Reliability of a Series Chain for Time Dependent Stress – Strength Models of Weibull Distribution

A.I. Shawky[1] F. H. Al-Gashgari[2]

[1]Department of Statistics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia.
[2]Department of Statistics, Faculty of Science for Girls, King Abdulaziz University, P.O. Box 53873, Jeddah 21593, Saudi Arabia.
aishawky@yahoo.com

Abstract: In this work, we study consider the problem of determining the reliability of a series chain consisting of k identical links. The stress acting on the chain is deterministic. We consider the case of repeated applications of stresses, i.e., cycles of stresses. We also consider the change of the distribution of strengths of the links with time, i.e., during different cycles of stresses. We find an expression of the reliability function after m cycles of stresses. The strengths of the links of the chain could be random- independent, random- fixed or deterministic. We introduce a two-sided confidence interval for the reliability. As an application, the case of weibull distribution is studied. Finally the system is applied to simulated data and real data for numerical illustration.
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1. Introduction:

In stress – strength models a component fails at any time the applied stress X is greater than its strength Y and there is no failure if Y > X. Thus P(Y > X) is a measure of the reliability of the component.

The problem of estimating R= P(Y > X) has been studied in the literature in both distribution free and parametric frameworks. However in this paper we are concerned with the parametric case.

Church and Harris [1] derived the maximum likelihood estimator (MLE) of R assuming X and Y are independent normal and that the distribution of X is completely known. Downton [2] obtained the MVUE of R in the case of independent normal where the parameters of X and Y are unknown. Reiser and Guttman [3] presented two approximate methods for obtaining confidence intervals and an approximate Bayesian probability interval. Owen *et al.* [4] discussed the normal case for equal standard deviation and presented non parametric confidence limits for this problem, in addition to the normal case. The problem considered here has been extensively studied for many other models including exponential [5], Gamma [6] and Burr distributions [7]. Nassar *et al.* [8] obtained confidence intervals for R= P(Y > X), where Y and X follow Rayleigh and normal distribution distributions respectively.

If the stress and strength change with time, we call it time dependent stress- strength model. Kapur and Lamberson [9] stated that time dependent stress-strength (SST) are models that consider the repeated application of stresses and also, consider the change of the distribution of strength with time, which may caused by aging and/or cumulative damage. Such models are frequently observed in practice. Shaw *et al.* [10] discussed a time dependent stress-strength models for non-electrical and electrical systems. Furthermore, Schartz *et al.* [11] studied an application of time dependent stress-strength models of non-electrical and electrical systems. Xue and Yang [12] obtained formula for estimating upper and lower bounds for stress – strength interference reliability when X and Y are s-independent normally distributed. However, not too much work is done on time dependent models. Mokhles and Khayar [13] studied the time dependent stress- strength model with Rayleigh and exponential distributions.

In this paper, we obtained an explicit expression for the reliability function of a series consisting of k links after m cycles of stress. The repeated stress, in our case, is deterministic. To derive the reliability under three strength forms of the links of the chain: random- independent, random- fixed and deterministic. As an application, Weibull distribution is considered. we find a two- sided confidence intervals for the reliability in case of random- independent and random fixed strength. We apply our results in both simulation study and real data.

2. Assumption and Notation.

1- The system is a series chain consisting of k links.
2- The links are identical and independent.
3- The chain is subjected to cycles of common repeated stresses. These stresses are the main cause to break the chain and are independent of the strength of the links of the chain.
4- The chain will break (fail) if the stress on the chain exceeds the strength of the chain for the first time.

5- The repeated stress acting on the chain is deterministic, i.e., the stress during cycle j is given by x_0, for all j, $j = 1, 2, \dots, m$, where x_0 is known value.
6- Y_{ij} is the strength of link i during cycle j, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, m$.
7- $E_{k,j}$ event that no failure occurs on jth cycle.
8- $R_{k,m}$ is the reliability of the chain of k links after m cycles.

3. The System Reliability

Mokhles and Khayar [13] and Kapur and Lamberson [9] discussed the reliability of the system assuming three different models .

Model I: Random-Independent Strength

In this model, the strength of i^{th} link during the j^{th} cycle is a random variable , which denoted by Y_{ij}, $i = 1,2,\dots,k$ and $j = 1,2,\dots,m$, are non-identical independent distributed random variables having c.d.f $G_j(y)$ and p.d.f $g_j(y)$. Since the chain consists of k links connected in series, the strength of the chain on the j^{th} cycle is given by

$$Y_j^* = min(Y_{1j}, Y_{2j}, \dots, Y_{kj}),$$

with c.d.f $G_{Y_j^*}(y) = [1 - G_j(y)]^k$.

The system reliability is

$$R_{k,m} = \prod_{j=1}^{m} \Pr(E_{k,j}) = \prod_{j=1}^{m} \Pr(Y_j^* > x_0) = \prod_{j=1}^{m} [1 - G_j(x_0)]^k. \quad (1)$$

Model II: Random-Fixed strength

In this model, the random variable of strength varies in time (during cycle j) in a known manner, i.e., the strength of i^{th} link on the j^{th} cycle Y_{ij} is given by

$$Y_{ij} = Y_{i0} - a_j, \quad i = 1, 2, \dots, k; \quad j = 1, 2, \dots, m, \quad (2)$$

where Y_{i0} is the initial random strength of the i^{th} link, and a_j is a known non-decreasing function in j .

Assuming that Y_{i0} are i.i.d, i= 1,2,…,k having c.d.f. $G_0(y)$ and p.d.f. $g_0(y)$, then the strength of the chain during the j^{th} cycle is

$$Y_j^* = Y_0^* - a_j, \quad (3)$$

where,

$$Y_0^* = min(Y_{10}, Y_{20}, \dots, Y_{k0}),$$

having c.d.f. $G_{Y_0^*}(y) = [1 - G_0(y)]^k$.

Thus, the system reliability is

$$R_{k,m} = P_r(E_{k,m}) = P_r(Y_m^* > x_0) = [1 - G_0(x_0 + a_m)]^k . \quad (4)$$

Model III: Deterministic Strength

In this model, the strength of i^{th} link on the j^{th} cycle is deterministic given by y_{ij} , $1 \le i \le k$; $1 \le j \le m$. Since the chain consists of k links connected in series, the strength of the chain on the j^{th} cycle will be

$$Y_j^* = min(y_{1j}, y_{2j}, \dots, y_{kj}).$$

Since,

$$R_{k,m} = P_r(E_{k,1}, E_{k,2}, \dots, E_{k,m}) ,$$

where, $E_{k,j}$ is the event that $(y_j^* > x_0)$, we get

$$R_{k,m} \begin{cases} 1 & if\ y_j^* > x_0 \quad for\ all\ j,\ 1 \le j \le m \\ 0 & if\ y_j^* < x_0 \quad for\ some\ j,\ 1 \le j \le m. \end{cases}$$

Remarks.

1. Taking k= 1, we obtain the reliability of an item after m cycles of stress.
2. Taking $a_m = 0$, we obtain the reliability of the system in the static case.

4. Confidence Intervals for System Reliability.

We obtain confidence intervals (C.I.) for system reliability under Models I and II in considering the Weibull distribution.

For Weibull distribution, the distribution function is

$$G(y) = 1 - e^{-\frac{y^p}{\theta}}, \quad y > 0,\ p,\ \theta > 0 \quad (5)$$

Assume that

$$G_j(y) = 1 - exp\left(-\frac{y^p}{\theta_j}\right), \text{ for Model I,}$$

$$G_0(y) = 1 - exp\left(-\frac{y^p}{\theta_0}\right), \text{ for Model II.}$$

Using Equations (1) and (4), we obtain

$$R_{k,m} = \begin{cases} e^{-k\,\eta\, x_0^p}, & \text{for Model I} \\ e^{-k\,\eta_0\,(x_0+a_m)^p}, & \text{for Model II,} \end{cases} \quad (6)$$

where $\eta = \sum_{j=1}^{m} \frac{1}{\theta_j}$ and $\eta_0 = \frac{1}{\theta_0}$.

If the parameters θ_0 and θ_j, j=1, 2,…, m are known, then by equation (6), we obtain the exact reliability .

If the parameters θ_0 and θ_j, j=1, 2,…, m are unknown, we can then replace these parameters by their MLEs, to get MLE, $\hat{R}_{k,m}$ of $R_{k,m}$ for the two models as follows

$$\hat{R}_{k,m} = \begin{cases} e^{-k\,\hat{\eta}\, x_0^p}, & \text{for Model I} \\ e^{-k\,\hat{\eta}_0\,(x_0+a_m)^p}, & \text{for Model II,} \end{cases} \quad (7)$$

where $\hat{\eta} = \sum_{j=1}^{m} \frac{1}{\hat{\theta}_j}$ and $\hat{\eta}_0 = \frac{1}{\hat{\theta}_0}$. $\hat{\theta}_0 = \sum_{i=1}^{n_0} \frac{y_{0i}^p}{n_0}$ and $\hat{\theta}_j = \sum_{i=1}^{n_j} \frac{y_{ij}^p}{n_j}$, p is known parameter.

Let $y_{j1}, y_{j2}, \dots, y_{jn_j}$ and $y_{01}, y_{02}, \dots, y_{0n_0}$ be random samples of sizes n_j and n_0 drawn from $G_j(y)$, j=1, 2,…, m, and $G_0(y)$ respectively. For simplicity, we shall set $n_j = n$.

It can be easily shown that $\hat{\theta}_j$, j=1, 2, …, m and $\hat{\theta}_0$ have Gamma distribution with parameters $\left(n, \frac{\theta_j}{n}\right)$, j=1,2, …, m and $\left(n_0, \frac{\theta_0}{n_0}\right)$ respectively, or $\left(\frac{2n\hat{\theta}_j}{\theta_j}\right), j = 1, 2, \dots, m$ and $\left(\frac{2n_0\hat{\theta}_0}{\theta_0}\right)$, have a Chi-square distribution with n and n_0 degrees of freedom respectively, therefore

$$E(\hat{\theta}_j) = \theta_j, \qquad var\,(\hat{\theta}_j) = \frac{\theta_j^2}{n}.$$
$$E(\hat{\theta}_0) = \theta_0, \qquad var\,(\hat{\theta}_0) = \frac{\theta_0^2}{n_0}.$$

Define $W_j = \hat{\theta}_j - \theta_j$, j= 1,2,…, m and $U = \hat{\theta}_0 - \theta_0$. It is clear that W_j, j= 1,2,…, m and U are asymptotically normally distributed with zero means and variance as $\frac{\theta_j^2}{n}$, $\frac{\theta_0^2}{n_0}$, respectively.

Therefore, $\hat{R}_{k,m}$ can be rewritten as:

$$\hat{R}_{k,m} = \begin{cases} \prod_{j=1}^{m} e^{-kx_0^p \eta_j^*}, & from\ Model\ I \\ e^{-k(x_0+a_m)^p \eta_0^*}, & from\ Model\ II, \end{cases} \tag{8}$$

where $\eta_j^* = \sum_{j=1}^{m} \frac{1}{W_j+\theta_j}$ and $\eta_0^* = \frac{1}{U+\theta_0}$.

Using Taylor's expansion and Equation (8), we obtain

$$\hat{R}_{k,m} = \begin{cases} R_{k,m} + k\,x_0^p\,R_{k,m} \sum_{j=1}^{m} \frac{W_j}{\theta_j^2} x + \mathbb{R}_1, from\ Model\ I \\ R_{k,m} + \frac{1}{\theta_0^2} k\,(x_0 + a_m)^p R_{k,m} U + \mathbb{R}_2, from\ Model\ II, \end{cases} \tag{9}$$

where $\mathbb{R}_1$ and $\mathbb{R}_2$ are remainder terms.

For the models I and II, $\hat{R}_{k,m}$ are asymptotically normal with means $R_{k,m}$ as given by Equation (6), and variances

$$\sigma^2_{\hat{R}_{k,m}} = \begin{cases} (k\,x_0^p\,R_{k,m})^2 \sum_{j=1}^{m} \frac{1}{n_j\theta_j^2}, & from\ Model\ I \\ \frac{1}{n_0}\left(\frac{k}{\theta_0}(x_0 + a_m)^p\right)^2 R^2_{k,m}, & from\ Model\ II. \end{cases} \tag{10}$$

Hence, $\hat{R}_{k,m}$ in (7) is a consistent estimator of $R_{k,m}$.

4.1. Two- Sided Approximate Confidence Interval for $R_{k,m}$

Since $\hat{R}_{k,m}$ in (7) is asymptotically normal with mean $R_{k,m}$ and variance given by Equation (10), thus a two-sided approximate (1 - α)100% confidence intervals of $R_{k,m}$ for the two models are obtained by

$$\Pr\{\hat{R}_{k,m} - z_{\alpha/2}\,\hat{\sigma}_{\hat{R}_{k,m}} < R_{k,m} < \hat{R}_{k,m} - z_{\alpha/2}\,\hat{\sigma}_{\hat{R}_{k,m}}\} = 1 - \alpha, \tag{11}$$

where $\hat{R}_{k,m}$ is given by (7), $\Pr(Z > z_{\alpha/2}) = \alpha/2$ and Z has a standard normal distribution,

$$\hat{\sigma}^2_{\hat{R}_{k,m}} = \begin{cases} (k\,x_0^p\,\hat{R}_{k,m})^2 \sum_{j=1}^{m} \frac{1}{n_j\theta_j^2}, & from\ Model\ I \\ \frac{1}{n_0}\left(\frac{k}{\theta_0}(x_0 + a_m)^p\right)^2 \hat{R}^2_{k,m} & from\ Model\ II. \end{cases} \tag{12}$$

4.2. Exact Two- Sided Confidence Interval for $R_{k,m}$

Since, $\left(\frac{2n\,\hat{\theta}_j}{\theta_j}\right)$, $j = 1, 2, \dots, m$ and $\left(\frac{2n_0\hat{\theta}_0}{\theta_0}\right)$ have a Chi-square distribution with $2n$ and $2n_0$ degrees of freedom respectively. Thus, the two-sided approximate $(1-\alpha)100\%$ confidence intervals of $R_{k,m}$ for the two models are obtained by:

For Model I

$$\Pr\left\{\chi^2_{(2n_j,\alpha)} < \frac{2n_j}{\theta_j}\hat{\theta}_j < \chi^2_{(2n_j,1-\alpha)}\right\} = 1 - \alpha,$$

$$\Pr\left\{\exp\left(-kx_0^p \sum_{j=1}^{m} \frac{\chi^2_{(2n,1-\alpha)}}{2n\,\hat{\theta}_j}\right) < R_{k,m} < \exp\left(-k\,x_0^p \sum_{j=1}^{m} \frac{\chi^2_{(2n,\alpha)}}{2n\,\hat{\theta}_j}\right)\right\} = 1 - \alpha. \tag{13}$$

For Model II

$$\Pr\left\{\chi^2_{(2n_0,\ \alpha)} < \frac{2n_0}{\theta_0}\hat{\theta}_0 < \chi^2_{(2n_0,1-\alpha)}\right\} =$$

$$\Pr\left\{\exp\left(-k(x_0 + a_m)^p \frac{\chi^2_{(2n_0,\ 1-\alpha)}}{2n_0\hat{\theta}_0}\right) < R_{k,m} < \exp\left(-k\,(x_0 + a_m)^p \frac{\chi^2_{(2n_0,\alpha)}}{2n_0\hat{\theta}_0}\right)\right\} = 1 - \alpha, \tag{14}$$

where $1-\alpha$ is the confidence coefficient.

When p=1 and p=2, we get the exponential and Rayleigh cases, respectively, which are discussed in [13].

5. Special Case

If the strength of link i, $i = 1, 2, \dots, k$, during repeated cycles of stress are independent but identical random variables, we have in Model I, $G_j(y) = G(y)$ for all j. Then equation (1) becomes

$$R_{k,m} = [1 - G(x_0)]^{km}. \tag{15}$$

For Weibull distribution, using Equations (5)- (7), we obtain

$$R_{k,m} = e^{-kmx_0^p/\theta}, \tag{16}$$

$$\hat{R}_{k,m} = e^{-kmx_0^p/\hat{\theta}}, \qquad (17)$$

where $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} y_i^p$, $y_1, \ldots, y_n$ is a random sample drawn from G(y) in (5).

Also,

$$\sigma^2_{\hat{R}_{k,m}} = \frac{1}{n}\left(\frac{kmx_0^p}{\theta}\right)^2 R^2_{k,m}. \qquad (18)$$

Therefore, an exact two-sided (1 - α)100% confidence interval for $R_{k,m}$ is given by

$$\Pr\left\{\exp\left(-km\, x_0^p \frac{\chi^2_{(2n,\ 1-\alpha)}}{2n\,\hat{\theta}}\right) < R_{k,m} < \exp\left(-km\, x_0^p \frac{\chi^2_{(2n,\ \alpha)}}{2n\,\hat{\theta}}\right)\right\} = 1 - \alpha. \qquad (19)$$

For Model II, putting $a_j = 0$ in Equations (2-3), we return to the static case. Hence

$$Y_{ij} = Y_{i0},$$

$$Y_j^* = Y_0^* = min(Y_{10}, Y_{20}, \ldots, Y_{k0}),$$

and $R_{k,m} = [1 - G_0(x_0)]^k$.

Therefore, for Weibull distribution, we get

$$R_{k,m} = e^{-\frac{k\,x_0^P}{\theta_0}}, \quad \hat{R}_{k,m} = e^{-\frac{k\,x_0^P}{\hat{\theta}_0}}, \quad \text{and}$$

$$\hat{\sigma}^2_{\hat{R}_{k,m}} = \frac{1}{n_0}\left(\frac{k\,x_0^P}{\hat{\theta}_0}\right)^2 (\hat{R}_{k,m})^2.$$

When p=1 and p=2, we obtain the exponential and Rayleigh cases which are discussed in [13].

6. Illustrative Examples and Simulation Study.

Example 1. Simulated Data.

A simulation study is made by taking the average of 1000 generated samples drawn from Weibull distribution with parameters $\theta_1 = 10000, \theta_2 = 8000$ and $\theta_0 = 9000$ while for the identical case, we take $\theta_1 = \theta_2 = 9000$. $R_{k,m}$, $\hat{R}_{k,m}$, $\hat{\sigma}^2_{\hat{R}_{k,m}}$, exact and approximate (1 - α)100% confidence interval (C.I.) for Model I and Model II are calculated and presented in Tables 1-9. For simplicity, we take k=1, m=2, a=0.01, $a_m = a.m$, p=3 and $x_0 = 10$. The results are presented in Tables 1-9.

Table 1. Non-Identical Random Independent Strength, $R_{1,2} = 0.798516$

n	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	10127	8102	0.7667	0.00430
15	09966	7973	0.7869	0.00122
25	10150	8120	0.7943	0.00065
50	10048	8038	0.7962	0.00034
75	10004	8003	0.7964	0.00022
100	10028	8023	0.7974	0.00017
250	09974	7979	0.7975	0.00007

Table 2. Approximate C.I. for $R_{1,2}$ in case Non-Identical Random Independent Strength.

$1-\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.6637,0.8697)	0.2060	(0.6444, 0.8890)	0.2447	(0.6057, 0.9277)	0.3220
15	(0.7303,0.8436)	0.1133	(0.7196, 0.8542)	0.1346	(0.6984, 0.8755)	0.1771
25	(0.7516,0.8370)	0.0854	(0.7436, 0.8451)	0.1015	(0.7275, 0.8611)	0.1336
50	(0.7661,0.8262)	0.0601	(0.7605, 0.8319)	0.0714	(0.7492, 0.8432)	0.0940
75	(0.7718, 0.8209)	0.0491	(0.7672, 0.8255)	0.0583	(0.7580, 0.8348)	0.0767
100	(0.7763,0.8186)	0.0423	(0.7723, 0.8226)	0.0503	(0.7644, 0.8306)	0.0662
250	(0.7840,0.8108)	0.0268	(0.7815, 0.8133)	0.0318	(0.7764, 0.8183)	0.0419

Table 3. Exact C.I. for $R_{1,2}$ in case Non-Identical Random Independent Strength.

$1-\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.6597, 0.8765)	0.2168	(0.6233, 0.8983)	0.2750	(0.5541, 0.9324)	0.3783
15	(0.7257, 0.8479)	0.1222	(0.7059, 0.8622)	0.1563	(0.6676, 0.8869)	0.2193
25	(0.7479, 0.8405)	0.0926	(0.7333, 0.8518)	0.1185	(0.7049, 0.8719)	0.1670
50	((0.7634, 0.8288)	0.0654	(0.7533, 0.8372)	0.0839	(0.7340, 0.8523)	0.1183
75	(0.7696, 0.8230)	0.0534	(0.7615, 0.8300)	0.0685	(0.7460, 0.8428)	0.0968
100	(0.7744, 0.8205)	0.0461	(0.7674, 0.8266)	0.0592	(0.7541, 0.8377)	0.0836
250	(0.7828, 0.8120)	0.0292	(0.7784, 0.8159)	0.0375	(0.7703, 0.8233)	0.0530

Table 4. Random –Fixed Strength, $R_{1,2} = 0.894242$

n	$\hat{\theta}_0$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	9051	0.872905	0.00318343
15	9054	0.887508	0.00078603
25	9018	0.890439	0.00043954
50	8866	0.890785	0.00021521
75	8965	0.892493	0.00013871
100	8965	0.892945	0.00001029
250	8984	0.893692	0.00004047

Table 5. Approximate C.I. for $R_{1,2}$ in case Random –Fixed Strength

$1\text{-}\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.7870, 0.9588)	0.1718	(0.7709, 0.9749)	0.2040	(0.7386, 1.0072)	0.2686
15	(0.8426, 0.9324)	0.0898	(0.8342, 0.9408)	0.1067	(0.8173, 0.9577)	0.1404
25	(0.8564, 0.9244)	0.0680	(0.8500, 0.9308)	0.0808	(0.8373, 0.9436)	0.1063
50	(0.8668, 0.9148)	0.0480	(0.8623, 0.9193)	0.0570	(0.8532, 0.9283)	0.0751
75	(0.8732, 0.9118)	0.0386	(0.8695, 0.9154)	0.0459	(0.8623, 0.9227)	0.0604
100	(0.8762, 0.9096)	0.0333	(0.8731, 0.9127)	0.0396	(0.8669, 0.9190)	0.0521
250	(0.8832, 0.9042)	0.0210	(0.8812, 0.9061)	0.0249	(0.8773, 0.9101)	0.0328

Table 6. Exact C.I. for $R_{1,2}$ in case Random –Fixed Strength

$1\text{-}\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.8066, 0.9354)	0.1288	(0.7826, 0.9472)	0.1646	(0.7348, 0.9653)	0.2305
15	(0.8523, 0.9212)	0.0689	(0.8405, 0.9289)	0.0884	(0.8173, 0.9421)	0.1248
25	(0.8637, 0.9162)	0.0525	(0.8551, 0.9224)	0.0673	(0.8382, 0.9333)	0.0951
50	(0.8720, 0.9091)	0.0371	(0.8661, 0.9138)	0.0477	(0.8547, 0.9221)	0.0674
75	(0.8774, 0.9073)	0.0299	(0.8727, 0.9111)	0.0384	(0.8638, 0.9181)	0.0543
100	(0.8799, 0.9058)	0.0259	(0.8759, 0.9091)	0.0332	(0.8683, 0.9152)	0.0469
250	(0.8855, 0.9018)	0.0163	(0.8831, 0.9040)	0.0209	(0.8785, 0.9080)	0.0295

Table 7. Identical Random Independent Strength, $R_{1,2} = 0.800737$ and $\theta_1 = \theta_2 = 9000$.

n	$\hat{\theta}$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	9170	0.77027	0.00419
15	8917	0.78827	0.00120
25	9095	0.79528	0.00067
50	9008	0.79764	0.00033
75	9021	0.79896	0.00022
100	8991	0.79906	0.00016
250	9016	0.80040	0.00006

Table 8. Approximate C.I. for $R_{1,2}$ in case Identical Random Independent Strength

$1\text{-}\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.6687, 0.8719)	0.2032	(0.6496, 0.8909)	0.2413	(0.6114, 0.9291)	0.3177
15	(0.7323, 0.8443)	0.1120	(0.7217, 0.8548)	0.1331	(0.7007, 0.8758)	0.1751
25	(0.7530, 0.8375)	0.0845	(0.7451, 0.8455)	0.1004	(0.7292, 0.8614)	0.1322
50	(0.7679, 0.8273)	0.0594	(0.7624, 0.8329)	0.0705	(0.7512, 0.8441)	0.0929
75	(0.7748, 0.8231)	0.0483	(0.7703, 0.8276)	0.0573	(0.7612, 0.8366)	0.0754
100	(0.7782, 0.8199)	0.0417	(0.7742, 0.8238)	0.0496	(0.7664, 0.8317)	0.0653
250	(0.7873, 0.8136)	0.0263	(0.8812, 0.9061)	0.0249	(0.7799, 0.8210)	0.0411

Table 9. Exact C.I. for $R_{1,2}$ in case Identical Random Independent Strength

$1\text{-}\gamma$	0.90		0.95		0.99	
n	C. I.	D	C. I.	D	C. I.	D
5	(0.6639, 0.8789)	0.2150	(0.6275, 0.9004)	0.2729	(0.5585, 0.9339)	0.3754
15	(0.7274, 0.8489)	0.1215	(0.7077, 0.8632)	0.1555	(0.6695, 0.8877)	0.2182
25	(0.7491, 0.8412)	0.0921	(0.7344, 0.8525)	0.1181	(0.7062, 0.8725)	0.1663
50	(0.7651, 0.8300)	0.0649	(0.7551, 0.8384)	0.0833	(0.7358, 0.8534)	0.1176
75	(0.7724, 0.8253)	0.0529	(0.7644, 0.8322)	0.0678	(0.7490, 0.8448)	0.0958
100	(0.7761, 0.8219)	0.0458	(0.7692, 0.8280)	0.0588	(0.7560, 0.8390)	0.0830
250	(0.7859, 0.8148)	0.0289	(0.7817, 0.8187)	0.0370	(0.7736, 0.8259)	0.0523

Example 2. Real Data.

As an another example we choose the real data set proposed by Lawless [14] (1982, p. 185) and Nelson [15], referring to which the time breakdown of an insulating fluid between electrodes at a voltage of 36 kV (minutes), 32 kV (minutes) and 30 kV (minutes). The data shown below are breakdown times for 3 groups of specimens, each group involving a different voltage level.

Voltage Level (kV)	n	Breakdown Times
36	15	{1.97, 0.59, 2.58, 1.69, 2.71, 25.50, 0.35, 0.99, 3.99, 3.67, 2.07,0.96, 5.35, 2.90, 13.77}
32	15	{0.40, 82.85, 9.88, 89.29, 215.10, 2.75, 0.79, 15.93, 3.91, 0.27, 0.69,100.58, 27.80,13.95, 53.24};
30	11	{17.05, 22.66, 21.02, 175.88,139.07,144.12, 20.46, 43.40, 194.90, 47.30, 7.74}

A models suggested by engineering consideration are that, for a fixed voltage level, time to breakdown have a Weibull distributions. Furthermore, distributions corresponding to different voltage levels are thought to differ only with respect to their scale parameters, the shape parameter being the same for different levels.

The computations in this example are done with k=1, m=2, a= 0.01, $a_m = a.m$, p=3, $x_0 = 10$, θ_1= 10000,$\theta_2 = 8000$ and $\theta_0 = 9000$, $n_1 = n_2 = 15, n_0 = 11$, $R_{1,2} = 0.798516$ for Model I and $R_{1,2} = 0.894242$ for Model II.
The results are presented in Tables 10-12.

Table 10. $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_0$, $\hat{R}_{k,m}$ and , $\hat{\sigma}^2_{\hat{R}_{k,m}}$ in case Non-Identical Random Independent Strength

$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_0$	$\hat{R}_{1,2}$		$\hat{\sigma}^2$	
			Model I	Model II	Model I	Model II
1303	828701	170452	0.46360	0.99941	0.00844	0.0000003

Table 11. Approximate C.I. for $R_{1,2}$ in case Non-Identical Random Independent Strength

1- γ	0.90		0.95		0.99	
Model	C. I.	D	C. I.	D	C. I.	D
I	(0.3120, 0.6152)	0.3032	(0.2835, 0.6437)	0.3601	(0.2266, 0.7006)	0.4740
II	((0.9991, 0.9997)	0.0006	(0.9991, 0.9998)	0.0007	(0.9990, 0.9999)	0.0009

Table 12. Exact C.I. for $R_{1,2}$ in case Non-Identical Random Independent Strength

1- γ	0.90		0.95		0.99	
Model	C. I.	D	C. I.	D	C. I.	D
I	(0.3565, 0.5899)	0.2334	(0.3257, 0.6226)	0.2969	(0.2714, 0.6817)	0.4103
II	(0.9992, 0.9996)	0.0004	(0.9991, 0.9997)	0.0006	(0.9990, 0.9997)	0.0007

7. Conclusions

In this paper we presented the problem of determining the reliability of a series chain consisting of k identical likes. Our computational results were computed by using Mathematica 8.0. Our observation concerning the results are stated in the following points:

1- From Tables (1), (4) , (7) and (10), we see that $\hat{\sigma}^2_{\hat{R}_{k,m}}$ decreases as the sample size n increases, i.e. $\hat{R}_{k,m}$ is consistent estimator of $R_{k,m}$.

2- From another Tables, we see that the length of the C.I.s decreases by increasing the sample size.

3- We find that $\hat{R}_{1,2}$ under Model II is greater than that under Model I. This means that the value of reliability change by varying the type of strength.

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