Generalized Projective Synchronization for Four Scroll attractor

M. M. El-Dessoky[1, 2] and E. M. Elabbasy[2]

[1.] Department of Mathematics, Faculty of Science, King Abdulaziz University, P. O. Box 80203, Jeddah 21589, Saudi Arabia
[2.] Department of Mathematics, Faculty of science, Mansoura University, Mansoura, 35516, Egypt
dessokym@mans.edu.eg

Abstract: This paper investigates an active control method is proposed to generalize projective synchronize two identical chaotic dynamical systems by constructing the response system no matter whether they are identical. The proposed technique is applied to achieve generalized projective synchronization for the Four - scroll attractor, where all state variables are in a proportional way. A strategy for practical implementation of a secure communication strategy is also discussed. Finally computer simulations are done to verify the proposed methods, and the results show that the obtained theoretic results are feasible and efficient.
[El-Dessoky MM, Elabbasy EM. **Generalized Projective Synchronization for Four Scroll attractor**. *Life Sci J* 2012;9(3):2477-2483] (ISSN:1097-8135). http://www.lifesciencesite.com. 358

Keywords: Projective synchronization; Chaotic system; Active control; Four-scroll attractor, Secure communication.

1. Introduction

Researchers from different areas, such as mathematicians, physicists, chemist, as well as control engineers have devoted themselves to examine the issue of synchronization over the past decade (Pacora and Carroll, 1990; Carroll and Pacora, 1991; Kocarev et al.,1992). Chaotic systems, in particular, have been applied to the development of secure communications, chemical reactions, biological systems and so on (Kocarev et al.,1992; Yuan and Jun, 2009; Zhu, 2009; Lu et al., 2002; Hua et al., 2005; Juan et al., 2012; Rafael and Yu, 2008). The system which received the most attention among chaotic communication systems perhaps is the Chua oscillator (Kocarev et al.,1992). This system belongs to general class of Lure systems (Khalil, 1996).

Chaos control and synchronization have attracted a great deal of attention from various fields since Huber published the first paper on chaos control in 1989 (Hubler, 1989). Over the last decades, many methods and techniques have been developed, such as OGY method (Ott et al., 1990), PC method (Pacora and Carroll, 1990; Carroll and Pacora, 1991), feedback approach, nonfeedback control methods, adaptive method, nonlinear control, active control, and backstepping design technique, etc (Wang et al., 2001, Elabbasy et al., 2004; Wang and Ge, 2001; Jiang et al., 2002; Sun and Zhang 2004).

There are many applications to chaotic communication (Fallahi and Leung, 2010; Elabbasy and El-Dessoky, 2010) and chaotic network synchronization (Chow et al., 2001). The techniques of chaotic communication can be divided into three categories, (i) chaos masking (Kocarev et al., 1992), the information signal is added directly to the transmitter; (iii) chaos modulation (Boutayeb et al., 2002), it is based on the drive – response (master-slave) synchronization, where the information signal is injected into the transmitter as a nonlinear filter; (ii) chaos shift keying (Parlitz et al., 1992), the information signal is supposed to be binary, and it is mapped into the transmitter and the receiver. In these three cases, the information signal can be recovered by a receiver if the transmitter and the receiver are synchronized.

In 1963, Lorenz found the first classical chaotic attractor. In 1999, Chen found another similar but not topological equivalent chaotic attractor the Chen attractor (Chen and Ueta, 1999). In 2002, Lü and Chen found a new critical chaotic system (Lü and Chen, 2002), bearing the name of Lü system. It is noticed that these systems can be classified into three different types by the definition of Vaněček and Čelikovsky (Vaněček and Čelikovsky, 1996): the Lorenz system (Lorenz, 1963) satisfies the condition $a_{12}a_{21} > 0$, the Chen system (Chen and Ueta, 1999) satisfies $a_{12}a_{21} < 0$ and the Lü system (Lü and Chen, 2002) satisfies $a_{12}a_{21} = 0$, where a_{12} and a_{21} are the corresponding elements in the linear part matrix $A = [a_{ij}]_{3\times 3}$ of the dynamical system.

The early projective synchronization (PS) is usually observable only in a class of systems with partial-linearity (Xu et al., 2001; Xu and Chee, 2002; Xu and Li, 2002, Wen and Xu 2005), but recently some researchers (Zhigang and Daolin, 2001; El-Dessoky, 2010) have achieved control of the projective synchronization in a general class of chaotic systems including non-partially-linear systems, and termed this projective synchronization as generalized projective synchronization (GPS) (Yan and Li, 2005; Changpin and Jianping, 2006; El-Dessoky and Salah, 2011).

In this paper, we generalize active control to GPS, and demonstrate this technique by some typical chaotic systems, for example, the chaotic Lorenz system and the chaotic Chen system such that GPS is achieved. The results from numerical simulations show that the method works well.

The paper is organized as follows. In Section 2, the generalized projective synchronization with active control is applied to synchronize two identical Four-scroll attractor and numerical simulations are presented to show the effectiveness of the proposed method. In Section 3, a scheme of secure communication based on the active control of Four-scroll chaotic system is presented. Conclusions are finally given in Section 4.

2. Generalized projective synchronization (GPS) of two identical Four-scroll chaotic attractor

The projective synchronization means that the drive and response vectors synchronize up to a scaling factor a, that is, the vectors become proportional. First, we define the GPS below.
Consider the following chaotic system:

$$\dot{x} = f(x, t) \qquad (1)$$

$$\dot{y} = g(y, t) + u(x, y, t) \qquad (2)$$

where $x, y \in R^n$ are the state vector of the systems (1) and (2), respectively ; $f, g \in R^n \times R \to R^n$ are two continuous nonlinear vector functions, $u(x, y, t)$ is the vector control input. If there exists a constant α $(\alpha \neq 0)$, such that $\lim_{t\to\infty} |y - \alpha x| = 0$, then the GPS of the systems (1) and (2) is achieved, and we call α is a scaling factor.

Now, we apply the adaptive feedback control method for generalized projective synchronization of two identical Four-scroll attractor (Lü et al., 2002 & 2004; Liu and Chen 2004; Elabbasy et al., 2006; El-Dessoky, 2010) which can be described by:

$$\begin{aligned}\dot{x} &= ax - yz \\ \dot{y} &= -by + xz \qquad (3) \\ \dot{z} &= -cz + xy\end{aligned}$$

where *a, b* and *c* are positive control parameters. This system exhibits a strange attractor at the parameter values *a=0.4, b=12* and *c=5*. This system bridges the gap between the Lorenz (Lorenz, 1963) and Chen attractors (Chen and Ueta, 1999), i.e. $a_{12}\,a_{21} = 0$. The divergence of the flow (3) is given by

$$\nabla.F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = a - b - c < 0,$$

Where

$$\begin{aligned}F &= (F_1, F_2, F_3) \\ &= (a\,x - y\,z, -b\,y + x\,z, -c\,z + x\,y).\end{aligned}$$

Hence the system is dissipative when:
$a < b + c.$

Science the system of Four-scroll attractor (3) is a dissipative system thus the solutions of the system of equations (3) are bounded as $t \to \infty$ for $a < b + c.$ If $a\,b > 0, a\,c > 0$ and $b\,c > 0$ then the system has five equilibrium points:

$$E_0 = (0,0,0), E_1 = (\sqrt{bc}, \sqrt{ac}, \sqrt{ab}),$$
$$E_2 = (-\sqrt{bc}, -\sqrt{ac}, \sqrt{ab}), E_3 = (-\sqrt{bc}, \sqrt{ac}, -\sqrt{ab}),$$
$$E_4 = (-\sqrt{bc}, -\sqrt{ac}, -\sqrt{ab}).$$

Differing from other known similar systems, system (3) has five equilibrium, and does not have Hopf and pitch bifurcations [38, 39]. Of most interesting is the observation that this chaotic system not only can display a two-scroll chaotic attractor when *a=4.5, b=12* and *c=5* (Figure 1), but also can display a Four-scroll chaotic attractor when *a=0.4, b=12* and *c=5* (Figure 2).

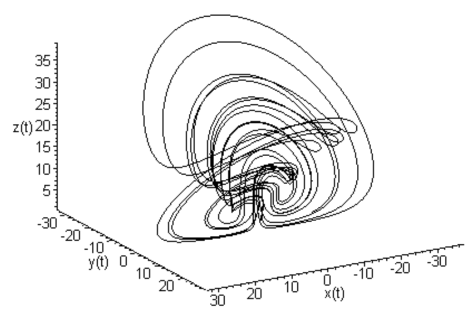

Figure 1: The chaotic attractor of two- scroll attractor at *a=4.5, b=12* and *c=5* in 3-dimensional.

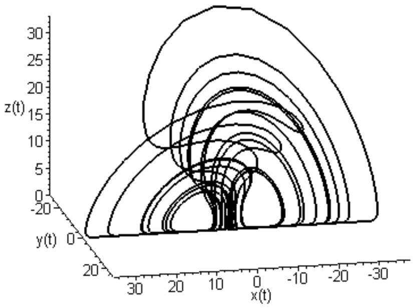

Figure 2: The chaotic attractor of Four- scroll attractor at *a=0.4, b=12* and *c=5* in 3-dimensional.

In this section we apply the generalized projective synchronization of two identical Four-scroll chaotic attractor. In order to observe the generalized projective synchronization behavior in the Four-scroll system, we have two Four-scroll systems where the drive system with three state variables denoted by the subscript 1 drives the response system having identical equations denoted by the subscript 2. However, the initial condition on the drive system is different from that of the response system. The two Four-scroll systems are described, respectively, by the following

$$\dot{x}_1 = ax_1 - y_1 z_1$$
$$\dot{y}_1 = -by_1 + x_1 z_1 \qquad (4)$$
$$\dot{z}_1 = -cz_1 + x_1 y_1$$

and

$$\dot{x}_2 = ax_2 - y_2 z_2 + u_1$$
$$\dot{y}_2 = -by_2 + x_2 z_2 + u_2 \qquad (5)$$
$$\dot{z}_2 = -cz_2 + x_2 y_2 + u_3$$

There are three control functions u_i, (*i=1, 2, 3*) to be determined later.

Define the error vector as

$e_x = x_2 - \alpha\, x_1, e_y = y_2 - \alpha\, y_1 \; and \; e_z = z_2 - \alpha\, z_1$

where α is a desired scaling factor. Then one obtains the error dynamical system by subtracting (4) from (5)

$$\frac{de_x}{dt} = ae_x - y_2 z_2 + \alpha\, y_1 z_1 + u_1$$
$$\frac{de_y}{dt} = -be_y + x_2 z_2 - \alpha\, x_1 z_1 + u_2 \qquad (6)$$
$$\frac{de_z}{dt} = -ce_z + x_2 y_2 - \alpha\, x_1 y_1 + u_3$$

Referring to the original methods of active control, so we choose the three control functions u_i, (*i=1, 2, 3*) as follows:

$$u_1 = y_2 z_2 - \alpha\, y_1 z_1 + v_1$$
$$u_2 = \alpha\, x_1 z_1 - x_2 z_2 + v_2 \qquad (7)$$
$$u_3 = \alpha\, x_1 y_1 - x_2 y_2 + v_3$$

then the error dynamical system (6) is described by

$$\frac{de_x}{dt} = ae_x + v_1$$
$$\frac{de_y}{dt} = -be_y + v_2 \qquad (8)$$
$$\frac{de_z}{dt} = -ce_z + v_3$$

The error system (8) to be controlled is a linear system with a control input $v_1,\, v_2 \; and \; v_3$ as function of the error $e_x, e_y \; and \; e_z$. As long as these feedbacks stabilize the system (8), $e_x, e_y \; and \; e_z$ converge to zero as time tends to infinity, which implies that GPS of two identical Four-scroll systems is achieved with a scaling factor α. There are many possible choices for the control $v_1,\, v_2 \; and \; v_3$. In order to make the closed loop system (8) be stable, the proper choice of the control should guarantees that the feedback system must have all eigenvalues with negative real parts. For simplify, we choose

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} -2a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \qquad (9)$$

In this particular choice, the three eigenvalues of the closed loop system (8) are *-a, -b* and *-c*. Since the closed loop system has all eigenvalues that are found to have negative real parts, the system will be convergence. In other words, this choice will result in a stable system and the GPS of two identical Four scroll systems. What deserves to be mentioned is that the values of the eigenvalues play an important role in the stability of the error system. In order to quicken the rate of convergence, we should make them get smaller.

2.1 Numerical Results

By using Maple 12, we select the parameters of the Four-scroll attractor as *a=0.4, b=12* and *c=5*. The initial values of the drive system and response system are taken as $x_1(0) = 0.23, y_1(0) = 0.1, z_1(0) = 0.32,$ $x_2(0) = 1.77, y_2(0) = 0.9 \, and \, z_2(0) = 0.44$ respectively. If we take the scaling factor $\alpha = 2$ hence the error system has the initial values $e_x(0) = 1.29, e_y(0) = 0.7$ and $e_z(0) = -0.$ then the generalized projective synchronization between two identical Four-scroll attractor are shown in Figure 3. If we take the scaling factor $\alpha = -0.2$ hence the error system has the initial values $e_x(0) = 1.818,$ $e_y(0) = 0.94$ and $e_z(0) = -0.504$ then the GPS between two identical Four-scroll attractor are shown in Figure 4. If we take the scaling factor $\alpha = 1$ hence the error system has the initial values $e_x(0) = 1.54,$ $e_y(0) = 0.8$ and $e_z(0) = 0.12$ then the complete synchronization between two identical Four-scroll attractor are shown in Figure 5. If we take the scaling factor $\alpha = -1$ hence the error system has the initial values $e_x(0) = 2,$ $e_y(0) = 1$ and $e_z(0) = 0.76$ then the anti synchronization between two identical Four-scroll attractor are shown in Figure 6.

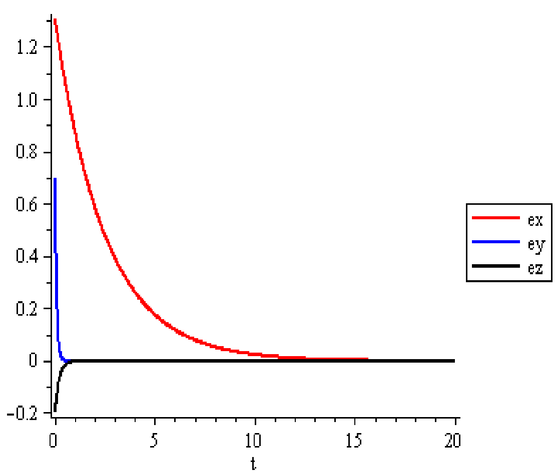

Figure 3: Shows the trajectories of $e_x, e_y \; and \; e_z$ of two identical Four- scroll attractor with scaling factor $\alpha = 2$ for generalized projective synchronization.

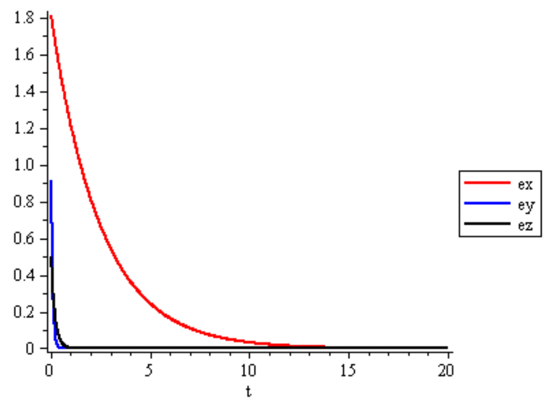

Figure 4 : The trajectories of $e_x, e_y \; and \; e_z$ of two identical Four- scroll attractor with scaling factor $\alpha = -0.2$ for generalized projective synchronization.

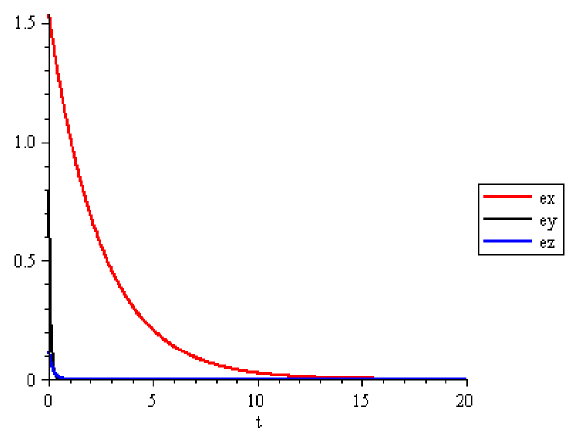

Figure 5: The trajectories of $e_x, e_y \; and \; e_z$ of two identical Four- scroll attractor with scaling factor $\alpha = 1$ for complete synchronization.

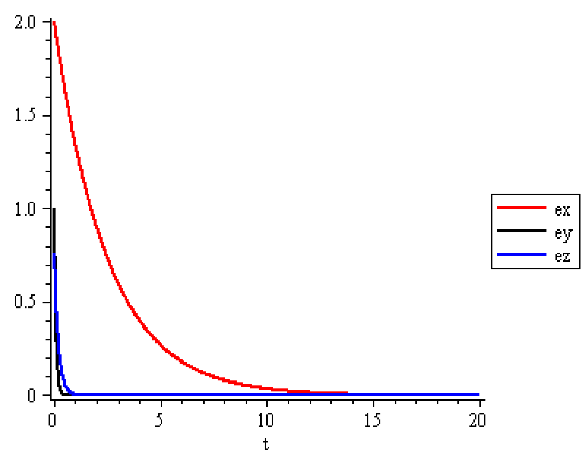

Figure 6: The trajectories of $e_x, e_y \; and \; e_z$ of two identical four- scroll attractor with scaling factor $\alpha = -1$ for anti synchronization.

3. The application in secure communication

In this section, we will apply the adaptive scheme derived above to secure communication using the Four-scroll attractor. Assume that $m(t)$ is the message signal, adding it to the right hand side of the second equation for the transmitter (drive system), then we have

$$\begin{aligned} \dot{x}_1 &= ax_1 - y_1 z_1 \\ \dot{y}_1 &= -by_1 + x_1 z_1 + m(t) \\ \dot{z}_1 &= -cz_1 + x_1 y_1 \end{aligned} \qquad (9)$$

Select the output $y(t)$ of the system (9) as the transmitted signal, then construct the receiver as follows:

$$\begin{aligned} \dot{x}_2 &= ax_2 - y_2 z_2 \\ \dot{y}_2 &= -by_2 + x_2 z_2 + p(t) + u_1 \\ \dot{z}_2 &= -cz_2 + x_2 y_2 + u_2 \\ \dot{p} &= -k(y_2 - \alpha y_1) \end{aligned} \qquad (10)$$

where k is a positive parameter.

Let

$e_1 = y_2 - \alpha y_1, e_2 = z_2 - \alpha z_1 \; and \; e_3 = p - \alpha m$.

Then the error system can be described by

$$\begin{aligned} \dot{e}_1 &= -be_1 + x_1 e_2 + e_3 + u_1 \\ \dot{e}_2 &= -ce_2 + x_1 e_1 + u_2 \\ \dot{e}_3 &= -ke_1 - \alpha \frac{dm}{dt} \end{aligned} \qquad (11)$$

Referring to the original methods of active control, so we choose the three control functions $u_i, (i = 1, 2)$ as follows:

$$u_1 = -x_1 e_2 \; and \; u_2 = -x_1 e_1 \qquad (12)$$

then the error dynamical system (11) is described by

$$\begin{aligned} \dot{e}_1 &= -be_1 + e_3 \\ \dot{e}_2 &= -ce_2 \\ \dot{e}_3 &= -ke_1 - \alpha \frac{dm}{dt} \end{aligned} \qquad (13)$$

Then we take the Lyapunov function :

$$V(e) = \frac{1}{2}(ke_1^2 + e_2^2 + e_3^2) \qquad (14)$$

It is clear that the Lyapunov function $V(e)$ is a positive definite function. Now, taking the time derivative of equation (14), then we get

$$\begin{aligned} \frac{dV(e)}{dt} &= ke_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= -ke_1^2 + ke_1e_3 - ce_2^2 - ke_1e_3 - \alpha\frac{dm}{dt}e_1 \\ &= -ke_1^2 - ce_2^2 - \alpha\frac{dm}{dt}e_1 \end{aligned}$$

Since the eigen-frequency of the message signal $m(t)$ is much less than the oscillating frequency of the chaotic system in practice $\alpha\frac{dm}{dt} \approx 0$. It is easy to have $\frac{dV(e)}{dt} < 0$. This can derive $e_3 = p - \alpha m \to 0$ as $t \to \infty$, that is $\frac{p(t)}{\alpha}$ can recover the message signal $m(t)$.

Taking $m(t) = 0.1\sin(0.1t), k = 100,$ initial values $x_1(0) = 0.23, y_1(0) = 0.1, z_1(0) = 0.32,$ $y_2(0) = 0.9, z_2(0) = 0.4 \; and \; p(0) = 6$.

Figures 7(a, b) show shows that the trajectory of e_3 of the error system with scaling factor $\alpha = 0.3$ and $\alpha = -0.4$, respectively.

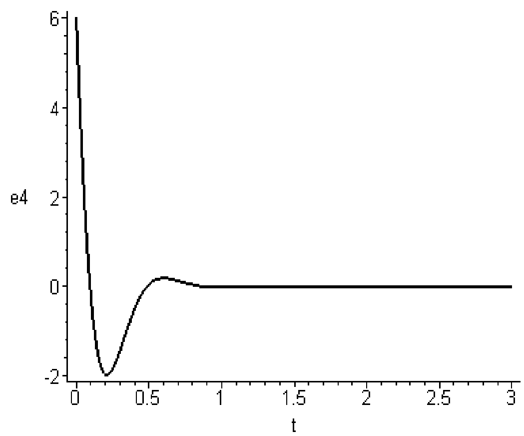

Fig. 7 (a): display the trajectory of e_3 of the error system with scaling factor $\alpha = 0.3$.

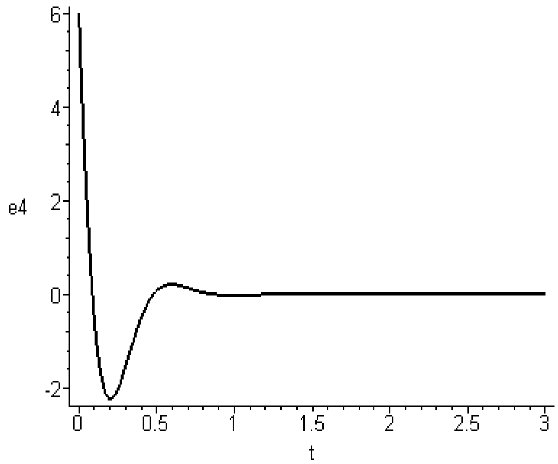

Fig. 7 (b): display the trajectory of e_3 of the error system with scaling factor $\alpha = -0.4$.

4. Conclusion

In this paper, an active control method is proposed for manipulating generalized projective synchronization in a general class of chaotic systems. This method is effective and convenient to generalized projective synchronize two identical systems and two different chaotic systems. Also we have proposed a scheme for a practical implementation of secure communication based on a active control method. Numerical simulations are also given to validate the proposed synchronization approach.

Acknowledgements:

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant no. 341/130/1432. The authors, therefore, acknowledge with thanks DSR technical and financial support. The authors would like to thank the editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

Corresponding Author:
Dr. M. M. El-Dessoky
Department of Mathematics
Faculty of Science, King Abdulaziz University,
P. O. Box 80203, Jeddah 21589, Saudi Arabia.
E-mail: dessokym@mans.edu.eg

References

1. Pecora LM, Carroll TM. Synchronization of chaotic systems. Phys. Rev. Lett., 1990, 64(8): 821-30.
2. Carroll TM, Pecora LM. Synchronizing a chaotic systems. IEEE Trans. Circuits Systems,1991. 38: 453-56.
3. Wu CW, Chua LO. A unified framework for synchronization and control of dynamical systems. Int J Bifurcat Chaos, 1994, 4(4): 979-89.
4. Kocarev L, Halle KS, Eckert K, Chua LO, Parlitz U. Experimental demonstration of secure communications via chaotic synchronization. Int J Bifurcat Chaos, 1992, 2(3): 709-13.
5. Wang Xing-Yuan, Wang Ming-Jun. A chaotic secure communication scheme based on observer. Commu. Nonlinear Sci. Numer. Simulat., 2009, 14: 1502-8.
6. Fanglai Zhu, Observer-based synchronization of uncertain chaotic system and its application to secure communications. Chaos, Solitons and Fractals,2009, 40: 2384-91.
7. Lu JA, Wu XQ, Lü JH. Synchronization of unified chaotic system and the application in secure communication. Phys. Lett. A, 2002, 305: 365-70.
8. Changchun Hua, Bo Yang, Gaoxiang Ouyang, Xinping Guan. A new chaotic secure communication scheme. Phys. Lett. A, 2005, 342: 305-8.
9. Juan L. Mata-Machuca, Rafael Martinez-Guerra, Ricardo Aguilar-Lopez, Carlos Aguilar-Ibanez, A chaotic system in synchronization and secure communications, Commun Nonlinear Sci Numer Simul, 2012, 17: 1706-13.
10. Rafael Martinez-Guerra, Yu W. Chaotic synchronization and secure communication via sliding-mode observer. Int J Bifur Chaos 2008;18:235-43.
11. Khalil H. K. Nonlinear systems. 2nd ed. Englewood Cliffs, NJ: Prentice-Hall; 1996.
12. Hubler AW. Adaptive control of chaotic system [J]. Helv Phys Acta 1989; 62: 343-6.
13. Ott E, Grebogi C, Yorke JA. Controlling chaos [J]. Phys Rev Lett 1990; 64: 1196-9.
14. Wang G, Yu X, Chen S. Chaos control, synchronization and its application. China: National Defense Industry Publishing House; 2001.
15. Elabbasy EM, Agiza HN and El-Dessoky MM. Synchronization of Modified Chen System. Int. J. Bifur. Chaos, 2004, 14(11): 1-11.
16. Wang C, Ge SS. Adaptive synchronization of uncertian chaotic system via backstepping design. Chaos, Solitons & Fractals, 2001, 12:1119-206.
17. Jiang G-P, Chen G, Tang K. S. A new criterion for chaos synchronization using linear state feedback control. Int. J. of Bifur. Chaos, 2002, 13(8): 2343-51.
18. Sun J, Zhang Y. Some simple global synchronization criterion for coupled time-

varying chaotic systems. Chaos, Solitons & Fractals, 2004, 19: 93-98.
19. Fallahi K, Leung H. A chaos secure communication scheme based on multiplication modulation. Commun Nonlin Sci Numer Simul 2010;15: 368-83.
20. Elabbasy EM, El-Dessoky MM. Adaptive feedback control for projective synchronization for Lu dynamical system and application in secure communication. Chinese Journal of Physics, 2010, 48(6): 863-72
21. Chow TW, Feng JC, Ng KT. Chaotic network synchronization with application to communications. Int J Commun Syst, 2001;14:217-30.
22. Kocarev L, Halle KS, Eckert K, Chua LO, Parlitz U. Experimental demonstration of secure communication via chaotic synchronization. Int J Bifur Chaos 1992;2:709-13.
23. Boutayeb M, Darouach M, Rafaralahy H. Generalized state observers for chaotic synchronization and secure communication. IEEE Trans Circuits Syst I 2002;49:345-9.
24. Parlitz U, Chua LO, Kocarev L, Halle KS, Shang A. Transmission of digital signals by chaotic synchronization. Int J Bifur Chaos 1992;2:973-7.
25. Chen G, Ueta T. Yet another chaotic attractor. Int. J. of Bifur. and Chaos 1999, 9: 1465-6.
26. Lü J, Chen G. A new chaotic attractor coined. Int. J. Bifur. and Chaos, 2002, 12(3): 659-61.
27. Vaněõek and Čelikovskŷ, Control Systems: From Linear Analysis to Syntheis of chaos, Prentice-Hall, London, 1996.
28. Lorenz EN. Deterministic non-periodic flows. J. Atmos. Sci. 1963, 20: 130-41.
29. Xu D, Li Z, Bishop R. Manipulating the scaling factor of projective synchronization in three-dimensional chaotic systems. Chaos, 2001, 11(3): 439-42.
30. Xu D, Chee CY. Controlling the ultimate state of projective synchronization in chaotic systems of arbitrary dimension. Phys. Rev. E, 2002, 66: 046218.
31. Xu D, Li Z. Controlled projective synchronization in nonpartially-linear chaotic systems. Int. J. Bifurcation and Chaos,2002, 12: 1395-402.
32. Wen G, Xu D. Nonlinear observer control for full-state projective synchronization in chaotic continuous-time systems. Chaos, Solitons & Fractals, 2005, 26: 717.
33. Zhigang Li, Daolin Xu. Stability criterion for projective synchronization in three-dimensional chaotic systems. Phys. Lett. A, 2001, 282: 175-9.
34. El-Dessoky MM, " Adaptive feedback control for modified projective synchronization of Liu chaotic dynamical system " International Journal of Control Theory and Applications, 2010, 3(2), 93-102.
35. Yan J, Li C. Generalized projective synchronization of a unified chaotic system. Chaos, Solitons & Fractals, 2005, 26: 1119-24.
36. Changpin Li, Jianping Yan. Generalized projective synchronization of chaos: The cascade synchronization approach. Chaos, Solitons and Fractals, 2006, 30: 140-46.
37. El-Dessoky, MM, Salah, E. Generalized Projective Synchronization for different some Hyperchaotic Dynamical Systems. Discrete Dynamics in Nature and Society, 2011, Vol. 2011, Article ID 437156, 19 pages, doi:10.1155/2011/437156.
38. Lü J, Chen G, Chen DZ, Čelikovský S. Bridge the gap between the Lorenz system and the Chen system. Int. J. Bifur. and Chaos, 2002, 12: 2917-26.
39. Lü J, Chen G, Chen DZ. A new chaotic system and beyond: The generalized Lorenz-like system. Int. J. of Bifur. and Chaos, 2004, 14(5): 1507-37.
40. Liu W, Chen G. Can a Three-Dimensional Smooth Autonomous Quadratic Chaotic System Generate a Single Four-Scroll Attractor?. Int. J. Bifur. and Chaos, 2004, 14(4): 1395-403.
41. Elabbasy EM, Agiza HN and El-Dessoky MM. Adaptive synchronization for Four-scroll attractor with fully unknown parameters. Phys. Lett. A, 2006, 349, 187-91.
42. El-Dessoky MM, " Anti-synchronization of Four scroll attractor with fully unknown parameters", Nonlinear Analysis Series B: Real World Applications 2010, 11, 778-83.

9/6/2012