies differently in different directions) as demonstrated in Fig. 7.

It can be observed from Fig. 7 that data in each image is distributed differently along four directions i.e. EW, NS, NW-SE, NE-SW and vice versa. Since image data is distributed differently in each direction, we aim direction based semivariance calculations may influence punctual kriging results and consequently may have an effect on image restoration. In this paper, we calculate semivariance at various lags in different directions for all 8 pixels around the central pixel in local window of size 3x3 and form matrix $A$ to be used in equation (8). Further, we also calculate semivariance vector $b$ which is the semivariance of the central pixel from all neighboring pixels at lag 1, square root 2 in four directions.
Figure 6. Experimental variograms of three different images

Figure 7. Direction based experimental variograms of four different images
Various image quality measures as explained in section 2 are applied to find out the quality of the processed image as compared to the original image. We have carried out analysis of directional variograms in punctual kriging and its effect on image restoration. In this experimental study, we have considered five different images Lena, Cameraman, Blood Cells, Boat and Baboon as test data. We have investigated the performance of direction based SAFK-II in comparison with PWFK, AWF, SAFK and SAFK-II by corrupting the images through white Gaussian noise of variance ranging from 0.01 to 0.1. This is because noise effect may change with the variance of noise as regards the visual distortion for an image. On the other hand, same noise may affect different images differently as regards the visual distortion. Typical results from the fuzzy decider type-I and type-II shown in Fig. 5. The white pixels are the ones that need kriging.

4.2. Performance Analysis by Varying Variance of Gaussian Noise

We have considered Lena, Cameraman, Blood Cells, Baboon and Boat image as test data. Images are degraded with Gaussian white noise of variance ranging from 0.01 to 0.1. Results obtained through direction based SAFK-II has been compared with that of the AWF and non-directional punctual kriging based image restoration methods PWFK, SAFK and SAFK-II. The effect of additive Gaussian noise and its removal by various approaches is shown in Fig. 8. Experimental variograms of resulted image through various approaches are illustrated in Fig. 9. It can be observed from Fig. 9 that variogram of the resulted image through directional SAFK-II is far away from variogram of the original image. Whereas, non-directional (simple) SAFK-II produces an experimental variogram very close to the original image and outperforms all methods. Further, Table 1 gives a quantitative comparison among different methods in terms of PSNR, SSIM and VMSE at test images against various variances. It can be observed from Table 1 that directional SAFK-II outperforms PWFK and AWF but cannot achieve better results than simple SAFK and SAFK-II approaches. Image data is highly correlated and pixels are located at micro level (micro meter). In geo-statistic ore contents analysis, the available data points are located at macro level (distance in meters). This characteristic of image data may also be observed from Fig. 6. In addition to this, matrix \( A \) contains ‘0’ diagonals and is an ill conditioned matrix. Further, directional semivariance among various neighboring pixels may be zeros or close to zero because of lying in smooth region. These factors make the matrix \( A \) singular or close to singular matrix and results in matrix inversion failure. As described in section 3.4, if punctual kriging fails, the original pixel value is taken as the processed pixel value. While both SAFK and SAFK-II use omni semivariance (average semivariance at different lags irrespective of direction) in matrix \( A \) and try to avoid from producing singular matrix. This fact differentiates directional SAFK-II from SAFK and SAFK-II. Performance comparison of directional SAFK-II, PWFK, AWF, SAFK and SAFK-II approaches in terms of PSNR and SSIM is also shown graphically in Fig. 10 & 11. From Fig. 10 & 11, it can be examined that directional semivariance does not play an effective role in punctual kriging based image restoration.

![Fig.8. Original, noisy and estimated images obtained through AWF, PWFK, SAFK, SAFK-II, and directional SAFK-II.](image-url)
Table 1. Comparison of de-noising methods in terms of PSNR, SSIM and VMSE for various images degraded with Gaussian white noise

<table>
<thead>
<tr>
<th>Images</th>
<th>Methods</th>
<th>Noise Variance</th>
<th>Qualitative Measures</th>
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<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>0.04</td>
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<tr>
<td>Blood</td>
<td>PWFK</td>
<td>24.50</td>
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<tr>
<td></td>
<td>AWF</td>
<td>26.66</td>
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<td>Baboon</td>
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Figure 9. Comparison of the variograms of the original and processed Boat image
Fig. 10. Performance comparison in terms of PSNR for different images

Figure 11. Performance comparison in terms of SSIM for different images
5. Conclusions

An experimental analysis has been carried out to evaluate the role of directional semivariance in punctual kriging based image restoration. Performance comparison in terms of PSNR, SSIM and VMSE of directional SAFK-II method with PWFK, AWF, SAFK and SAFK-II techniques has been made. The results show that directional variograms does not contribute significantly in punctual kriging to estimate a noisy image. In future work, we intend to use evolutionary algorithms in addition to fuzzy sets to find an optimum/near optimum semivariance matrix A and vector b as used in equation (8) which may produce unbiased kriging estimate for noisy image data.

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