Overcoming Actuators Saturation Problem in Structural Active Control

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Abstract: Since the actuator capacity is limited, in the real application of active control systems under severe earthquakes, it is conceivable that the actuators saturate, hence the actuator saturation should be considered as a constraint in the design of optimal controllers. In this paper, a new procedure for structural active control is proposed to overcome the actuators saturation. This approach is based on elimination of structural response as early as possible to save the high control force required later due to the response generated from the small recent response taking into consideration the actuator capacity. The proposed approach is formulated and applied to single and multi-story buildings subjected to ground motion. Two types of ground excitations are considered. The first is sinusoidal and in resonance with building. The second type of excitation represented by several real earthquakes. The proposed approach is compared with the traditional optimal control in two manners, when the maximum control force in the two approaches is maintained the same and when the response reduction level is the same. It is found that the proposed approach does not only overcome the actuator saturation, but it also reduces the response for all cases considered, namely, single or multi-story building, light or heavy damped structures, and when buildings are subjected to sinusoidal or real ground motion.

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Nomenclature

[M]: mass matrix of the structure [C]: damping matrix of the structure [K] : stiffness matrix of the structure

{y} : displacement response vector of the structure $\{\dot{\mathbf{y}}\}$: velocity vector of the structure

$\{\ddot{\mathbf{y}}\}$: acceleration vector of the structure {m}: mass vector of the structure $\ddot{\mathbf{y}}_{\mathbf{g}}$: ground motion acceleration,

{b}: control force location vector u: control force

1. Introduction

During the last three decades, various methods have been developed and applied to suppress structural vibration in numerous areas. Vibration control of lightly damped and flexible structures such as high-rise buildings, long-span bridges, and so on has been widely researched. To improve their inherent low damping ratios, various kinds of passive, active, and semi-active devices have been developed and applied to many real-world structures. The idea of applying active control as a means of hazard reduction has become increasingly popular [1, 2]. In the design of an active control system, the objective is to reduce the structural response (accelerations, velocities, and displacements) under the limitation of both the control force level (limited by the actuator capability and the required amount of energy) and the number of measured signals. In active control of real-world building structures, active tendons, active mass damper, active tuned mass damper, and hybrid mass damper have been widely used and verified as promising devices [3- 5].

One matter that is encountered in controlling structures is the actuator saturation problem, which has been a topic of interest over the past several years [6–11].

Actuator saturation may lead to instability, and may also lead to serious deterioration in the performance of the closed-loop system. Numerous proposed solutions to the actuator saturation problem include Riccati and Lyapunov-type local and semi-global stabilization methods, the anti-windup technique, and absolute stability theory. In recent research [6,12–14], methods that can employ large gains of control input and provide guaranteed performance levels were developed.

Most controllers have been designed based primarily on linear control theories like optimal linear quadratic regulator formulation, H_2 control technique, direct velocity feedback method [15- 17]. However, linear control theories may not be effective in producing significant peak response reduction which is practically important for buildings safety. In addition, in some approaches the response of the controlled system exceeds the uncontrolled system. In order to overcome these problems of linear controllers and saturation, a new approach is developed. This approach is summarized in that it is preferred to eliminate the structure response as early as possible by exerting control force within its limit because any small response may cause later higher response which require more control force. The new approach is applied to single story structures controlled actively by tendons and the results are compared with that of the traditional optimal linear approach. The approach is extended to apply on the multi-story structures.

2. THEORY:

2.1. Optimal active control approach:

The equations of motion of an active controlled multi-story structure shown in Figure 1, when subjected to ground motion are given by:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-\{m\}\ddot{y}_g+\{b\}u \quad (1)$$

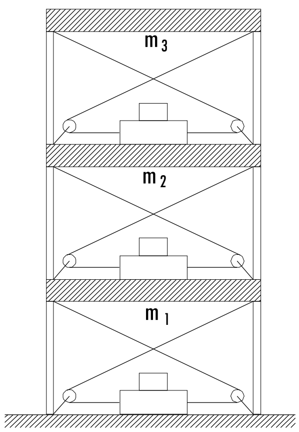

Figure 1. Controlled Structure

The response vector {y} is given by;

$$\{y\}=[\mathbf{y_1} \quad \mathbf{y_2} \quad \cdots \quad \mathbf{y_n}]^{\mathbf{T}} \quad (2)$$

where y_i is the displacement response of the ith level.

The equations of motion can be recasted in the *State Space Formulation* as follow;

$$\dot{\mathbf{X}}=\mathbf{A\,X}+\mathbf{B_u\,u}+\mathbf{B_f F} \quad (3)$$

where {X} is the state vector which defined as

$$\mathbf{X}=\left\{\begin{array}{c}\mathbf{y}\\ \hline \dot{\mathbf{y}}\end{array}\right\}$$

and the matrices A, B_u, B_f and F are given by:

$$\mathbf{A}=\left[\begin{array}{c|c}\mathbf{0} & \mathbf{I}\\ \hline -[\mathbf{M}]^{-1}[\mathbf{K}] & -[\mathbf{M}]^{-1}[\mathbf{C}]\end{array}\right],\mathbf{B_u}=\left\{\begin{array}{c}\mathbf{0}\\ \hline \mathbf{b}\end{array}\right\},$$

$$\mathbf{B_f}=\left\{\begin{array}{c}0\\ \hline \{-1\}_{n\times1}\end{array}\right\} \text{ and } \mathbf{F}=\{\ddot{\mathbf{y}}_g\}_{1\times1}$$

The output response can be expressed using the state vector as follows;

$$\mathbf{Y}=\mathbf{H\,X}+\mathbf{D\,F} \quad (4)$$

where {Y} is the output response vector containing y, $\dot{\mathbf{y}}$ and $\ddot{\mathbf{y}}$ as follows;

$$\mathbf{Y}=\left\{\begin{array}{c}\mathbf{y}\\ \hline \dot{\mathbf{y}}\\ \hline \ddot{\mathbf{y}}\end{array}\right\} \quad (5)$$

and the matrices H and D are given by;

$$\mathbf{H}=\left[\begin{array}{c|c}\mathbf{I} & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{I}\\ \hline -[\mathbf{M}]^{-1}[\mathbf{K}] & [\mathbf{M}]^{-1}[\mathbf{C}]\end{array}\right], \textbf{and } \mathbf{D}=\left\{\begin{array}{c}\mathbf{0}\\ \hline \mathbf{0}\\ \hline -\mathbf{1}_{n\times1}\end{array}\right\}$$

In linear optimal control theory, the control force (u) is represented as a linear function of the state vector X, i.e.

$$\mathbf{u}=-\,\mathbf{G\,X} \quad (6)$$

and the constant time invariant gain vector (G) is obtained by minimizing the quadratic objective function (J), where J is given by;

$$\mathbf{J}=\int_0^\infty\left(\mathbf{x}^*\mathbf{Q\,x}+\mathbf{u}^*\mathbf{R\,u}\right)\,\mathbf{dt} \quad (7)$$

where Q is a positive-definite (or positive-semi-definite) Hermitian or real symmetric matrix and R is a positive-definite Hermitian or real symmetric matrix and they consider the relative importance factors for controlling the response and the control force.

The gain vector is given by [18] :

$$\mathbf{G}=\mathbf{R}^{-1}\mathbf{B}^*\;\mathbf{P} \quad (8)$$

where the matrix P is obtained using the Riccati equation ;

$$\mathbf{A}^*\,\mathbf{P}+\mathbf{P\,A}-\mathbf{P\,B\,R}^{-1}\,\mathbf{B}^*\;\mathbf{P}+\mathbf{Q}=\mathbf{0} \quad (9)$$

2.2. The proposed approach:

The state space form of the equation of motion (Eq. 3) can be converted to discrete system and solved numerically as follow;

$$\mathbf{X_{k+1}}=\mathbf{e}^{\mathbf{A\,dt}}\mathbf{X_k}+\left(-\mathbf{B_u\;u_k}+\mathbf{B_f F_k}\right)\mathbf{dt} \quad (10)$$

The negative sign before the control force is due to the fact that its direction is opposite to the motion direction.

To eliminate the response in the next step, the left hand side of equation (10) should be zero, then the control force (u_k) can be calculated from the following equation;

$$\mathbf{u_k}=\mathbf{B_u^*}\left[\frac{\mathbf{1}}{\mathbf{dt}}\mathbf{e}^{\mathbf{A\,dt}}\mathbf{X_k}+\mathbf{B_f F_k}\right] \quad (11)$$

where $\mathbf{B_u^*}$ is the Moore-Penrose pseudoinverse of the matrix (Bu).

The calculated control force may be within its limit, then applying this force will eliminate the response as possible as in the next step. If the calculated control force exceeds its limit, the control force is applied as its upper limit and the response is calculated according to Equation (10).

Two Matlab programs are developed. The first follows the traditional optimal active control approach when variable weighting matrices are available. The second program apply the proposed approach to the same structures in two cases, the first case when the control force limit is adjusted as the maximum force obtained from the traditional approach to show that the same control force is reduced drastically in the structure response for the proposed approach. Then many indices are constructed as follow:

First Response Index (RI1): the ratio between the peak response of the traditional controlled and the uncontrolled.

Second Response Index (RI2): the ratio between the peak response of the proposed controlled and the uncontrolled.

Third Response Index (RI3): the ratio between the peak response of the proposed controlled and the traditional controlled.

Control Force Index (CFI): the ratio between the peak control force and the structure weight.

The second case when the same response reduction is adjusted to compare the control force and compare also the quadratic objective function (J). Then a control force index (CFI2) is constructed as a ratio between the control force of the proposed controlled and that of the traditional controlled. Also quadratic objective function index (JI) is created as ratio between the quadratic objective value of the proposed controlled and that of the traditional controlled. These indices are obtained for a constant level of response reduction (RI).

3. IMPLEMENTATION AND RESULTS

3.1. Single story structure:

A single story structures have a varied dynamic properties as shown in Table 1. These properties are selected to cover wide range of structures. The structures are controlled by the two approaches when subjected to many sinusoidal ground motion records with resonance frequencies of amplitude 0.3 g. The weighting matrices in the traditional control approach is adjusted as $Q_{ij}=0$; $Q_{ii}=100$; and R=0.1. The response indices are shown in Table 2. It is shown that the proposed approach is better than the traditional approach especially in case of flexible structures than the rigid ones, and in case of light damped than the heavy ones. This conclusion is shown in Figure 2, where the third response index (RI3) is plotted for all structures.

To emphasize the benefit of the proposed approach in reducing the control force, the eight selected structures are controlled by the two approaches to maintain the same level of response reduction. The control force is calculated by the two approaches. Table 3 shows the control force index and the quadratic objective function index when the response reduction level is adjusted to 0.1. It is clear that the proposed approach leads to lower control force (about 77 %) and in most cases lower quadratic index. The value of the quadratic objective function index (JI) less than unit means that the proposed approach is better than the traditional approach from the view point of traditional approach. This also means that the procedure that follow in optimal control approach does not lead to the optimal solution because there is another approach get better solution.

Table 1. Dynamic Properties of the Studied Structures

Structure No	Mass (kN/m.sec2)	Natural Frequency	Damping ratio
1	10	5	2
2	10	5	5
3	10	10	2
4	10	10	5
5	10	15	2
6	10	15	5
7	10	20	2
8	10	20	5

Table 2. Response indices for single story structure

Structure No	RI1	RI2	RI3	CFI (%)
1	0.101	0.001	0.007	28
2	0.172	0.010	0.057	25
3	0.147	0.003	0.018	26
4	0.303	0.109	0.358	21
5	0.197	0.008	0.039	24
6	0.428	0.271	0.633	17
7	0.250	0.039	0.155	23
8	0.534	0.406	0.761	14

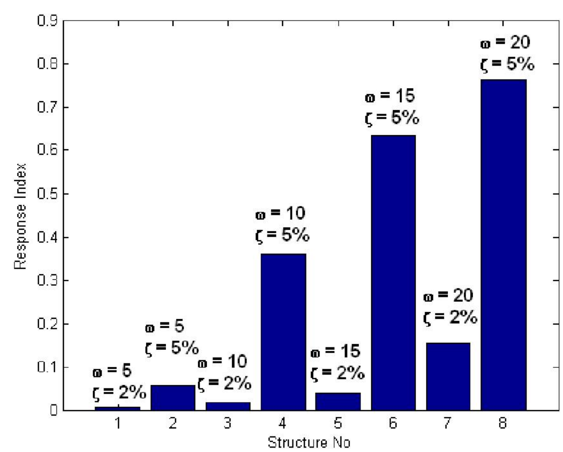

Figure 2. Third Response index (RI3) for single story structures

Table 3. Indices of single story structure for the same response reduction

Structure No	RI1=RI2	FI	JI
1	0.1	0.75	0.88
2	0.1	0.78	0.92
3	0.1	0.77	0.91
4	0.1	0.78	1.02
5	0.1	0.78	0.93
6	0.1	0.78	1.05
7	0.1	0.79	0.94
8	0.1	0.79	1.06

One of the problems that face the active control techniques is that the good results obtained in the case of ideal ground motion is diminished when applied to real ground motions. The structures number three and four are subjected to six earthquakes where their information are shown in Table 4. These earthquakes are scaled to 0.3 g. The two selected structures are subjected to these earthquakes when uncontrolled and when controlled by the two approaches when the same control force is adjusted. Table 5 shows the maximum displacement of the uncontrolled and the response indices for each earthquake. It is shown from the results of the third response index (RI3) that the proposed approach is more useful than the traditional approach in all cases and this benefit is clear in case of light damped structures than the heavy damped structures. It must be noticed that in some cases the proposed approach eliminates the structure vibration (zero values) with the same control force of the traditional approach, because the proposed approach apply the control force in early time. This early application of the force eliminates the small response which in the case of traditional approach causes additional response which require more control force later.

Table 4. Information of the applied six earthquakes

Earthquake name	Date	Magnitude	Location
El-centro	May 18, 1940	7.1	El centro, California
Hachinohe	May 16, 1968	7.9	Hachinohe city
Kobe	Jan. 17, 1995	7.2	Kobe japanese
Northridge	Jan. 17, 1994	6.8	Sylmar, California
Pacoima	Feb. 9, 1971	6.2	San fernando
Parkfield	Jun. 27, 1966	7.3	Parkfield, California

Table 5. Response Indices for Control approaches

Earthquake	Zeta = 2%				Zeta = 5%			
	Max. Disp.	RI1	RI2	RI3	Max. Disp.	RI1	RI2	RI3
El-centro	0.07	0.22	0.00	0.00	0.06	0.27	0.00	0.00
Hachinohe	0.06	0.29	0.00	0.00	0.04	0.42	0.02	0.04
Kobe	0.11	0.32	0.11	0.34	0.09	0.42	0.17	0.40
Northridge	0.03	0.49	0.00	0.00	0.02	0.63	0.00	0.00
Pacoima	0.03	0.37	0.22	0.59	0.02	0.51	0.32	0.63
Parkfield	0.13	0.21	0.02	0.09	0.10	0.27	0.04	0.14

3.2. Multi-story structure:

Two multi-story buildings are considered where they have the dynamic properties that shown in Table 1 for structures number three and four but for all stories and they have ten stories. The structures are controlled by the two approaches when subjected to sinusoidal ground motion with the lowest fundamental resonance frequency. Figure 3 shows the maximum drift of the uncontrolled, traditional controlled and proposed approach when the control force is adjusted as 2 % of the individual slab weight when the damping ratio for all modes is 2%. It is found that the new approach has a good reduction than the traditional approach in all stories by the same control force that calculated with the traditional approach.

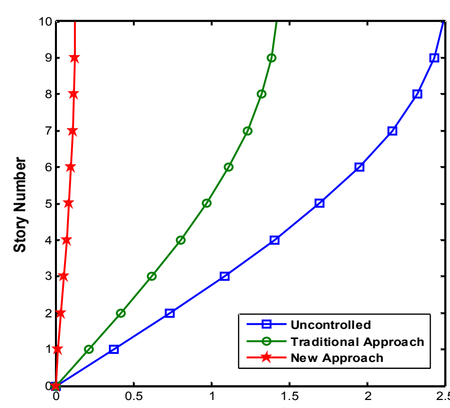

Figure 3. Maximum Drift for Ten story building

The ten story building with damping ratio 2% is subjected to Kobe earthquake when its record is adjusted to 0.3 g. Figure 4 shows the top story displacement of the uncontrolled, traditional controlled and new controlled structures. It is found that the new approach approximately eliminate the response.

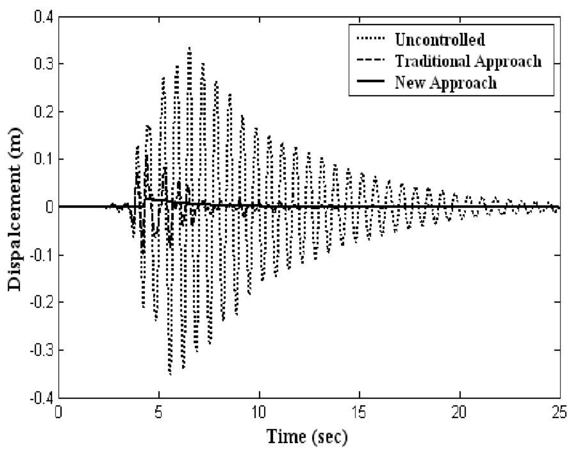

Figure 4. Top story drift of ten-stories building (zeta=2%) subjected to Kobe earthquake.

The multi-story structures are subjected to the records of the six earthquakes where the weighting factors in the traditional approach is adjusted as Q_{ii} = 10, R_i=0.01 when the control force in the new approach is adjusted within the limit of that obtained from the traditional approach. Table 6 shows the maximum drift ratio of the two buildings when controlled with the two approaches. It is shown that the new approach make more reduction in the response than the traditional approach with the same control force. It is found that the reduction of light damped structure is more than that of the heavy damped structure. The benefit of the new approach is not limited to the reduction of the structural displacement but it also reduces the velocity of the building as shown in Table 7.

Table 6. Displacement peak ratio of traditional and proposed approach

Earthquake	Zeta = 2%		Zeta = 5%	
	Trad.	New	Trad.	New
El-centro	0.50	0.17	0.55	0.24
Hachinohe	0.28	0.21	0.37	0.31
Kobe	0.32	0.05	0.48	0.11
Northridge	0.35	0.08	0.42	0.15
Pacoima	0.29	0.07	0.34	0.07
Parkfield	0.33	0.15	0.47	0.24

Table 7. Velocity peak ratio of traditional and proposed approach

Earthquake	Zeta = 2%		Zeta = 5%	
	Trad.	New	Trad.	New
El-centro	0.57	0.18	0.65	0.25
Hachinohe	0.36	0.21	0.40	0.28
Kobe	0.44	0.06	0.60	0.11
Northridge	0.36	0.10	0.41	0.16
Pacoima	0.31	0.08	0.36	0.09
Parkfield	0.36	0.13	0.52	0.21

To make a fair comparison between the traditional and proposed approach, a study was conducted to investigate the relation between the response ratio of the multi-story building and the required control force for the two approaches. Figure 5 shows the relation between Response ratio (which indicated the ratio between the maximum drift of the controlled and the uncontrolled structures) and the control force ratio (which indicated the ratio between the maximum control force and the individual slab weight) for ten story building with 2 % damping ratio. It is shown that the proposed technique is better than the traditional where the same control force makes more reduction and the same level of response reduction require smaller control force. The same conclusion is noticed for building with 5% damping ratio as shown in Figure 6.

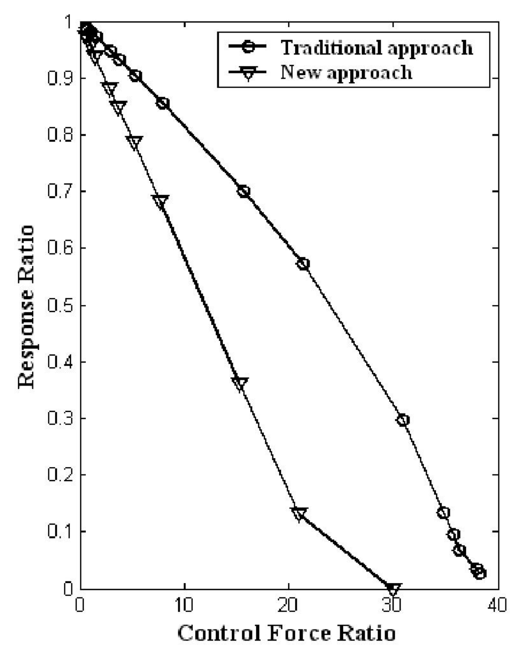

Figure 5. Relation between Response ratio and control force ratio for ten story building with 2 % damping ratio

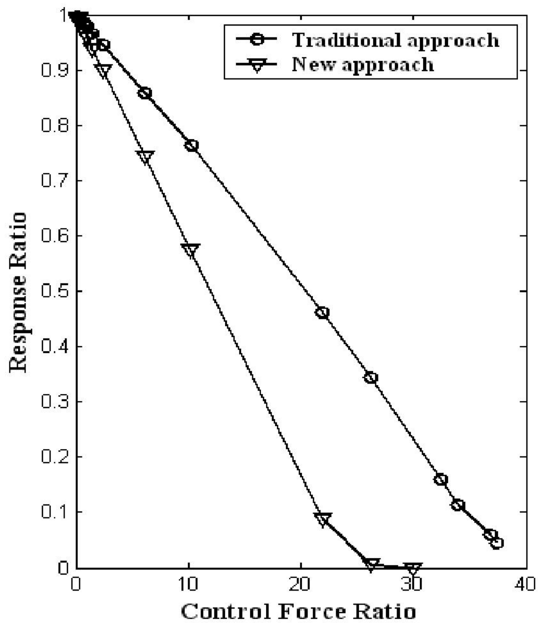

Figure 6. Relation between Response ratio and control force ratio for ten story building with 5 % damping ratio

4. Conclusions:

A new approach for structural active control is proposed to overcome actuators saturation problem. This approach is based on eliminating the structural response as early as possible to save the high control force required later due to the response generated from the small recent response. This approach is formulated and applied to single story and multi-story buildings. Firstly when the ground motion is ideal and in resonance with building and secondly when the ground motion is real earthquakes. The proposed approach is compared with the traditional optimal control in two manners; when the control force in the two approaches is maintained constant and when the response reduction level is the same. The main findings from this research are:

1. The proposed approach overcomes efficiently actuators saturation problem in structural active control.
2. The proposed approach decreases the structure response more than the optimal active control approach especially in case of flexible structures than the rigid ones, and in case of light damped than the heavy ones when the maximum control force in each approach is remained constant.
3. The proposed approach requires smaller control force than that of the traditional approach to make the same level of response reduction.
4. The benefit of the proposed approach is cleared when real ground motion is applied to the structure when highly response reduction is obtained.
5. The proposed approach reduces the velocity of the structure as that of the displacement.

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