# Finite Groups With At Most Nine Non T-Subgroups

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**Abstract.** In this short note, we extend the characterization of soluble groups by using the number of their non-T-subgroups and also we classify finite groups having exactly nine non-T-subgroups.
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## 1. Introduction

Group is an algebraic structure provides information about operations and relations satisfying certain algebraic conditions. Group and its algebraic properties provides many application in various fields like chemisty [8], Physics [9] and Computer sciences [10-23]. In this paper we concentrate on A group $G$ is T- group when its every subnormal subgroup is normal. Thus T-groups are exactly those groups in which normality is a transitive relation. This structure has been studied by many authors such as Gaschutz [5], Zacher [4] and Robinson [6]. Robinson characterized finite soluble groups by T-groups. He proved that if all subgroups of a finite group are T-groups then $G$ is soluble. For details we refer to Robinson [2].

Arif et all characterized finite soluble groups by number of non-T-subgroups. In [6] this has been proven that if a finite group $G$ has at most 4 non-T-subgroups then the group will be soluble. It has also been shown there that finite groups having exactly five non-T-subgroups may not be soluble.

The purpose of this present note is to extend the characterization of finite groups up to 9 non-T-subgroups of group $G$. Thus we prove if $G$ is a finite group all whose proper subgroups are T-groups except for $n = 6,7,8,$ then $G$ is soluble see Theorem 1. We also prove that if $G$ is a finite group such that $G$ contains exactly 9 non-T-subgroups then either $G$ is soluble or $G$ is isomorphic to $SL(2,8)$ see Theorem 2.

Since our characterization is based on the number of non- T-subgroups so in Table 1 we present the details of the number of non-T-subgroups of each non-soluble group up to order 900 for the sake of completeness and for future use. We have used the following two GAP [3] functions to construct this table. In Table 1, $O$ denotes the order of non-soluble groups, $N_1$ denotes the number of non-soluble groups of the order given in column first and $N_2$ denotes the number of non-T-subgroups of each non-soluble subgroup of the group given in column second.

```
function( G ):= NrNonTSubgroups
local allsubgrps, i;
allsubgrps:=Subgroups( G );
for i in allsubgrps do
if IsSubnormal(G,i) then
if not IsNormal(G,i) then
return false;
break;
fi;
fi;
od;
return true;
end ;
function( G ):= NrNonTSubgroups
local allsubgrps;
allsubgrps:= Subgroups( G );
return Number(allsubgrps, t → not IsTGroup( t
));
end;
```

## 1. Preliminaries

The following results from [7] are needed.

(1). If $G$ is a finite simple group such that $G$ has exactly $n$ non-T-subgroups $(n \geq 1)$, then $G$ is isomorphic to a subgroup of $S_n$ [7, Theorem 2.2].

(2). Let $G$ be a finite group with exactly $n$ non-T-subgroups and suppose that any finite group with exactly $m$ non-T-subgroups is soluble for $1 \leq m \leq n-1$ [7, Theorem 2.3].

Table 1. Number of non-T-subgroups of each non-soluble subgroup.

| $O$ | $N_1$ | $N_2$ |
|---|---|---|
| 60 | 01 | 05 |
| 120 | 03 | 05,25,10 |
| 168 | 01 | 49 |
| 180 | 01 | 30 |
| 240 | 08 | 25,50,30,15,30,10,100,25 |
| 300 | 01 | 16 |
| 336 | 03 | 49,91,175 |
| 360 | 06 | 30,115,90,160,70,70 |
| 420 | 01 | 25 |
| 480 | 26 | 35,75,55,20,55,35,175,371,126,220,141,246,100, 225,371,115,201,40,145,271,190,30,259, 25,515,80 |
| 504 | 02 | 09,154 |
| 540 | 02 | 55,160 |
| 600 | 05 | 16,62,142,58,38 |
| 660 | 02 | 55,10 |
| 672 | 08 | 217,161,91,301,231,175,385,833 |
| 720 | 23 | 115,90,150,130,200,160,270,110,90,380,110,110, 130,70,455, 215,190,405,340,333,600,210,210 |
| 780 | 01 | 10 |
| 840 | 06 | 10,98,50,170,50,20 |
| 900 | 01 | 72 |

## 2. Main results

The following guarantees the solubility of a finite group having exactly $n$ non-T-subgroups, $6 \le n \le 8$.

Theorem 1. *If $G$ is a finite group all of whose proper subgroups are $T$ -groups except for $n = 6,7,8$ , then $G$ is soluble.*

Proof. Let $G$ be an insoluble group containing exactly $n$ proper non T-subgroups, $L = \{H_1, H_2, \cdots, H_n\}$. Clearly each $H_i$ contains fewer than $n$ proper non T-subgroups and so is soluble by the above result 2, no $H_i$ can be normal. By the result 1, there is a homomorphism $f : G \to S_n$. Let $K$ be the kernel of $f$. Then $K$ cannot contain any of the $H_i$ (since $K \le \cap H_i$) and so $K$ is soluble and in particular is a T-group. Then we must have $G/K$ isomorphic to an insoluble subgroup of $S_n$. But $S_6$, $S_7$, $S_8$ has no non soluble subgroups which have $6,7,8$ non T-subgroups respectively, a contradiction. Hence $G$ is soluble for $n = 6,7,8$.

Theorem 2. *If $G$ is a finite group such that $G$ contains exactly 9 non T-subgroups. Then either $G$ is soluble or $G$ is isomorphic to $SL(2,8)$.*

Proof. Suppose that there is an insoluble group with at most 9 non-T-subgroups and let $G$ be a minimal counterexample. Since the property is preserved under taking subgroups and quotient groups, $G$ must be simple. So, by result 1, $G$ is isomorphic to a non soluble subgroup of $S_9$ and since $SL(2,8)$ is the only non-soluble subgroup of $S_9$ which have 9 non-T-subgroups, therefore $G$ is isomorphic to $SL(2,8)$.

## References


[1]. G. Karpilovsky, Group Representations Vol 2, North-Holland Mathematical Studies, 1993, Elsevier, Amsterdam, London, New York.

[2]. D. J. S. Robinson, A course in the theory of groups, Graduate text in Mathematics 80, Springer, New York 1982.

[3]. The GAP, http://www.gap-system.org. GAP, Groups Algorithms and Programming, Version 4.4.12, 2008.

[4]. G. Zacher, Caratterizzione dei t-grouppi .niti risolubili, Ricerche Math. 1 (1952) 287-294.

[5]. W. Gaschutz, Gruppen in denen das Normalteilersein transitiv ist, J. Reine Angew. Math. 198(1957), 87-92.

[6]. W. Gaschtitz, Groups in which normality is intransitive relation, P. Journal of Math. Vol. 313 (1969), 777-785.

[7]. M. Arif, A. Ali and S. I. Ali Shah, Finite groups with at most five non-T-subgroups, Acta Universitatis Apulensis, 25(2011), 245-248.

[8]. March, Jerry (1985), Advanced Organic Chemistry: Reactions, Mechanisms, and Structure (3rd ed.), New York: Wiley, ISBN 0-471-85472-7

[9]. Dresselhaus, MS., Applications of Group Theory to the Physics of Solids, Springer, 2002.

[10]. Khan, S. A., N. A. Zafar, et al. (2011). "Petri net modeling of railway crossing system using fuzzy brakes." International J. Phy. Sci 6(14): 3389-3397.

[11]. M, F. and S. A. Khan (2012). "Specification and Verification of Safety Properties along a Crossing Region in a Railway Network Control." Applied Mathematical Modelling, http://dx.doi.org/10.1016/j.apm.2012.10.047

[12]. Raza, M. I., Q. J. Zaib, et al. (2012). "Meticulous analysis of Semantic Data Model -An optimal approach for ERD." J. Basic. Appl. Sci. Res. 8(2): 8344-8354.

[13]. Yousaf, S., N. A. Zafar, et al. (2010). Formal analysis of departure procedure of air traffic control system, IEEE.

[14]. Zafar, N. A., S. A. Khan, et al. (2012). "Formal Modeling towards the Context Free Grammar." Life Science Journal 9(4).
[15]. Zafar, N. A., S. A. Khan, et al. (2012). "Towards the safety properties of moving block railway interlocking system." Int. J. Innovative Comput., Info & Control.
[16]. Ahmad, F. and S. A. Khan (2012). "Module-based architecture for a periodic job-shop scheduling problem." Computers & Mathematics with Applications.
[17]. Ali, G., S. A. Khan, et al. (2012). "Formal modeling towards a dynamic organization of multi-agent systems using communicating X-machine and Z-notation." Indian Journal of Science and Technology 5(7).
[18]. Gul Afzal Khan, S. A. K., Nazir Ahmad Zafar, F.A.S.I. (2012). "A Review of different Approaches of Land Cover Mapping." Life Sci J 9(4).
[19]. Khan, S. A., A. A. Hashmi, et al. (2012). "Semantic Web Specification using Z-Notation." Life Science Journal 9(4).
[20]. Khan, S. A. and N. A. Zafar (2007). "Promotion of local to global operation in train control system." Journal of Digital Information Management 5(4): 231.
[21]. Khan, S. A. and N. A. Zafar (2009). Towards the formalization of railway interlocking system using Z-notations, IEEE.
[22]. Khan, S. A. and N. A. Zafar (2011). "Improving moving block railway system using fuzzy multi-agent specification language." Int. J. Innov. Computing, Inform. Control 7(7).
[23]. Khan, S. A., N. A. Zafar, et al. (2011). "Extending promotion to operate controller based on train's operation." International J. Phy. Sci 6(31): 7262 - 7270.

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