Trust Region Algorithm for Multi-objective Transportation, Assignment, and Transshipment Problems

Yousria Abo-Elnaga[1,2]; Bothina El-Sobky[3] and Hanadi Zahed[4]

[1]Department of Basic Science, Tenth of Ramadan City, Higher Technological Institute, Egypt
[2]Department of Mathematics, Faculty of Science, Taibah University, Kingdom of Saudi Arabia)
[3]Department of Mathematics, Faculty of Science, Alexandria University, Egypt
[4]Department of Mathematics, Faculty of Science, Taibah University, Kingdom of Saudi Arabia
bothinaelsobky@yahoo.com

Abstract: In this paper, we present a trust-region globalization strategy to solve a multi-objective transportation (MOT) problem, which is one of great interest to many researchers and several local methods have been proposed to solve it. A weighting approach is used together with an active set strategy and a multiplier method to transform (MOT) problem to unconstrained optimization problem and we used a trust-region algorithm to solve it. In this work, the effect of changing weights on (MOT) problem is studied to show the degree of satisfaction of each objective. We also make a comparative study between our proposed approach and different approaches treated the multi-objective transportation problem before. The proposed approach is carried out on two multi-objective transportation test problems.

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1. Introduction:

In real-world cases transportation problems can be formulated as multi-objective transportation problems because the complexity of a social and economic environment requires explicit considerations of criteria other than cost. Examples of additional concerns include: average delivery time of commodities, reliability of transportation, accessibility to the user, product deterioration, and many others. Thus, multiple penalty criteria may exist concurrently, which leads to the research work on multi-objective transportation (MOT) problems.

The multi-objective transportation problem is of great interest to many researchers and several local methods have been proposed to solve it [1, 2, 11, 12, 14, 15, 18, 19].

Most of the methods which are used to solve (MOT) problem are local methods. By local method we mean that the method is designed to converge to optimal solution from closest starting point whether it is local or global one. For a local method, there is no guarantee that it converge if it starts from remote.

In this paper we will use a trust-region globalization strategy to solve (MOT) problem. Globalizing strategy means modifying the local method in such a way that it is guaranteed to converge at all even if the starting point is far away from the solution.

In this work, we convert the multi-objective transportation problem to a single-objective constrained optimization problem with equality and inequality constraints (SCOEI) problem, by using a weighting approach. The weighting approach is considered as one of the most useful algorithms in treating multi-objective optimization problems to generate a wide set of optimal solutions (pareto set), for more detail see [17]. Here, an active set strategy is used to convert (SCOEI) problem to a single-objective equality constrained optimization problem (SECO) problem and a multiplier method is used to convert (SECO) problem to unconstrained optimization problem.

The trust-region strategy for solving (SCOEI) problem, (SECO) problem, and unconstrained optimization problem has proved to be very successful, both theoretically and practically [6-10]

In this current study, the effect of changing weights on (MOT) problem was studied to show the degree of satisfaction of each objective. We also make a comparative study between our proposed algorithm and different approaches treated the multi-objective transportation problem before.

Subscripted functions denote function values at particular points; for example, $f_k = f(x_k)$, $\nabla f_k = \nabla f(x_k)$, $l_{k+1} = l(x_{k+1}, \mu_{k+1}, \upsilon_{k+1})$, $\nabla_x l_k = \nabla_x l(x_k, \mu_k, \upsilon_k)$ and so on. However, the arguments of the functions are not abbreviated when emphasizing the dependence of the functions on their arguments. The matrix H_k denotes the Hessian of the Lagrangian function $l(x_k, \mu_k, \upsilon_k)$ or an approximation to it. Finally, all norms are l_2 -norms.

This paper is organized as follows: In section two we introduce the mathematical form of multi-objective transportation problem and how (MOT) problem transform to unconstrained optimization problem. In section three we give a detailed discussion of the trust region algorithm for solving (MOT) problem. Furthermore, we then discuss in detail the two test problems with all their possible solutions in section four. Finally, the conclusion is discussed in section five, and we come to acknowledgments in section six.

2 Mathematical Formulation of (MOT) Problem.

The mathematical model of (MOT) problem can be stated as follows:

$$minimize\ f^{\hat{k}} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{\hat{k}} x_{ij}, \quad \hat{k}=1,2,...,p$$

$$subject\ to \quad \sum_{j=1}^{n} x_{ij} = b_i, \qquad (2.1)$$

$$\sum_{i=1}^{m} x_{ij} = c_j,$$

$$x_{ij} \geq 0,$$

where m and n stands for the number of sources and the number of destinations, respectively, and $\hat{k}=1,2,...,p$. Positive constants b_i are the amount of homogeneous product for i^{th} origin which are transported to n destinations. Positive constants c_j represent the demand of homogeneous product for the j^{th} destination. Positive constants $a_{ij}^{\hat{k}}$ represent the coefficients of the $\hat{k}^{th}$ objective functions which are associated with transportation of unit of the product from source i to destination j . Variables x_{ij} are the unknown quantity to be transported from origin i to destination j .

The first set of constraints $\sum_{j=1}^{n} x_{ij} = b_i$ stipulates that the sum of shipments from the source must equal its supply and the second set of constraints $\sum_{i=1}^{m} x_{ij} = c_i$ requires that the sum of the shipments to the destination must satisfy its demand. Since the total supply is equal to total demand, this formulation is called a balanced transportation problem. In this paper, we study the case of balanced transportation problem because the unbalanced transportation problem can be converted to balanced transportation problem after including a dummy origin or a dummy destination.

Definition 2.1 :(Nondominated solution). A feasible vector x^0 in a feasible region S , yields a nondominated solution of (MOT) problem, iff there is no other vector such that $x \in S$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{\hat{k}} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{k} x_{ij}^{0} \qquad \forall \hat{k}=1,2,...,p,$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{\hat{k}} x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{k} x_{ij}^{0} \qquad for\ some\ \hat{k}.$$

For more detail see [16].

Definition 2.2:(Efficient Solution). A point $x^0 \in S$ is efficient, iff there does not exist another $x \in S$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{\hat{k}} x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{k} x_{ij}^{0} \qquad \forall \hat{k}=1,2,...,p,$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{\hat{k}} x_{ij} \neq \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{k} x_{ij}^{0} \qquad for\ some\ \hat{k}.$$

Otherwise x^0 is an inefficient solution. For example, the point $x^0 \in S$ is efficient if its criterion vector is not dominated by the criterion vector of another point in the feasible region S . In this paper we will use efficient solution. [17].

Definition 2.3:(Compromise Solution). A feasible vector $x^* \in S$ is called a compromise solution of (MOT) problem iff $x^* \in E$ and $f(x^*) \leq \min\{f(x)\forall x \in S\}$ where E is the set of efficient solutions. [13].

From the above definition a compromise solution must meet two conditions

The solution should be efficient. 1-

2- The feasible solution vector x^* should have the minimum deviation from the ideal point than any other point in S .

The compromise solution which maximizes the underlying utility function is the closest one to the ideal solution. While knowledge of the set of efficient solutions E is not always necessary in real world cases, the decision maker's preferences should be considered in the determination of the final compromise solution.

Definition 2.4:(Preferred Compromise Solution).

The solution is called the preferred compromise solution if the compromise solution satisfies the decision maker's preferences.

By using the weighting approach, the multi-objective optimization problem (2.1) is converted to the following single-objective constrained optimization problem with equality and inequality constraints (SCOEI) problem.

$$\begin{aligned} \text{minimize } & f(x)=\sum_{\hat{k}=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n} w^{\hat{k}} \tilde{a}_{ij}^{k} x_{ij}, \\ \text{subject to } & \sum_{j=1}^{n} x_{ij}=b_i, \qquad (2.2) \\ & \sum_{i=1}^{m} x_{ij}=c_j, \\ & x_{ij}\ge 0, \end{aligned}$$

where $\sum_{\hat{k}=1}^{p} w^{\hat{k}}=1$ and $w^{\hat{k}}\ge 0$ *for all* $\hat{k}$.

The above problem can be written as follows:

$$\begin{aligned} \text{minimize } & f(x) \\ \text{subject to } & y(x)=0, \qquad (2.3) \\ & z(x)\le 0, \end{aligned}$$

$j=1,2,\ldots,n.$, $z(x)=[x_{ij}]^T$, $i=1,2,\ldots,m$,

and where $y(x)=[\sum_{j=1}^{n} x_{ij}-b_i, \sum_{j=1}^{n} x_{ij}-c_j]^T$

The functions $f(x):\mathbb{R}^{n\times m}\to\mathbb{R}$, $y(x):\mathbb{R}^{n\times m}\to\mathbb{R}^{n+m}$, *and* $z(x):\mathbb{R}^{n\times m}\to\mathbb{R}^{n\times m}$ are twice continuously differentiable.

The Lagrangian function associated with problem (2.3) is the function

$$l(x,\mu,\upsilon)=f(x)+\mu^T y(x)+\upsilon^T z(x), \qquad (2.4)$$

where $\mu\in\mathbb{R}^{n+m}$ and $\upsilon\in\mathbb{R}^{n\times m}$ are the Lagrange multiplier vectors associated with equality and inequality constraints, respectively.

Following [6], we define a 0-1 diagonal indicator matrix $U(x)\in\mathbb{R}^{(n\times m)\times(n\times m)}$,

whose diagonal entries are

$$u_e(x)=\begin{cases}1 & \text{if } z_e(x)\ge 0, \\ 0 & \text{if } z_e(x)<0,\end{cases} \qquad (2.5)$$

where $e=1,2,\ldots,m\times n$.

Using the above matrix, we transform problem (2.3) to the following equality constrained optimization problem

$$\begin{aligned} \text{minimize } & f(x) \\ \text{subject to } & y(x)=0, \qquad (2.6) \\ & \frac{1}{2} z(x)^T U(x) z(x)=0. \end{aligned}$$

U sing a multiplier method, we transform the equality constrained optimization problem (2.6) to the following unconstrained optimization problem

$$\begin{aligned} \text{minimize } & \Phi(x,\mu,\upsilon,s;r)=l(x,\mu,\upsilon)+\frac{s}{2}\|U(x)z(x)\|^2+\frac{r}{2}\|y(x)\|^2 \\ \text{subject to } & x\in\mathbb{R}^{n\times m}, \qquad (2.7) \end{aligned}$$

where s is the positive parameter and $r>0$ is a parameter usually called the penalty parameter.

A detailed description of the main steps of the trust-region algorithm for solving the above problem and it's an algorithmic framework is presented in the following section.

3. Trust Region Algorithm Outline

This section is devoted to presenting the detailed description of the trust-region algorithm for solving problem (2.7).

3.1. Computing a Trial Step

We compute the trial step d_k by solving the following trust-region sub problem

$$\begin{aligned} \text{minimize } & l_k+\nabla_x l_k^T d+\frac{s_k}{2}\|U_k(z_k+\nabla z_k^T d)\|^2+\frac{r_k}{2}\|y_k+\nabla y_k^T d\|^2, \\ \text{subject to } & \|d\|\le\delta_k, \qquad (3.1) \end{aligned}$$

where H_k is the Hessian matrix of the Lagrangian function $l(x_k,\mu_k,\upsilon_k)$ or an approximation to it. Since our convergence theory is based on the fraction of Cauchy decrease condition, therefore a dogleg method can be used to compute the trial step. [5].

3.2. Testing the Step and Updating δ_k

To test the step, estimates for the two Lagrange multipliers μ_{k+1} and υ_{k+1} are needed. Our way of evaluating the two Lagrange multipliers μ_{k+1} and υ_{k+1} is presented in Step 5 of Algorithm (3.1) below.

To test whether the point $(x_k+d_k,\mu_{k+1},\upsilon_{k+1})$ will be taken as a next iterate, an actual reduction and a predicted reduction are needed and defined as follows:

The actual reduction in the merit function is defined as

$$Ared_k = l(x_k,\mu_k,\upsilon_k) - l(x_{k+1},\mu_k,\upsilon_k) - \Delta\mu_k^T y_{k+1} - \Delta\upsilon_k^T z_{k+1} + \frac{s_k}{2}[z_k^T U_k z_k - z_{k+1}^T U_{k+1} z_{k+1}] + \frac{r_k}{2}[\|y_k\|^2 - \|y_{k+1}\|^2], \quad (3.2)$$

where $\Delta\mu_k = (\mu_{k+1} - \mu_k)$ and $\Delta\upsilon_k = (\upsilon_{k+1} - \upsilon_k)$.

The predicted reduction in the merit function is defined to be

$$Pred_k = q_k(0) - q_k(d_k) - \Delta\mu_k^T(y_k + \nabla y_k^T d_k) - \Delta\upsilon_k^T U_k z_k + \frac{r_k}{2}[\|y_k\|^2 - \|y_k + \nabla y_k^T d_k\|^2], \quad (3.3)$$

where

$$q_k(d) = l_k + \nabla_x l_k^T d + \frac{1}{2} d^T H_k d + \frac{s_k}{2}\|U_k(z_k + \nabla z_k^T d)\|^2. \quad (3.4)$$

After computing a trial step and updating the Lagrange multipliers, the penalty parameter is updated to ensure that $\Pr ed_k \geq 0$. To update r_k, we use a scheme that has the flavor of the scheme Proposed by El-Alem [7] . This scheme is described in Step 6 of Algorithm (3.1) below. After that, the step is tested to know whether it is accepted. This is done by comparing $\Pr ed_k$ against $A\,red_k$.

If $\frac{A\,red_k}{\Pr ed_k} < \eta_1$ where $\eta_1 \in (0,1)$ is a small fixed constant, then the step is rejected. In this case, the radius of the trust region δ_k is decreased by setting $\delta_k = \alpha_1 \|d_k\|$, where $\alpha_1 \in (0,1)$ and another trial step is computed using the new trust-region radius. If $\frac{A\,red_k}{\Pr ed_k} \geq \eta_1$ then the step is accepted.

Our way of evaluating the trial steps and updating the trust-region radius is presented in Step 7 of Algorithm (3.1) below. After accepting the step, we update the parameter s_k and the Hessian matrix H_k. To update s_k, we use a scheme suggested by Yuan [20]. This scheme is described in Step 8 of Algorithm (3.1) below.

Finally, the algorithm is terminated when either $\|d_k\| \leq \varepsilon_1$ or

$$\|\nabla_x l_k\| + \|\nabla z_k U_k z_k\| + \|y_k\| \leq \varepsilon_2, \quad \textit{for some } \varepsilon_1, \varepsilon_2 > 0.$$

3.3. Main Algorithm

A formal description of the trust-region algorithm for solving problem (2.7) is presented in the following algorithm.

Algorithm 3.1. (The Main Algorithm)

Step 0. (Initialization)

Given $x_1 \in \mathbb{R}^{n\times m}$. Compute U_1. Evaluate μ_1 and υ_1 (see Step 5 with $k = 0$ and $\mu_0 = (0,0,...,0)^T$). Set $s_1 = 1$, $r_0 = 1$, $\sigma_1 = 1$, and $\beta = 0.1$. Choose $\varepsilon_1 = \varepsilon_2 = 10^{-8}$, $\alpha_1 = 0.05$, $\alpha_2 = 2$, $\eta_1 = 10^{-4}$, and $\eta_2 = 0.5$ such that $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $0 < \alpha_1 < 1 < \alpha_2$, and $0 < \eta_1 < \eta_2 < 1$. Set $\delta_{\min} = 10^{-3}$ and $\delta_{\max} = 10^5 \delta_1$ such that $\delta_{\min} \leq \delta_1 \leq \delta_{\max}$.

Set $k = 1$.

Step 1. (Test for convergence)

then terminate the algorithm. $\|\nabla_x l_k\| + \|\nabla z_k U_k z_k\| + \|y_k\| \leq \varepsilon_2$, If

Step 2. (Compute a trial step)

a) Compute the step d_k by solving (3.1)(

b) Set $x_{k+1} = x_k + d_k$.(

Step 3. (Test for termination)

If $\|d_k\| \leq \varepsilon_1$, then terminate the algorithm.

Step 4. (Update the active set)

Compute U_{k+1}.

Step 5. (Compute the Lagrange multipliers μ_{k+1} and υ_{k+1})

a) Compute μ_{k+1} by solving(

$$\begin{array}{ll} \min imize & \|\nabla f_{k+1} + \nabla y_{k+1}\mu_k + \nabla z_{k+1} U_{k+1} \upsilon\|^2 \\ subject\ to & U_{k+1}\upsilon \geq 0, \end{array}$$

and set the rest of the components of υ_{k+1} to zero .

$\mu_{k+1} = \mu_k$. then set

$$\|\nabla f_{k+1} + \nabla y_{k+1}\mu_k + \nabla z_{k+1} U_{k+1} \upsilon_{k+1}\| \leq \varepsilon_1,$$

(b) If

Else, compute μ_{k+1} by solving

$$\min imize \quad \|\nabla f_{k+1} + \nabla y_{k+1}\mu + \nabla z_{k+1} U_{k+1} \upsilon_{k+1}\|^2.$$

End if.

Step 6. (Update the penalty parameter r_k)

If $\quad \Pr ed_k \leq \frac{r_k}{4}[\|y_k\|^2 - \|y_k + \nabla y_k^T d_k\|^2],$

then set

$$r_k = \frac{4[q_k(d_k) - q_k(0) + \Delta\mu_k^T(y_k + \nabla y_k^T d_k) + \Delta\upsilon_k^T U_k z_k]}{\|y_k\|^2 - \|y_k + \nabla y_k^T d_k\|^2} + \beta.$$

End if.

Step 7. (Test the step and update the trust-region radius)

If $\frac{Ared_k}{\Pr ed_k} < \eta_1$.

Reduce the trust-region radius by setting $\delta_k = \alpha_1 \|d_k\|$ and go to step 2

accept the step $x_{k+1} = x_k + d_k$. Else if $\eta_1 \le \frac{Ared_k}{\Pr ed_k} < \eta_2$, then

Set the trust-region radius: $\delta_{k+1} = \max(\delta_k, \delta_{\min})$.

Else, accept the step: $x_{k+1} = x_k + d_k$.

Set the trust-region radius: $\delta_{k+1} = \min\{\delta_{\max}, \max\{\delta_{\min}, \alpha_2\delta_k\}\}$.

End if

Step 8. (To update the parameters s_k and σ_k)

(a) Set $s_{k+1} = s_k$ and $\sigma_{k+1} = \sigma_k$.

(b) Compute $Tpred_k = q_k(0) - q_k(d_k) - \Delta\mu_k^T(y_k + \nabla y_k^T d_k) - \Delta\upsilon_k^T U_k z_k$.

(c) If $Tp\,red_k < \sigma_k \|\nabla z_k U_k z_k\| \min\{\|\nabla z_k U_k z_k\|, \delta_k\}$,

then set $s_{k+1} = 2s_k$ and $\sigma_{k+1} = \frac{1}{2}\sigma_k$.

End if.

Step 9. Set $k = k+1$ and go to Step 1

In the following section, we introduce two multi-objective transportation test problems to obvious the goal of our paper.

4. Multi-objective Transportation Test Problems

In this section, we introduce two test problems for the multi-objective transportation optimization problem. The proposed algorithm was implemented on 2.7 MHZ PC using MATLAB 7 to confirm the effectiveness of the algorithm. The two multi-objective transportation optimization test problems are presented in the following subsections.

4.1. Multi-objective Transportation Test Problem 1

Let us consider the following numerical example presented by many [1-4, 16, 21]

to illustrate the application of the proposed algorithm. The problem has the following characteristics:

supplies: $b_1 = 8$, $b_2 = 19$, and $b_3 = 17$.

demands: $c_1 = 11$, $c_2 = 3$, $c_3 = 14$, and $c_4 = 16$.

penalties: $a^1 = \begin{pmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{pmatrix}$ and

$$a^2 = \begin{pmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{pmatrix}.$$

This problem could be written as follows:

$$\begin{aligned}
\min imize\ f_1 &= x_{11} + 2x_{12} + 7x_{13} + 7x_{14} \\
&\quad + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} \\
&\quad + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}, \\
\min imize\ f_2 &= 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} \\
&\quad + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} \\
&\quad + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}, \\
\textit{subject to}\quad & x_{11} + x_{12} + x_{13} + x_{14} = 8, \\
& x_{21} + x_{22} + x_{23} + x_{24} = 19, \\
& x_{31} + x_{32} + x_{33} + x_{34} = 17, \\
& x_{11} + x_{21} + x_{31} = 11, \\
& x_{12} + x_{22} + x_{32} = 3, \\
& x_{13} + x_{23} + x_{33} = 14, \\
& x_{14} + x_{24} + x_{34} = 16, \\
& x_{ij} \ge 0, \quad \forall i = 1,2,3, \quad j = 1,2,3,4.
\end{aligned}$$

4.2.1Results and Discussions of (MOT) Test Problem

A weighting approach is used together with the trust-region algorithm (3.1) to solve the above problem at several values of weighting values based on $w^1 = \{0, 0.1, ..., 1\}$ and $w^2 = \{1, 0.9, ..., 0\}$. By discussing the effect of changing weights on the two objective functions f_1 and f_2, we note from Figure (1) that the best value of w^1 is 0.4 and w^2 is 0.6, which give $f_1 = 173$ and $f_2 = 173$ as the best compromise solution.

To evaluate the performance of the suggested approach we show a schematic comparison in table (1) between our results and the results of researchers who have used other approaches (Interactive approach [16], Fuzzy approach [1], Fuzzy approach [4] and IFGP approach [2]). It becomes evident from the table that the value of $f_2 = 173$ is the best result, whereas the value of $f_1 = 173$, while still acceptable, is not the best

Table 1. Comparison between different approaches.

The name of approach	f_1	f_2
Interactive approach [16]	186	174
Fuzzy approach [1]	170	190
Fuzzy approach [4]	160	195
IFGP approach [2]	168	185
Proposed approach	173	173

4.3. Multi-objective Transportation Test Problem 2

Let us consider the following numerical example presented by Aneja and Nair[3] ; Ringuest and Rinks [16] to illustrate the goal of our paper. The problem has the following characteristics

supplies: $b_1 = 5$, $b_2 = 4$, $b_3 = 2$, and $b_4 = 9$.

demands: $c_1 = 4$, $c_2 = 4$, $c_3 = 6$, $c_4 = 2$, and $c_5 = 4$.

penalties: $$a^1 = \begin{pmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{pmatrix},$$

$$a^2 = \begin{pmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{pmatrix}$$

and

$$a^3 = \begin{pmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{pmatrix}.$$

This problem could be written as follows:

$$\begin{aligned}
\min imize\ f_1 = {} & 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} \\
& + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} \\
& + 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} \\
& + 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45,} \\
\min imize\ f_2 = {} & 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} \\
& + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25} \\
& + 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} \\
& + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45,} \\
\min imize\ f_3 = {} & 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} \\
& + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25} \\
& + 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} \\
& + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45,} \\
subject\ to\quad & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5, \\
& x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4, \\
& x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2, \\
& x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9, \\
& x_{11} + x_{21} + x_{31} + x_{41} = 4, \\
& x_{12} + x_{22} + x_{32} + x_{42} = 4, \\
& x_{13} + x_{23} + x_{33} + x_{43} = 6, \\
& x_{14} + x_{24} + x_{34} + x_{44} = 2, \\
& x_{15} + x_{25} + x_{35} + x_{45} = 4, \\
& x_{ij} \ge 0, \quad \forall i = 1,2,3,4, \quad j = 1,2,3,4,5.
\end{aligned}$$

4.4.2 Results and Discussions of (MOT) Test Problem

Similar to the (MOT) test problem 1, the weighting approach is used together with the trust-region algorithm (3.1) to solve the above problem and discuss the effects of changing weights on it. As one weight is changed linearly in each case, the other two weights are generated randomly, such that $\sum_{\hat{k}=1}^{3} w^{\hat{k}} = 1$ and $w^{\hat{k}} \ge 0$ for all $\hat{k} = 1,2,3$. The values of the weights which are used for three cases are illustrated in three tables (2-4).

Figures (2-4), show the objective functions obtained from six solutions corresponding to the six weights compared to the weights for three cases. We observe that the best compromise solutions are $f_1 = 144$, $f_2 = 104$, and $f_3 = 73$ which are occur at $w^1 = w^2 = w^3 = 0.6$.

When we compare the results of our suggested approach and the results of researchers (Interactive approach [2] and Fuzzy approach [1]) who have used other approaches it becomes evident from table (5) that the value of $f_3 = 73$ is the best result, the value of $f_2 = 104$ is in agreement with the value obtained by using the Interactive approach [2] and better than the result of Fuzzy approach [1], whereas the value of $f_1 = 144$, is comparatively higher than the results obtained by other approaches.

Table 2. Different weights (w^1 is changed linearly)

Run	w^1	w^2	w^3
1	0.0000	0.5721	0.4279
2	0.2000	0.6205	0.1795
3	0.4000	0.2118	0.3882
4	0.6000	0.1636	0.2364
5	0.8000	0.1759	0.0241
6	1.0000	0.0000	0.0000

Table 3. Different weights (w^2 is changed linearly)

Run	w^1	w^2	w^3
1	0.6028	0.0000	0.3972
2	0.5676	0.2000	0.2324
3	0.4573	0.4000	0.1427
4	0.2718	0.6000	0.1282
5	0.1468	0.8000	0.0532
6	0.0000	1.0000	0.0000

Table 4. Different weights (w^3 is changed linearly)

Run	w^1	w^2	w^3
1	0.7477	0.2523	0.0000
2	0.5994	0.2006	0.2000
3	0.4576	0.1424	0.4000
4	0.1354	0.2646	0.6000
5	0.1076	0.0924	0.8000
6	0.0000	0.0000	1.0000

Table 5.Comparison between different approaches.

The name of approach	f_1	f_2	f_3
Fuzzy approach [1]	112	106	80
Interactive approach [2]	127	104	76
Proposed approach	144	104	73

5. Conclusions

In the present work we propose a new approach by using the trust-region globalization strategy to solve a multi-objective transportation (MOT) problem, which is interested to many researchers and several local methods have been proposed to solve it. Globalizing strategy means modifying the local method in such a way that it is guaranteed to converge at all even if the starting point is far away from the solution. The trust-region strategy for solving (SECOP) and unconstrained optimization problem has proved to be very successful, both theoretically and practically.

A weighting approach is used together with an active set strategy and a multiplier method to transform (MOT) problem to unconstrained optimization problem and we used a trust-region algorithm to solve it.

We have arrived at the conclusion that this new numerical technique has shown itself to be suitable for the numerical and parametric study of (MOT) problem after having been tested in the work with two test problems. Also, this approach consider as interactive approach, because it allows the decision maker to specify the weights of the criterion importance which show the degree to which the objectives have been satisfied. Finally, the success of our approach on most of the test problems not only provides confidence, but also stresses the importance of numerical parametric studies to investigate the best weighting values of each objective function which leads directly to the best compromise solution in solving multi-objective transportation problems.

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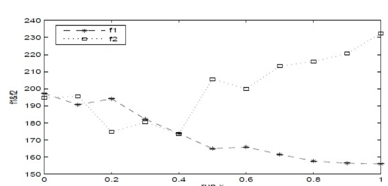

Figure 6.1: Plot showing the values of f_1 and f_2 as w_1 or w_2 changes linearly.

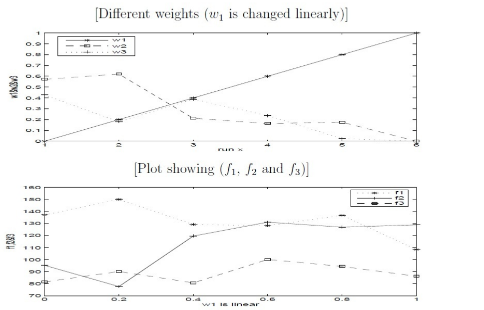

Figure 6.2: Plot showing the values of f_1, f_2 and f_3 solution for different weights in 6 runs of table 2.

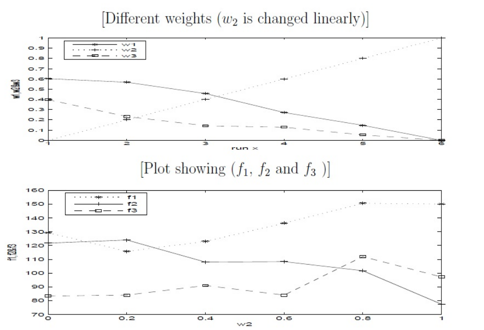

Figure 6.3: Plot showing the values of f_1, f_2 and f_3 solution for different weights in 6 runs of table 3.

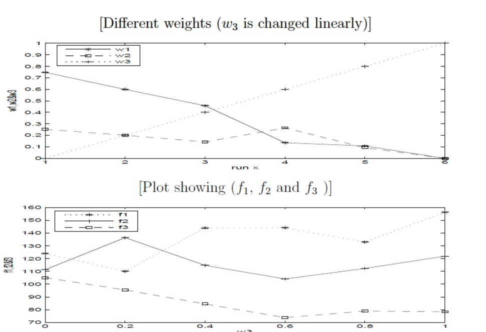

Figure 6.4: Plot showing the values of f_1, f_2 and f_3 solution for different weights in 6 runs of table 4.

Corresponding author
Bothina El-Sobky
Department of Mathematics, Faculty of Science, Alexandria University, Egypt

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