ZERO B-SPLINEPOLYNOMIAL APROXIMATION METHOD FOR CHAOTIC FUNCTIONAL INTEGRAL EQUATIONS AND CONTROLING BY PARAMETERS COIFFICIENT ARRAY

Ali Moharrami [a]

[a]Faculty of Engineering, University of Zanjan, P.O. Box 45195-313, Zanjan, Iran
ali.moharrami@ymail.com

Abstract. In this article, we study an approximation of a functional integral equations by zero B-Spline polynomial. We use the polynomialapproximation we have n unknown determined coefficients. We used zero B-Spline function. For n knots of x_j we have a linear system. When we solved this system we could found determined coefficients. And this method error was little when n be enough large.

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1. Introduction

In many natural and real phenomenons, differential ofIntegral equations can be seen.These systems may have different behavior in different qualifications [2]. Many of these equations are called functional integral equations. Fredholm and Voltrra integral equations are special type of these equations [3]. In nearest year everybody can to solve these equations by differential numerical method such that polynomial approximation [6].We used B-Spline equation in this paper [5]. When we have linear functions, we can approximateunknown function by a polynomial function. So we must found their coefficients. We solved the linear system and found this determined coefficient.A Integral equations can be chaotic when we give differentparameters coefficient. We study these systems and controlling it.

2. Chaos

A dictionary definition of chaos is a disordered state of a collection; a confused mixture. This is an accurate description of dynamical systems theory today or of any other lively field of research. (Morris W. Hirsch).

When a system in nature is mathematically modeled, we find that their graphical representations are not straight light lines and the system behavior is not so easy to predict. After researches on complex systems, now we know that noise is actually important information about the experiments. When noise is inserted in to the result graph, the graph no longer appear as straight line, neither its point are predictable. Once, this noise was referred to as the chaos in an experiment. For chaos applications we can mention, much like physics, chaos theory provides a foundation for the study of all other scientific disciplines. It is actually a toolbox of methods for incorporating non-linear dynamics for the study of science.

3. functional integral eqquations

functional integral equation of the second kind is given by:

$$y(x) + p(x)y(h(x)) + \lambda \int_a^b k(x,t)y(t)dt = g(x), \qquad a \le x \le b \tag{2.1}$$

Also, Voltra functional integral equation of the second kind is given by:

$$y(x) + p(x)y(h(x)) + \lambda \int_a^x k(x,t)y(t)dt = g(x), \qquad a \le x \le b \tag{2.2}$$

But, functional integral equation of first kind is given by:

$$p(x)y(h(x)) + \lambda \int_a^b k(x,t)y(t)\,dt = g(x), \qquad a \le x \le b \tag{2.3}$$

Also, Voltra functional integral equation of first kind is given by:

(2.4)

$$p(x)y(h(x)) + \lambda \int_a^x k(x,t)y(t)\,dt = g(x), \qquad a \le x \le b$$

4. Zero B-spline polynomial Aprroximation method

Definition 3.1. *Let $\Psi(x): \mathbb{R} \to \mathbb{R}$ be interpolation polynomial. So determined coefficients $\{\alpha_i\}_{i=1}^n$ such that*

$$\Psi(x) = \sum_{i=1}^n \alpha_i \phi_i(x)$$

We let $\Psi(x)$ be an approximation of $y(x)$. This method given an approximation of functional integral when our function are orthogonal.

Definition 3.2. *Let h be a partition of the interval $[a,b]$; defined by the knots $\{x_j\}_{j=1}^n$, such that*

$$a = x_1 \le x_2 \le \cdots \le x_n = b$$

We approximate $y(x)$ of (2.1) by a polynomial of $\Psi(x)$ such that (2.5). So we have:

$$\Psi(x) + p(x)\Psi(h(x)) + \int_a^b k(x,t)\Psi(t)dt = g(x)$$

When we use Definition.2.1 we have:

$$\sum_{i=1}^n \alpha_i\phi_i(x) + p(x)\sum_{i=1}^n \alpha_i\phi_i(h(x)) + \int_a^b k(x,t)\sum_{i=1}^n \alpha_i\phi_i(t)\,dt = g(x)$$

Then

$$\sum_{i=1}^n \alpha_i\phi_i(x) + p(x)\sum_{i=1}^n \alpha_i\phi_i(h(x)) + \sum_{i=1}^n \alpha_i \int_a^b k(x,t)\phi_i(t)dt = g(x)$$

And for (2.2) in homogenous temperament by Voltra functional integral equation we have:

$$\sum_{i=1}^n \alpha_i\phi_i(x) + p(x)\sum_{i=1}^n \alpha_i\phi_i(h(x)) + \sum_{i=1}^n \alpha_i\phi_i(t) \int_a^x k(x,t)\phi_i(t)dt = g(x)$$

Definition 3.3. *Let $B_i^{(0)}(x): \mathbb{R} \to \{0,1\}$ be a zero B-Spline such that*

$$B_i^{(0)}(x) = \begin{cases} 1 & x \in [t_i, t_{i+1}] \\ 0 & otherwise \end{cases} \qquad (3.7)$$

When we used $B_i^{(0)}(x)$ in (3.2) then we have:

$$\sum_{i=1}^n \alpha_i B_i^{(0)}(x) + p(x)\sum_{i=1}^n \alpha_i B_i^{(0)}(h(x)) + \sum_{i=1}^n \alpha_i \int_a^b k(x,t)B_i^{(0)}(t)dt = g(x) \qquad (3.8)$$

And for (3.3) we have:

$$\sum_{i=1}^n \alpha_i B_i^{(0)}(x) + p(x)\sum_{i=1}^n \alpha_i B_i^{(0)}(h(x)) + \sum_{i=1}^n \alpha_i \int_a^x k(x,t)B_i^{(0)}(t)dt = g(x) \qquad (3.9)$$

We stay to found coefficient $\{\alpha_i\}_{i=1}^n$, so we let $x = x_j$ for $j = 1,2,\ldots,n$. We have linear system such that

$$\sum_{i=1}^n \alpha_i B_i^{(0)}(x_j) + p(x)\sum_{i=1}^n \alpha_i B_i^{(0)}(h(x_j)) + \sum_{i=1}^n \alpha_i \int_a^b k(x_j,t)B_i^{(0)}(t)dt = g(x_j) \qquad (3.4)(3.10)$$

And

(3.11)

(3.12)

$$\sum_{i=1}^{n} \alpha_i B_i^{(0)}(x_j) + p(x) \sum_{i=1}^{n} \alpha_i B_i^{(0)}(h(x_j)) + \sum_{i=1}^{n} \alpha_i \int_a^x k(x_j,t) B_i^{(0)}(t)dt = g(x_j)$$

Definition 3.4. *Let* $\overline{I_n}(x), I_n(x): \mathbb{R} \to \mathbb{R}$ *be a operation integral for* $B_i^{(0)}(x)$ *such that:*

$$I_i(x) = \int_a^b k(x,t) B_i^{(0)}(t)dt$$

And

$$\overline{I_n}(x) = \int_a^x k(x,t) B_i^{(0)}(t)dt$$

Definition 3.5.*Let* A *be a squire matrix of* $n \times n$*, also* A *and* G *be a vector of* $n \times 1$*. So we can rewrite the linear systems (3.4) and (3.5) as follows*

$$AA = G$$

Where

$$A = \begin{pmatrix} B_1^{(0)}(x_1) + B_1^{(0)}(h(x_1)) + I_1(x_1) & \cdots & B_1^{(0)}(x_n) + B_1^{(0)}(h(x_n)) + I_1(x_n) \\ \vdots & \ddots & \vdots \\ B_n^{(0)}(x_1) + B_n^{(0)}(h(x_1)) + I_n(x_1) & \cdots & B_n^{(0)}(x_n) + B_n^{(0)}(h(x_n)) + I_n(x_n) \end{pmatrix}$$

And

$A = (\alpha_1, \alpha_2, \dots, \alpha_i)^T, \qquad G = (g(x_1), g(x_2), \dots, g(n))^T$

When we solve this linear system, we found determined coefficients $\{\alpha_i\}_{i=1}^{n}$ *and* $\Psi(x)$ *is a interpolation polynomial for* $y(x)$*.*

5. error analysis

Definition 4.1. *Let f be a scalar function or an operator, also* $\Phi = (\phi_1, \phi_2, \dots, \phi_n)$ *be a vector. We define* $f(\Phi)$ *as follows*

$$f(\Phi) = (f(\phi_1), f(\phi_2), \dots, f(\phi_n))^T$$

Definition4.2. *for all knots of* $\{x_j\}_{j=1}^{n}$ *we let* e_j *and* $\hat{e_j}$ *be an error of* $\phi(x_j)$ *and* $\phi(h(x_j))$ *such that:*

$$e_j = y(x_j) - \phi(x_j)$$

And

$$\hat{e_j} = y\left(h(x_j)\right) - \phi(h(x_j)). \qquad (4.3)$$

Theorem 4.1. *This approximate method is convergent.*

If $\det(A) = 0$ when increase n, we have the computational error. So n not be very large.
When we study error of this method, we see that this equation can be in chaotic mod by parameter coefficient.

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