A new approach for the facility layout design in manufacturing systems

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Abstract: Facility Layout Design (FLD) problems are concerned with the arrangement of a number of facilities in a given space to satisfy an objective function; for example, minimizing total interaction. Data envelopment analysis (DEA) has been used in order to extract the necessary information for selection the optimal layout and arrangement of a number of facilities. In order to eliminate the inconsistency caused by using different frontier facets to calculate efficiency, common set of weights' DEA models have been developed, under which a group of DMUs can be ranked for a specific period.A new approach to determine the optimal distribution of process facilities based on the common set of weights DEA model is presented in this paper.
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1. Introduction

Process layout is a multidisciplinary area by nature that requires input from different specialists such as civil, mechanical, electrical, and instrument engineers. The layout problem can be defined as allocating a given number of facilities in a given land to optimize an objective function that depends on the distance measure between facilities, subject to a variety of constraints of distances. Thus, the objective of the process layout is the most economical spatial allocation of process units and their piping to satisfy their required interconnections. Starting with the full plant flow diagrams, this activity has been associated with the process design stage: the process design should not be declared as done if the plant layout has not been covered. Furthermore, facility layout problems also occur if there are changes in requirements of space, people or equipment.

Previous research on the topic includes, among others, Wilhelm et al. (1987), Raoot and Rakshit (1993), Arapoglu et al. (2001), Castillo and Westerlund (2005), Konak et al. (2006), Meller (2004), Meller et al. (1998, 2004, 2007 and 2009) and Norman et al. (2001a, 2001b).

DEA is a linear programming technique for assessing the efficiency and productivity of DMUs. Over the last decade, DEA has gained considerable attention as a managerial tool for measuring performance. It has been used widely for assessing efficiency, in the public and private sectors, of organizations such as banks, airlines, hospitals, universities and manufacturers (Charnes et al., 1978). As a result, new applications with more variables and more complicated models are being introduced (Emrouznejad et al., 2007, Firuzi and Jahanshahloo , 2012).

The DEA methodology has been used to solve the facility layout design (FLD) problem by simultaneously considering both the quantitative and qualitative data leading to the determination of robust layout design alternatives. Yang and Kuo (2003) introduced a hierarchical DEA methodology for the FLD problem. Basically, each DMU is allowed to select the most favorable weights, or multipliers, for calculating efficiency. Since DMUs treat an input/output factor with varying degrees of importance, the method only distinguishes efficient and inefficient DMUs, and is unsuitable for ranking DMUs (Doyle and Green, 1994).

The evaluation of the FLD by developing a robust layout framework based on the DEA/AHP methodology was addressed by Ertay et al. (2006). They followed the multiple criteria approach introduced to DEA by Li and Reeves (1999). The minimax proposed method by Ertay et al. (2006) was applied to a real data set consisting of the 19 facility layout alternatives. The objective function of their model contains a parameter needs to be selected on a trial-and-error method in a way to reach a single relative efficient DMU.

Many methods have been devised to rank DMUs under the framework of DEA. Most of them are based on different benchmarks, so results are not comparable with all DMUs. In order to eliminate the inconsistency caused by using different frontier facets to calculate efficiency, common set of weights' DEA models have been developed, under which a group of DMUs can be ranked with a common basis. In this paper, we use Zohrehbandian et al. (2010) approach for obtaining Common Set of Weight (CSW) and solving one FLD problem.

2. Methodology

In this section, we'll first describe DEA and Zohrehbandian et al's approach in a nutshell, and then we'll propose our approach.

2.1. DEA

DEA has become a standard approach to efficiency measurement. It is a family of methods for the evaluation of relative efficiency of decision making units (DMUs). The classical DEA models developed by Charnes et al. (1978) and Banker et al. (1984) classify each DMU to be evaluated as "efficient" or "inefficient". Moreover, for each inefficient DMU, a measure is computed, which indicates the proportional input reduction which is necessary, ceteris paribus, to change its classification from inefficient to (radially) efficient.

Let each DMU_k in the set of n DMUs be characterized by its input output data collected in the row vector (x_k, y_k), where we suppose all entries to be nonnegative and at least one input and one output to be positive. Each unit has m inputs and s outputs. Let (x_k, y_k), denote the matrix of input output data, where each row represents one DMU, and each column represents one input or output. We assume that each column contains at least one positive element. Following Charnes et al. (1978) we assume that the production possibility set (PPS) exhibits constant returns to scale and is thus given by $T = \{(x,y)|\lambda X \le x, \lambda Y \ge y, \lambda \ge 0\}$, Recall that DMU_k or, synonymously, the input output vector (x_k, y_k) is called (radially) input-efficient in T if it is impossible to reduce inputs proportionally without reducing any output, i. e. if there is no $\theta < 1$ such that $(\theta x_k, y_k) \in T$. The CCR efficiency measure E_k of DMU_k is defined as the optimal value of the linear program:

$$\begin{aligned} & E_k = \min \theta \\ & \text{s.t.} \lambda X \le \theta x_k \\ & \lambda Y \ge y_k \\ & \lambda \ge 0, \theta \textit{ is free in design} \end{aligned} \quad (1)$$

Where θ is a scalar.

The dual multiplier form of the linear program (1) is expressed as:

$$\begin{aligned} & E_k = \max u y_k \\ & \text{s.t.}\quad uY - vX \le 0 \\ & \qquad v x_k = 1 \\ & \qquad u, v \ge 0 \end{aligned} \quad (2)$$

Where v and u are vectors and E_k is scalar. The equivalent CCR fractional program is obtained from the dual program as:

$$\begin{aligned} & E_k = \max \frac{u^t y_k}{v^t x_k} \\ & s.t.\ \ \frac{u^t y}{v^t x} \le 1 \\ & u, v \ge 0 \end{aligned} \quad (3)$$

Models (1), (2) and (3) are often referred to as the envelopment form, multiplier form and ratio form of the CCR model in input oriented form, respectively.

2.2. Zohrehbandian et al's approach for finding the CSW

Here, we discuss Zohrehbandian et al's approach (2010) briefly:

They proposed to compute $\theta_j^*, j = 1,2,\dots,n$ from the model (1), when DMU_j is under consideration and then let $(\hat{x}_j, \hat{y}_j) = (\theta_j^* x_j, y_j)$, which $(\theta_j^* x_j, y_j)$ called the projection of DMU_j on the efficient frontier, is an efficient (virtual) DMU.

In the Zohrehbandian et al's approach has been shown that following model and model (3) have the same answer.

$$\begin{aligned} & \max u^t y_k - v^t(\theta_k^* x_k) \\ & s.t.\ \ u^t y_j - v^t(\theta_j^* x_j) \le 0,\ j = 1,2,\dots,n \\ & \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1 \\ & u, v \ge 0 \end{aligned} \quad (4)$$

Therefore, they used the following multi-objective linear programming (MOLP) problem for computation of CSW:

$$\begin{aligned} & \min\ \left[\sum_{j=1}^{n}\left(0 - (u^t y_j - v^t(\theta_j^* x_j))\right)^p\right]^{\frac{1}{p}} \\ & s.t.\ \ u^t y_j - v^t(\theta_j^* x_j) \le 0,\ j = 1,2,\dots,n \\ & \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1 \\ & v = (v_1, v_2, \dots, v_m)^T \ge 0 \\ & u = (u_1, u_2, \dots, u_s)^T \ge 0 \end{aligned} \quad (5)$$

Notice that for $p = 1, \infty$, this model is linear. Solving the model (5) give us CSW and then efficiency score of DMU_j, $j = 1,2,\dots,n$ can be obtained by using this common set of weights as:

$$E_j = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \quad (6)$$

If for (u^*, v^*) we have $E_p = \frac{\sum_{r=1}^{s} u_r^* y_{rp}}{\sum_{i=1}^{m} v_i^* x_{ip}} = 1$, then DMU_p is called efficient.

2.3. The proposed approach

In this paper, some basic principles from DEA are used in order to extract the necessary information for selection the optimal layout and arrangement of a number of facilities. In the design process of the layout, many objectives must be considered. The criteria that are to be minimized are viewed as inputs whereas the criteria to be maximized are considered as outputs and if only benefit attributes are considered, the case for pure benefit analysis and comparison, an input value of 1 can be assumed for every alternative. So we assume there are n DMUs that each DMU consumes varying amount of m different inputs to produce s different outputs. In the standard DEA models DMUs are not evaluated by common performance attribute weights, which may not lead to desirable consequences, since company management will typically wish to evaluate all units on a common set of weights. We use common set of weights' concept in DEA for obtaining criteria weights and choosing the best layout. At first, we form PPS by constant return to scale with these DMUs, and then we will obtain the CSW of these DMUs, and then we consider these set of weights as common performance attribute weights. We compute the efficiency score of each DMU, and we will consider the efficient DMU as the best layout.

3. An Illustrate Example

The Table 1 which was provided by Ertay et al. (2006) contain a real data of 19 DMUs (FLDs) that consume two inputs, cost and adjacency score, to produce four outputs, shape ratio, flexibility, quality and hand-carry utility.

DEA Inputs
I_1: Cost ($)
I_2: Adjacency score
DEA Outputs
O_1: Shape ratio
O_2: Flexibility
O_3: Quality
O_4: Hand-carry utility

Table 1. Inputs and outputs of FLDs

DMU	DEA Inputs		DEA Outputs			
NO.	I_1	I_2	O_1	O_2	O_3	O_4
1	20309.56	6405.00	0.4697	0.0113	0.0410	30.89
2	20411.22	5393.00	0.4380	0.0337	0.0484	31.34
3	20280.28	5294.00	0.4392	0.0308	0.0653	30.26
4	20053.20	4450.00	0.3776	0.0245	0.0638	28.03
5	19998.75	4370.00	0.3526	0.0856	0.0484	25.43
6	20193.68	4393.00	0.3674	0.0717	0.0361	29.11
7	19779.73	2862.00	0.2854	0.0245	0.0846	25.29
8	19831.00	5473.00	0.4398	0.0113	0.0125	24.80
9	19608.43	5161.00	0.2868	0.0674	0.0724	24.45
10	20038.10	6078.00	0.6624	0.0856	0.0653	26.45
11	20330.68	4516.00	0.3437	0.0856	0.0638	29.46
12	20155.09	3702.00	0.3526	0.0856	0.0846	28.07
13	19641.86	5726.00	0.2690	0.0337	0.0361	24.58
14	20575.67	4639.00	0.3441	0.0856	0.0638	32.20
15	20687.50	5646.00	0.4326	0.0337	0.0452	33.21
16	20779.75	5507.00	0.3312	0.0856	0.0653	33.60
17	19853.38	3912.00	0.2847	0.0245	0.0638	31.29
18	19853.38	5974.00	0.4398	0.0337	0.0179	25.12
19	20355.00	17402.00	0.4421	0.0856	0.0217	30.02

Using DEA-Solver, the basic CCR model was solved for Table 1 and results have been presented in Table 2.

Solving 19 LP models, there are nine CCR-efficient DMUs.

Table 2. Efficiency scores of DMUs

Rank	DMU	Efficiency score
1	5	1
1	7	1
1	10	1
1	12	1
1	14	1
1	15	1
1	16	1
1	17	1
1	19	1
10	11	0.998
11	2	0.988
12	3	0.997
13	1	0.985
14	6	0.973
15	4	0.949
16	9	0.889
17	8	0.857
18	18	0.852
19	13	0.776

Hence, we obtain CSW for these 19 units that it is same as common performance attribute weights, and so we evaluate all units with these common performances attribute weights. The weights generated by the proposed method for CSW is $\mathrm{CSW} = (v^*, u^*) = (v_1^*, v_2^*, u_1^*, u_2^*, u_3^*, u_4^*) = (0.0001, 0.0001, 0.8459, 0.0856, 0.0001, 0.0682)$.

Now we can evaluate all units with these attribute weights. Solving (6) for each layout gives the following efficiency scores and ranking of layouts shown in Table 3.

We have $E_{15} = \frac{\sum_{r=1}^{s} u_r^* y_{r15}}{\sum_{i=1}^{m} v_i^* x_{i15}} = 1$, so $\sum_{r=1}^{s} u_r^* y_{r15} - \sum_{i=1}^{m} v_i^* x_{i15} = 0.$ Notice that ${u^*}^t y - {v^*}^t x = 0$ is one supporting hyperplan that passing through the most efficient DMU (DMU_{15}) in which (v^*, u^*) is gradient vector for this hyperplan. We obtained the efficiency scores of other units relative to this single-facet.

In the Table 3, the results of our approach and in Table 4, the results of Ertay et al. (2006) approach have been brought. The Spearman Rank Correlation Coefficient is its analogue when the data is in terms of ranks. One can therefore, also call it correlation coefficient between the ranks.

Table 3. Efficiency scores of DMUs by our approach

Rank	DMU (layout)	Efficiency score by our approach
1	15	1
2	14	0.9893
3	17	0.9889
4	16	0.9811
5	2	0.9730
6	3	0.9533
7	1	0.9377
8	6	0.9364
9	12	0.9305
10	11	0.9286
11	4	0.9114
12	10	0.9081
13	7	0.8693
14	5	0.8371
15	8	0.8158
16	18	0.8085
17	9	0.7735
18	13	0.7517
1 9	19	0.6432

Table 4
Efficiency scores of DMUs by Ertay et al. approach

Rank	DMU (layout)	Efficiency score by our approach
1	16	1
2	15	0.994
3	14	0.970
4	2	0.952
5	1	0.932
6	3	0.926
7	17	0.924
8	11	0.900
9	6	0.897
10	4	0.872
11	12	0.868
12	10	0.811
13	19	0.806
14	5	0.794
15	7	0.793
16	18	0.785
17	13	0.776
18	8	0.776
19	9	0.775

The correlation coefficient is sometimes denoted by r_s. The numerical value of the correlation coefficient, r_s . ranges between -1 and +1. The correlation coefficient is the number indicating the how the scores are relating.

Correlation coefficient= $r_s = 1 - \frac{6\sum_{i=1}^{k} d_i^2}{n(n^2-1)}$

With d_i being the differences of the rank numbers and n being the number of rows of data. The equation is valid when n is greater than 4.

Correlation

1 = Perfect positive correlation

$0.7 \leq r_s < 1$= Strong positive correlation

$0.4 \leq r_s < 0.7$= Fairly positive correlation

$0 < r_s < 0.4$= Weak positive correlation

0 = No correlation

$-0.4 < r_s < 0$= Weak negative correlation

$-0.7 < r_s \leq -0.4$ = Fairly negative correlation

$-0.7 \leq r_s < -1$ = Strong negative correlation

-1 = Perfect negative correlation

There are data sets, which could be tested for correlation using Spearman's test. For this task, look at the Table 5, which shows the ranks of scores by approach of Ertay et al. (2006) and ranks of scores by our approach of nineteen layouts.

Table 5
Computational procedures for calculating r_s

DMU NO.	Ranks of scores by our approach	Ranks of scores by Ertay et al's approach	d	d^2
1	7	5	2	4
2	5	4	1	1
3	6	6	0	0
4	11	10	1	1
5	14	14	0	0
6	8	9	1	1
7	13	15	2	4
8	15	17	2	4
9	17	19	2	4
10	12	12	0	0
11	10	8	2	4
12	9	11	2	4
12	9	11	2	4
13	18	18	0	0
14	2	3	1	1
15	1	2	1	1
16	4	1	3	9
17	3	7	4	16
18	16	16	0	0
19	19	13	6	6

So we have:

$$r_s = 1 - \frac{6(92.5)}{19(360)} = 1 - \frac{555}{6840} = 0.9189$$

The answer of 0.9189 shows that there is a strong correlation between the two sets of data. So the DMUs were roughly in the same order for both approaches.

In Ertay et al. (2006) there is one efficient DMU and the range of the other scores is [0.775, 1). We have only one efficient DMU, and the range of scores for the other layouts is [0.6432, 1). So, our approach has the most widespread range of efficiency scores, and also its discriminating power is very excellent.

4. Conclusion

Facility layout design problem based on common performance attribute weights is discussed in this paper where this is one of the merits of this paper. The present paper proposes a robust practical common set of weight methodology for the evaluation of layouts based on the multiple inputs and multiple outputs. This approach improves discriminating power of DEA methods and also effectively yields more reasonable attribute weightswithout a priori information about the weights.Finally, to illustrate the model capability it is applied to 19 layouts borrowed from Ertay et al. (2006).

References

1. Arapoglu, R.A., Norman, B.A., Smith, A.E., 2001. Locating input and output points in facilities design-a comparison of constructive, evolutionary, and exact methods. IEEE Transactions on Evolutionary Computation, 5(3): 192–203.
2. Banker, R.D., Charnes, A., Cooper, W.W., 1984. Some models for estimating technical and scale inefficiency in data envelopment analysis. Management Science, 30: 1078–1092.
3. Castillo, I., Westerlund, T., 2005. An -accurate model for optimal unequal-are a block layout design. Computers and Operations Research, 32(3): 429–47.
4. Charnes, A., Cooper, W.W., Rhodes, E., 1978. Measuring the efficiency of decision making units, European Journal of Operational Research, 2: 429-444.
5. Doyle, J., Green, R., 1994. Efficiency and cross efficiency in DEA: derivations, meanings and uses. Journal of the Operational Research Society, 45(5): 567–578.
6. Emrouznejad, A., Tavares, G., Parker, B., 2007. A bibliography of data envelopment analysis (1978–2003). Socio-Economic Planning Sciences. in press.
7. Ertay, T., Ruan, D., Tuzkaya, U.R., 2006. Integrating data envelopment analysis and analytic hierarchy for the facility layout design in manufacturing systems. Information Sciences, 176: 237–262.
8. Firuzi, P., Jahanshahloo, G.R., 2012. Meta Malmquist Index Based On Trade Offs Models in Data Envelopment Analysis, Life science journal, 9(4): 2875-2883.
9. Konak, A., Kulturel-Konak, S., Norman, B.A., Smith, A.E., 2006. A new mixed integer programming formulation for facility layout design using flexible bays. Operations Research Letters, 34(6): 660–72.
10. Li, X.B., Reeves, G.R., 1999. A multiple criteria approach to data envelopment analysis. European Journal of Operational Research, 115: 507–517.
11. Meller, R.D., 2004. Facility layout survey report. Presentation made to Industrial Advisory Board, Center for High-Performance Manufacturing (Virginia Tech).
12. Meller, R.D., Narayanan, V., Vance, P.H., 1998. Optimal facility layout design, Operations Research Letters, 23(3–5): 117–27.
13. Meller, R.D., Kleiner, B.M., Nussbaum, M.A., 2004. The facility layout problem: a new model to support a bottom-up approach to facility design. In: Progress in material handling research: 2004, Charlotte, NC. Material Handling Institute, 399–414.
14. Meller, R.D., Chen, W., Sherali, H.D., 2007. Applying the sequence-pair representation to optimal facility layout designs, Operations Research Letters, 35(5): 651–9.
15. Meller, R.D., Kirkizoglu, Z., Chen, W., 2009. Detailed testing results of a new model to support a bottom-up approach to facility design. In: Proceedings of the, NSF engineering research and innovation conference, to appear.
16. Norman, B.A., Arapoglu, R.A., Smith, A.E. 2001. Integrated facilities design using a contour distance metric. IIE Transactions on Design & Manufacturing, 33(4): 337-44.
17. Norman, B.A., Smith, A.E., Yildirim, E., Tharmmaphornphilas, W., 2001b. An evolutionary approach to incorporating intradepartmental flow into facilities design, Advances in Engineering Software, 32(6): 443–53.
18. Raoot, A.D., Rakshit, A., 1993. A_linguistic pattern_ approach for multiple criteria facility layout problems, International Journal of Production Research, 31: 203–222.
19. Wilhelm, M.R., Karwowski, W., Evans, G.W., 1987. A fuzzy set approach to layout analysis, International Journal of Production Research, 25: 1431–1450.
20. Yang, T., Kuo, C.A., 2003. A hierarchical AHP/DEA methodology for the facilities layout design problem. European Journal of Operational Research, 147: 128–136.
21. Zohrehbandian, M., Makui, A., Alinezhad, A., 2010. A compromise solution approach for finding common weights in DEA: An improvement to Kao and Hung's approach. Journal of the Operations Research Society, 61: 604-610.

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