# A Novel Nanometric Reversible Four-bit Signed-magnitude Adder/Subtractor 

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#### Abstract

The reversible logic plays a significant role in the synthesis of circuits for quantum computing. Reversible gates have been widely used in low power CMOS design, optical information processing, and bioinformatics, quantum computing and nanotechnology-based systems. A new $3 \times 3$ reversible two's complement gate is suggested in this paper. Two quantum models are offered for two's complement gate. These quantum models differ from each other with respect to the quantum cost. Two novel reversible four-bit signed-magnitude adders/subtractors with HNG and ADD/SUB gates are also proposed for the first time. The proposed circuits detect overflow and produce a correct result for inputs in the range of [-7, 7]. The proposed two's complement gate is used in part of the reversible four-bit signed-magnitude adder/subtractor design. The proposed reversible 4-bit signed-magnitude adders/subtractors are evaluated in terms of number of reversible gates, number of garbage outputs, number of constant inputs, quantum cost and hardware complexity. All the scales are in the nanometric area. [Boroumand S. A Novel Nanometric Reversible Four-bit Signed-magnitude Adder/Subtractor. Life Sci J 2012;9(3):1646-1655]. (ISSN: 1097-8135). http://www.lifesciencesite.com. 240


Key words: nanotechnology based systems; quantum computing; reversible logic; reversible signed-magnitude adder/subtractor.

## 1. Introduction

One of the most prominent incentives for the study of reversible computing comes from the desire to reduce heat dissipation in computing machinery, and thus achieve higher density and speed (Toffoli, 1980). Reversible computing, in a general sense, means computing using reversible operations, that is, operations that can be easily and exactly reversed, or undone. Reversible operations realize bijective Boolean functions. Conventional gates such as AND, XOR, OR, etc., that are used in digital design are not reversible. Landauer (1961) proved that classical non-reversible circuits inevitably produce heat because of losses of information during the computation. Landauer's principle (Landauer, 1961) states that generated heat for each bit of information lost is KTln 2 joules of energy, Where $\mathrm{K}=$ $1.3806505 * 10^{-23} \mathrm{~m}^{2} \quad \mathrm{~kg}^{-2} \quad \mathrm{~K}^{-1} \quad$ (joule/Kelvin) is Boltzmann constant and T is the temperature in Kelvin degrees at which operation is performed. Bennett's (Bennett, 1973) theory was developed to compensate energy dissipation. Bennett's theorem suggests that every future binary technology will have to use some kind of reversible gates in order to reduce heat dissipation (Mozammel, 2008). Bennett (Bennett, 1973) showed that zero energy dissipation is possible in Reversible logic computing, that is a reversible logic gate has a one-to-one mapping between the inputs and outputs. Some restrictions of reversible circuits include Fan-out and loop or feedback. Recently, researchers illustrated that feedback is allowed in reversible computing in a case of sequential circuit's design (Thapliyal and

Ranganathan, 2010). Quantum technology is inherently reversible and it will become very important for future computing systems (Mozammel, 2008). In reversible logic the output logical states uniquely define the input logical states of the computational operation. Reversible logic circuits encompass the same number of input and output lines. Such circuits (gates) can produce inputs from the corresponding outputs and vice versa. On the other hand, the digital computer is a digital system that performs various computational tasks by arithmetic circuits such as Adders, Subtractors, Multipliers and Dividers from which Adders and Subtractors are the most essential blocks of a computing system. So some reversible Adders and Subtractors for unsigned numbers have been proposed in (6-14) and only one paper (Emam and Elsayed, 2010) has been proposed so far on reversible Adder/Subtractor for signed/unsigned binary numbers. The author in (Emam and Elsayed, 2010) proposed a circuit for addition and subtraction signed numbers represented in two's complement representation and no circuit for addition and subtraction signed numbers represented in the signedmagnitude representation so far, has been designed. For floating-point operations, most computers use the signed-magnitude representation for the mantissa, so addition and Subtraction floating-point numbers needed signed-magnitude Adder/Subtractor. Thus, in this article a circuit is proposed for Addition and Subtraction signed numbers represented in the signed-magnitude representation.

The paper is structured around the following sections: section 2 discusses the proposed reversible gate and some necessary reversible logic gates. Section 3 provides essential background on irreversible signed-magnitude full adder/subtractor. The proposed four-bit reversible signed-magnitude adders/subtractors with HNG and ADD/SUB gates and different parts of the proposed reversible signedmagnitude Adder/Subtractor are described in section 3 too. The simulation results with VHDL language and Quartus simulator is shown in section 4 and conclusions are contained in section 5.

## 2. Reversible Gates

The quantum $\operatorname{cost}(\mathrm{QC})$ of any reversible gate(circuit) is the number of $1 \times 1$ or $2 \times 2$ reversible gates and quantum logic gates such as $\mathrm{V}, \mathrm{V}^{+}(\mathrm{V}$ is also named square root of NOT gate $\sqrt{N O T}$ and $\mathrm{V}^{+}$ is hermitian of V ). The V and $\mathrm{V}^{+}$quantum gates have some properties that are shown in Equation (1) (Mohammadi et al., 2009).

$$
\text { (1) }\left\{\begin{array}{c}
V \times V=N O T \\
V \times V^{+}=V^{+} \times V=I \\
V^{+} \times V^{+}=N O T
\end{array}\right\}
$$

Any reversible logic gate (circuit) is realized by using mentioned gates above, NOT and FG gates. This section proposes a new reversible logic gate which is termed two's complement gate as shown in Figure 1, and its truth table is presented in Table 1. Then this section introduces NLG and ADD/SUB gates. For aware of other utilized reversible gates such as NOT, FG, F2G, FRG, HNFG, HNG, PG you can refer to (13-19) references.

## Two's complement Gate

A new $3 \times 3$ reversible logic gate is 2 's complement gate. This gate is very important in circuits design because Complements are used in digital computers for logical operation and simplifying the subtraction function. The proposed reversible 2's complement gate is depicted in Figure 1. The 2's complement gate can be represented as:
$I_{v}=(A, B, C)$
$O_{v}=(P=A, Q=A \oplus B, R=A \oplus B \oplus C \oplus A B)$


Figure 1. The proposed Two's Complement Gate


Figure 2. Characteristics of a self-inverse gate
Reversible 2's complement gate is a one through gate, and its truth table is shown in table 1. According to this table for every input pattern there is a unique output pattern. Reversible 2's complement gate is also a self-inverse gate. A function is selfinverse, If an input x into the function $f$ produces an output y , then putting y into the inverse function $f^{-1}$ produces the output $x$, and vice versa. Figure 2 shows the characteristics of a self-inverse gate.

Table 1. Truth table of 2's complement gate

| $\mathbf{C}$ | $\mathbf{B}$ | $\mathbf{A}$ | $\mathbf{R}$ | $\mathbf{Q}$ | $\mathbf{P}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |

Two quantum models are proposed for this gate which is demonstrated in Figure 3. The quantum cost of reversible 2's complement gate is 6 in Figure 3a. The QC is 5, in Figure 3b. The optimized QC is 5 for this gate.

Where, $\mathrm{I}_{\mathrm{V}}$ and $\mathrm{O}_{\mathrm{V}}$ are the input and output vectors.

(a)

(b)

Figure 3. Two quantum models for Two's Complement Gate

## NLG Gate

NLG gate is abbreviation for new reversible logic gate that has been proposed in (Lihui et al., 2010) for the first time. NLG gate is a one through gate as shown in figure 4. Quantum model and Quantum cost of this gate has not been reported in
(Lihui et al., 2010). So this article proposes two quantum models for this gate which are demonstrated in figure 5. Figure 5a uses quantum gates V and $\mathrm{V}^{+}$ and Figure 5 b uses FG and NOT gate. The quantum cost is four in Figure 5a and it is one in Figure 5b which is the lowest quantum cost.


Figure 4. NLG gate

(a)

(b)

Figure 5. Proposed equivalent quantum representation of NLG gate

## ADD/SUB Gate

$\mathrm{ADD} / \mathrm{SUB}$ gate is a reversible $4 \times 4$ gate. It has been proposed in (Emam and Elsayed, 2010) for the first time. This gate can be represented as:

$$
\begin{aligned}
& I_{v}=(F, A, B, C) \\
& O_{v}=(P=A \oplus B \oplus C, Q=C / B, R=A, S=F \oplus B)
\end{aligned}
$$

Where $\mathrm{I}_{\mathrm{V}}$ and $\mathrm{O}_{\mathrm{V}}$ are the input and output vectors. If $F$ input is set to zero then the ADD/SUB gate will work as a reversible full adder or else the gate will work as a reversible full subtractor as shown in figures $6 b$ and $6 c$ respectively.


Figure 6. Reversible ADD/SUB gate (a) symbol, (b) Full Adder, (c) Full Subtractor

## 3. Basic Concepts

There are two different common forms of representation for negative fixed-point numbers in a radix $r$ system.

1. Sign and magnitude representation, which is also called the signed-magnitude method.
2. Complement representation which comprises two alternatives:
(I) Two's complement in the binary system
(II) One's complement in the binary system

When an integer binary number is positive, the sign is represented by 0 and the magnitude by a positive binary number. When the number is negative, the sign is represented by 1 but the rest of the number may be represented in one of the three aforementioned ways (signed-magnitude, one's complement and two's complement representation).

Reversible Signed numbers Adder/Subtractor circuit design based on these methods will be different. In this article the main opinion is designing reversible signed-magnitude adder/subtractor. The algorithm for signed-magnitude adder/subtractor is more complex than addition and subtraction numbers in signedcomplement system, because not only it needs circuits to add and subtract but also its implementation needs circuits to compare the signs and the magnitudes of the numbers. The range of $n$ bit integer numbers in the signed-magnitude system is symmetric and equals
$-\left(2^{n-1}-1\right) \leq X \leq\left(2^{n-1}-1\right)$
So the range of 4 bits integer numbers in the signed-magnitude system is $[-7,7]$. The leftmost bit of the signed-magnitude number shows the number sign. As a result, there are two representations of
zero, a positive zero represented by $000 \ldots 0$, and a negative zero represented by $100 \ldots 0$, but in addition and subtraction for signed-magnitude numbers only positive zero is produced, so this is one advantages of the proposed design. The signed-magnitude adder/subtractor flowchart is presented in figure 7. A, $B$ determine the magnitude of the two numbers and $A_{s}, B_{s}$ show the signs of $A$ and $B$ respectively. According to the flowchart, $\mathrm{A}_{\mathrm{s}}$ and $\mathrm{B}_{\mathrm{s}}$ determine addition or subtraction operation. If $A_{S} \oplus B_{S}=0$ then the signs will be identical otherwise the signs will be different. Depending on the selected operation, adding or subtracting, the micro operations $E S \leftarrow A+B$ or $E S \leftarrow A-B$ is done (subtraction with two's complement method). E indicates the output carry when A and B are added. An overflow may occur if the two numbers added are both positive and both negative. So if the carry in E is equal to one then the overflow will occur. Since the addition and subtraction of the four bit signed-magnitude numbers are covered in the range of [-7, 7], the final result is considered 5 bits (because of the overflow). According to the existence flowchart in figure 7, If the micro operation $E S \leftarrow A+\bar{B}+1$ is done and E is equal to one $(\mathrm{E}=1)$ then A will be larger than or equal to $B(A \geq B)$. In this case if $S=0$ then $A$ will equal to $B$ and the sign $\mathrm{S}_{\mathrm{s}}$ must be positive to avoid a negative zero, otherwise $A$ will be larger than $B(A>B)$ and the number in $S$ is the correct result. If $E$ is equal to zero $(\mathrm{E}=0)$ then A will be less than $\mathrm{B}(\mathrm{A}<\mathrm{B})$, thus the sign $\mathrm{S}_{\mathrm{s}}$ is equivalent to complement the sign of A and the correct result is the two's complement of the value in S (Koren, 1945; Mano, 2001).

### 3.1 Proposed Design

Figure 8 shows the traditional block diagram of the proposed design. This block Adds/Subtracts two signed-magnitude four bit numbers. It has nine inputs consisting of A and B inputs that each are three bits and two signed bits $\mathrm{A}_{\mathrm{S}}$ and $\mathrm{B}_{\mathrm{S}}$, and $\mathrm{C}_{\mathrm{F} / \mathrm{S}}$ signal is also used as the control input. If $\mathrm{C}_{\mathrm{F} / \mathrm{S}}=0$, the operands are added otherwise they are subtracted. This block diagram has also four outputs consisting of $\mathrm{S}_{\mathrm{S}}$ which is considered as a signed bit for output bits and $E, S_{0}, S_{1}$ and $S_{2}$ outputs. Since the addition and subtraction of the four bit signed-magnitude numbers are covered in the range of [-7, 7], the final result is considered 5 bits. This article based on this block diagram and aforesaid flowchart in previous section proposes a Four-bit Reversible Signedmagnitude Full Adder/Subtractor. Therefore in next
sections, necessary parts are introduced for designing this circuit.

### 3.2 The proposed Circuit for Input Ctrl Signal of Reversible Ripple Adder/Subtractor with HNG gate

Two numbers are added If the signs are identical for an add operation or different for a subtract operation. If the signs are dissimilar for an add operation or similar for a subtract operation then two numbers will be subtracted. The existing circuit in figure 9 implements aforementioned characteristics and the output is the control input of the reversible ripple adder/subtractor. This proposed circuit and quantum representation are depicted in figure 9 a and 9 b respectively.


Figure 7. Flowchart for signed-magnitude adder/subtractor


Figure 8. Block Diagram of The Irreversible Four-bit Signed-magnitude Full Adder/Subtractor


Figure 9. Two symbols of Proposed Circuit For Control Input

The existence circuit in figure 9 has three outputs consist of two garbage outputs and Ctrl output that is connected to control input of the reversible ripple adder/subtractor. If Ctrl $=0$, the operands are added otherwise they are subtracted (two' s complement method). Reversible Four-bit Signed-
magnitude Adder/Subtractor design requires a three-bit reversible two's complement adder/subtractor. The reversible two's complement adder/subtractor with proposed circuit for Ctrl signal and its quantum implementation are shown in figure 10 and 11 respectively.


Figure 10. The reversible adder/subtractor with proposed Ctrl signal

As can be seen in figure 10 if $\mathrm{Ctrl}=0$, then the micro operation $S=A+B$ will be performed otherwise the micro operation $S=A+\bar{B}+1$ will be performed. The existing design in figure 10 has 9
garbage outputs and 4 constant inputs with $\mathrm{QC}=24$. The existing design in figure 11 requires 6 garbage outputs and 3 constant inputs with $\mathrm{QC}=23$.


Figure 11. Quantum implementation of reversible adder/subtractor with proposed Ctrl signal

### 3.3 Proposed Circuit for $A=B$ State

If Ctrl signal is set to one then micro operation $E S \leftarrow A+\bar{B}+1$ will be done. In this case if $E$ is equal to one and the number in $S$ is equal to zero then A will be equal to B. Therefore in this section a reversible circuit is proposed to compare the existing number in S with zero. Thus, when A is equal to B , the number in S is the correct result, but the sign $\mathrm{S}_{\mathrm{s}}$ must be positive to avoid a negative zero. For implementation circuit that compares the result with zero, we can use the following equation:
(2) $\left(S_{2} \odot 0\right)\left(S_{1} \odot 0\right)\left(S_{0} \odot 0\right)=1$

The proposed circuit Based on the equation (2) is demonstrated in figure 12. This proposed circuit have been designed using NLG and PG gates with $\mathrm{QC}=11$. It needs 5 constant inputs, 7 garbage outputs and 5 gates. DC inputs and outputs are an important figure of merit to evaluate a design. In this research, the quantum model is proposed to optimize garbage inputs and outputs. As it is shown in figure 12 b , the quantum circuit has 2 DC inputs, and 4 DC outputs and 5 gates. The quantum cost of the circuit is also 11 .


Figure 12. The reversible circuit for comparing the 3 bit number with zero

## 3.4 investigating Circuit In $A>B$ State

If Ctrl signal is set to one then micro operation $E S \leftarrow A+\bar{B}+1$ will be done. In this case if E is equal to one and the number in S is opposite of zero then $A$ will be larger than $B$. Therefore the number in $S$ is the correct result and the sign of the result is the same as the sign of A , so no change in $\mathrm{A}_{S}$ is required. Thus it is required to use multiplexer $2 \times 1$ in order to select the sign of $\mathrm{S}_{\mathrm{S}}$ between two states $\mathrm{A}=\mathrm{B}$ and $\mathrm{A}>\mathrm{B}$. Fredkin gate is applied as a multiplexer in figure 13. Note that this reversible multiplexer has 3 inputs ( $0, \mathrm{~A}_{\mathrm{S}}$ and control) and one main output and 2 garbage outputs. $\mathrm{F}(\mathrm{A}=\mathrm{B})$ signal in figure 12 is used as the control input for multiplexer. The reversible multiplexer applies 0 for $A=B(S=0)$ state or $A_{S}$ for $A>B(S \neq 0)$ state.


Figure 13. FRG as a MUX $2 \times 1$ for implementation sign bit

## 3.5 investigating Circuit In $\mathbf{A}<\mathbf{B}$ State

If the $E S \leftarrow A+\bar{B}+1 \quad$ operation is performed and E is equal to zero then A will be less than B. In this case the two's complement of the value in S must be obtained to create the correct result. In the proposed circuit this operation is done
by using Two's complement gate. In this case the sign of result is the complement of the original sign of A , so $\mathrm{A}_{\mathrm{S}}$ is complemented by using NOT gate for obtaining the correct sign.

### 3.6 Proposed Four-bit Reversible Signedmagnitude Adder/Subtractor with HNG gate

Four-bit reversible signed-magnitude adder/subtrator is proposed in figure 14. The circuit function is described, according to the existing flowchart in Figure 7 and table 2.

Table 2. Determining operations and carry out in
Adder/Subtractor

| ctrl | E |
| :---: | :---: |
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

The proposed design utilizes two FRG gates ( $\mathrm{F} 1 \& \mathrm{~F} 2$ ) as a multiplexer $4 \times 1$ to determine the sign S and also four FRG gates (F3, F4, F5 \& F6) to distinguish the final result (ES). In the first and second rows of Table 2, the Ctrl signal is zero and $\mathrm{A}_{S}$ passes to the second output of FRG (F2) gate $\left(\mathrm{S}_{\mathrm{S}}\right)$, and when the Ctrl signal is set to one in the next rows, depending on the value of E , signal $\bar{A}_{S}$ or $s_{S}^{*}$ passes to the second output of FRG (F2) gate ( $\mathrm{S}_{\mathrm{S}}$ ).


Figure 14. Proposed Four-bit Reversible Signed-magnitude Full Adder/Subtractor(design I)

In proposed Four-bit Reversible Signedmagnitude Full Adder/Subtractor, when $\mathrm{Ctrl}=0$ overflow may occur, therefore because of this overflow, FRG gate (F3) is considered. It means that F3 is used for detecting overflow and obtaining the correct result for inputs in the range of $[-7,7]$. K input is a control signal for three FRG gates (F4, F5 \& F6) which are located in the right side and down position of the proposed circuit. If the K input is zero then the result $\left(\mathrm{S}_{2} \mathrm{~S}_{1} \mathrm{~S}_{0}\right)$ is equal to the number in S ; otherwise, it is Two's complement of the value in S . logical equation (3) can be written for control K signal.
(3) $K=C t r l \times \bar{E}$

According to the equation $3, \mathrm{~K}$ is only one in 10 state of the Table 3 and it is zero in rest of states.
3.7 Proposed Four-bit Reversible
magnitude Adder/Subtractor with ADD/SUB Gate

This circuit(design II) has been implemented with ADD/SUB gates instead of HNG gates, so
subtraction operation is not performed using two's complement method. The borrow output of this design is different from previous design. In this circuit if $E$ is equal to one then $A$ will be less than $B$ otherwise $A$ will be larger than or equal to $B$ therefore the K control input is changed and obtained from equation (4).
(4) $K=\overline{(C t r l \times E)}$

On the other hand, inputs of F1 in previous design is swapped, it means that in $\mathrm{Ctrl} \mathrm{E}=10$ state the $s_{s}^{*}$ signal and $\mathrm{Ctrl} \mathrm{E}=11$ state $\bar{A}_{s}$ signal stand in final sign bit. Proposed Four-bit Reversible Signedmagnitude Adder/Subtractor with ADD/SUB Gates is shown in figure 15. A comparison between the two circuits for the Reversible Signed-magnitude Full Adder/Subtractor is shown in table 3.

Table 3. Comparison between Design I and design II

|  | NO. of <br> gates | NO. of Constant <br> inputs | NO. of Garbage <br> outputs | Quantum Cost | Total Logical Calculation |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Design I Figure 14 | 26 | 17 | 21 | 84 | $53 \alpha+38 \beta+14 \gamma$ |
| Design II Figure <br> 15 | 23 | 17 | 21 | $65+3(\mathrm{QC}(\mathrm{ADD} / \mathrm{SUB}))$ | $55 \alpha+36 \beta+14 \gamma$ |



Figure 15. Proposed Four-bit Reversible Signed-magnitude Full Adder/Subtractor(design II)

Quantum model and Quantum cost of $\mathrm{ADD} / \mathrm{SUB}$ gate has not been reported in (Emam and Elsayed, 2010), but according to its hardware complexity, the quantum cost of this gate is greater than HNG gate. The reversible circuit in design II is better than design I in terms of number of gates and design I is optimized in terms of quantum cost and hardware complexity in comparison with the reversible Signedmagnitude Full Adder/Subtractor in design II.

## 4. Simulation Results

Reversible proposed two's complement gate, Four-bit Signed-magnitude Adder/Subtractor with

HNG gates and Four-bit Signed-magnitude Adder/Subtractor with ADD/SUB gates are implemented using VHDL code and simulated using Quartus Simulator. The proposed circuits are coded using Structural style. In this style, components are as the main blocks, it means that every reversible gate is a component. Simulation results of two's complement gate, Four-bit Signed-magnitude Adder/Subtractor with HNG gates and Four-bit Signed-magnitude Adder/Subtractor with ADD/SUB gate are shown in figures $16,17,18$.


Figure 16. Simulation result of reversible two's complement gate


Figure 17. Simulation result of reversible Four-bit Signed-magnitude Adder/Subtractor with HNG gate


Figure 18. Simulation result of reversible Four-bit Signed-magnitude Adder/Subtractor with ADD/SUB gate

## 5. Conclusions and Future Works

In this paper, a new $3 \times 3$ reversible two's complement gate is presented. Proposed reversible two's complement gate can be used in implementation of the signed-magnitude Adder/Subtractor. For addition and Subtraction floating-point numbers is required to use signedmagnitude Adder/Subtractor because the mantissa in floating point is shown in signed-magnitude representation, Thus two designs for Reversible Four-bit Signed-magnitude Adder/Subtractor circuit has been proposed for the first time. Table 3 shows the results of two proposed schemes. According to obtained result from table 3 the first design with HNG gate is more optimal than second design with ADD/SUB gate in terms of Quantum cost and hardware complexity. If in designing circuit, minimum quantum cost was important then design I would be appreciate and if number of gates was important then design II must be used. Finally, the proposed circuits have been implemented using VHDL code and simulated using Quartus Simulator and obtained results of simulation shows the correct operation of circuits. This work is preface to design
more complex and efficient signed-magnitude adder/subtractor.

As future works, some optimization techniques such as genetic algorithm may be used to reduce the quantum cost of the circuits. The proposed reversible Four-bit Signed-magnitude Adder and Subtractor can be generalized for reversible n-bit Signed-magnitude Adder/Subtractor. All the circuits have nanometric scales.

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