Combined Distance-Reliability Model for Hazardous Waste Transportation and Disposal

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Abstract: A mathematical model that simultaneously locating a multiple disposal or a treatment facilities and determining a route for hazmat transportation network is presented. The objective is to minimize the distance traversed and population at risk. The route which minimizes a weighted hybrid metric path designation of accident probability and distance is significantly different from the minimum distance path. An adaption of Floyd Warshall's algorithm is used to find the hybrid path designation. An example is used to illustrate the applicability of the model.
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1. Introduction

The transportation of hazardous waste from generation sites to disposal sites or treatment sites has drawn considerable public attention. Over the last few decades there has been an increasing awareness of environmental matters, both by governing bodies and by the public. This includes a realization of the importance of the sensitive disposal of waste in its various forms (nuclear, chemical, domestic, etc.), each of which poses its own peculiar problems. Such terms as 'HAZMAT' (or HAZardous MATerials), 'noxious', 'obnoxious', 'semi-obnoxious' and 'undesirable' have been associated with waste, but here we will prefer merely to speak of high-, medium- or low-level waste [3].

Hazardous materials (hazmat) comprise explosives, flammables, oxidizing substances, poisonous gases, and radioactive materials. By definition, they can be extremely harmful to environment and to human health, since exposure to their toxic ingredients may cause injury or death to plants, animals, and humans. Their negative effects are an apparently inevitable consequence of industrial processes dictated by the life style of a modern society. It follows that transporting these materials, often in populated or environmentally sensitive areas, is also inevitable. Reducing the potential negative impacts of transporting hazmat is an important task faced by communities, governments, hazmat producers and shippers. Routing hazmat wisely and designing safer networks for doing so are powerful means to achieve this end. A fundamental requirement of route design and assignment is to assess the potential risk imposed by shipments traversing each link in a network [17].

Moreover the location of an obnoxious or potentially dangerous facility usually determines either the origin or the destination of obnoxious materials shipments, and therefore interacts with the routing decisions: the facility location and transportation logistics decisions are strictly interrelated within the context of obnoxious materials management systems. The problem of simultaneously locating obnoxious facilities and routing obnoxious materials between a set of built-up areas and the facilities is addressed.

This paper presents a model that combines siting and routing for hazardous material transportation and disposal, to minimize the sum of the weighted hybrid path designation over the planning horizon, and we use an adaptation of Floyd Warshall's algorithm to find the hybrid path designation for transportation of hazardous waste.

This paper is organized as follows: we present the relevant literature in section 2. Section 3 describes the transportation network with edge attribute which is the probability of accident and Section 4 presents the accident probability-distance hybrid metric for a path. In section 5, we developed a reliable hybrid multifacility location and routing model. An adaptation of Floyd Warshall's algorithm is described in section 6. An example for locating a single disposal facility and determining the routes of transport vehicles over the highway transportation network is presented in Section 7. Section 8 gives a summery and conclusion of this article.

2. Literature Review

The hazardous waste management problem is first handled in the location literature in locating treatment or disposal facilities. The treatment facilities, such as incinerators, and the disposal facilities, such as landfills, are usually termed as 'undesirable facilities' in this literature. There is a significant amount of literature on undesirable facility location. For an extensive discussion on undesirable facility location one can refer to Erkut and Neuman [4], which is the most recent review published in this area. In the location of undesirable facilities the aim is to minimize the nuisance and the adverse effects on the existing facilities or the population centers. Although the service cost of an undesirable facility increases when the facility is located far from the population centers, the undesirability of the facility usually seems to be more important.

Erkut et al. [6] presented an extensive study in a book chapter titled Hazardous Materials Transportation. They presented a high-level view of hazmat logistics research including a number of special issues of refereed academic journals that focus on hazmat transportation or location problems, books and book chapters, reports, web sites, and software. Also, they present different ways for risk assessment, routing and scheduling, and facility location and transportation for hazardous materials. Erkut and Verter [5] developed a review of the existing analytical approaches for strategic management of hazardous materials. ReVelle et al. [14] developed a model that locates storage facilities and selects routes for shipments of spent nuclear fuel. Zografos and Samara [18] presented a combined location and routing goal programming model for hazardous material transportation and disposal. Their model minimized travel time, transportation risk and disposal risk. List and Mirchandani [11] proposed a comprehensive model that simultaneously sited treatment facilities and made routing decisions for waste shipments. In their model, risk, cost, and equity were considered in a multiobjective framework. Stowers and Palekar [16] integrated routing decisions with the location of an undesirable facility using a single objective model, the minimized risk quantified by population exposure. This approach differed from previous work in that they considered both vertices and edges as feasible facility sites.

List et al. [12] presented a review of models for routing of obnoxious vehicles that is vehicles transporting undesirable materials. Giannikos [7] proposed a multiobjective model for locating disposal or treatment facilities and transporting hazardous waste along the links of a transportation network that minimizes the following four objectives: (1) total transportation cost and fixed cost of opening the treatment facilities; (2) total perceived risk due to the shipment of hazardous waste; (3) maximum individual risk (to force the risk equity); and (4) maximum individual disutility due to the treatment facilities.

Helander and Melachrinoudis [10] considered integrated location and routing models for minimizing the expected number of hazardous material transport accident. Two different routing policies are considered (1) most reliable route planning and (2) multiple routing with random selection. Path reliability measurements are used to derive the expected number of accidents over a given planning horizon. Also Melachrinoudis and Helander [13] presented the relisum location problem for siting a single facility on a tree in the presence of unreliable edges. Based on the objective of maximizing the expected number of reachable nodes from a service facility, they developed a number of analytical properties. They developed two polynomial algorithms for this problem a label-correcting procedure and a depth-first node traversal.

Boffey et al. [3] developed a model to locate a waste disposal site for low-level (domestic and nontoxic industrial) waste. Account is taken of nuisance caused to population along routes and of equity considerations. Not only is equity as regards effects on different population centers considered, but also between carriers of waste from different towns giving rise to the concept of routing fairness. Alumur and Kara [2] proposed a multiobjective location-routing model. The model has the objective of minimizing the total cost and the transportation risk and it includes some constraints. Sivakumar et al. [15] proposed the use of conditional risk (*i.e.,* expected consequence given the occurrence of the first accident).

3. Notation and Assumptions

It is assumed that the transportation network is described by an undirected graph $G(V,E)$, where $V=\{1,2,\ldots,n\}$ is the node set and $E=\{(i,h): i,h \in V\}$ is the edge set, where edge (i,h) represents a direct travel link between nodes i and h. Associated with each edge $(i,h) \in E$ is an attribute, $0 \le p(i,h) \le 1$ that denotes the probability of an accident on the edge during traversal by a hazmat transport vehicle. Throughout this paper, it is assumed that edge accident probabilities do not change over time, and that road conditions over an edge are uniform so that accident likelihood is approximately equivalent over points on the edge. In the case when this assumption is not practical, then a road segment can be represented in the transportation network $G(V,E)$ by two or more edges and connecting nodes instead of a single edge.

Hazmat transport is assumed to originate at nodes (origins) $i \in V$ and to be restricted along edges in the set E enroute to a storage facility (destination) $k \in G(V,E)$. Feasible sites for locating storage facilities are assumed to include the entire network including nodes and edges. Associated with each origin $i \in V$ is the attribute $w_i \ge 0$, denoting a weight for node i. The weights are general in the sense that they reflect frequency of transport shipments leaving the node, or the combined effect of frequency of transport shipments and virulence of the material to be transported. A number p of storage facilities are to be located. Each origin $i \in V$ is assigned to a unique destination k that receives hazmat shipments from i throughout the planning horizon.

For each edge $(i,h) \in E$, let $A(i,h)$ denote the event that an accident occurs while traversing that edge and $A(i,h)$ denote the event that an accident does not occur on the edge. These events are related to the edge attributes introduced earlier: $Pr\{A(i,h)\} = p(i,h)$ and $Pr\{\bar{A}(i,h)\} = q(i,h)$ for all $(i,h) \in E$, where $q(i,h) = 1 - p(i,h)$

Let $\Psi_{kj} = \{P_{kj}^1, P_{kj}^2, P_{kj}^3, \ldots, P_{kj}^{r_{kj}}\}$ be the set of all r_{kj} feasible paths from $k \in G$ to node j. Let $P_{kj}^l \in \Psi_{kj}, l = 1, 2, \ldots, r_{kj}$ denote a subset of edges from E that represents a loopless path from node j to node k in $G(V,E)$. For cyclic networks, several distinct loopless paths may exist between j and k, each differing by at least one edge. Associated with a path P_{kj}^l is an event that an accident occurs enroute from j to k during travel along the path, as well as an event that an accident does not occur on the path. These two events are denoted by $A(P_{kj}^l)$ and $\bar{A}(P_{kj}^l)$ respectively. It assumed that accidents are serious in the sense that vehicles are not able to continue (no accident recovery). Then clearly the probability of no accident during traversal of path P_{kj}^l is

$$Pr\{\bar{A}(P_{kj}^l)\} = \prod_{(i_h, i_{h+1}) \in P_{kj}^l} q(i_h, i_{h+1})$$

and the probability of an accident during traversal of path P_{kj}^l is

$$Pr\{A(P_{kj}^l)\} = 1 - \prod_{(i_h, i_{h+1}) \in P_{kj}^l} q(i_h, i_{h+1})$$

Following Melachrinoudis and Helander [13], we model the edge operational probability as an exponential function of the physical displacement, which allows us to determine the operational probabilities of the newly created edges and the exact edge location. The underlying assumption is that failures occur completely randomly along edges, according to the Poisson process. The longer the edge length (or physical displacement from a node), the higher the probability that a failure occurs, $i.e.$, the lower the operational probability is. The exponential model also allows us to calculate the operational probabilities of the edges of the network based on their lengths and failure rates.

The term $Pr\{\bar{A}(P_{kj}^l)\}$ is referred to the path reliability [10]. The most reliable route from node j to k is the path $P_{kj}^* \in \Psi_{kj}$ such as

$$Pr\{\bar{A}(P_{kj}^*)\} = \max_{P \in \Psi_{kj}} Pr\{\bar{A}(P)\}$$

When k is on edge $e = (i,h)$, the path reliability of the most reliable route from k to node j is $Pr\{\bar{A}(P_{kj})\} = \max[Pr\{\bar{A}(P)\} : P \in \Psi_{kj}] = R_{kj}^*$. Referring to Fig. 1, it is clear that the reliability of the most reliable route from k to node j is related to the path reliabilities of the most reliable routes from nodes i and h to node $R_{jk}^* = \max\{q(k,i)R_{ij}^*, q(k,h)R_{hj}^*\}$.

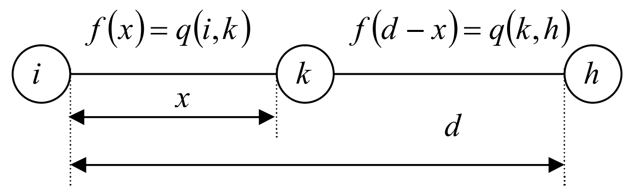

Figure 1: Facility location on an edge *(i, h)*

4 Hybrid Path Designations

ReVelle et al. [14] presented a multiobjective model in a problem dealing with storage siting and routing of spent nuclear fuel. They used two criteria: minimum transportation burden in ton-miles and minimum perceived risk as tons- past-people. Our model considers two objectives; minimizing the probability of an accident during traversal of path and minimizing the distance of that path between the origin and destination.

For each $P \in \Psi_{kj}$ define

$$\min_{P \in \Psi_{kj}} D(P) = \sum_{(i,h) \in P} d(i,h)$$

$$\max_{P \in \Psi_{kj}} R(P) = \prod_{(i,h) \in P} q(i,h)$$

where $\lambda(i,h)d(i,h) = -\ln q(i,h)$. Note that $d(i,h) \geq 0, \forall (i,h) \in E$, and $R(P) = \prod_{(i,h) \in \Psi_{kj}} q(i,h) = e^{-\sum_{(i,h) \in P} \lambda(i,h)d(i,h)}$. $D(P)$ is the sum of lengths of the edges of path P and $R(P)$ is the reliability of the path P which refers to the probability of traversal, $i.e.$, the probability that all edges along the path are operational.

We will use a constant α to relate the distance and the accident probability of any path between the origins and the destination. The accident probability-distance hybrid metric for the path $P_{kj}^l \in \Psi_{kj}$, traversed by a hazmat vehicle originating at node j with destination k, is defined as the convex combination of its accident probability and its length as follows:

$$H(\alpha, P_{kj}^l) = \alpha Pr\{A(P_{kj}^l)\} + (1-\alpha) D(P_{kj}^l)$$

where $D(P_{kj}^l)$ is the length of path P_{kj}^l and $0 \leq \alpha \leq 1$. The parameter α reflects the trade-off between population at risk and transportation cost. Let Ψ_{kj} be the set of all admissible paths between origin i and destination k. Then under the hybrid path designation policy, the designated path from j to k will always be $P_{kj}^* \in \Psi_{kj}$ defined by

$$H(\alpha, P_{kj}^*) = \alpha Pr\{A(P_{kj}^*)\} + (1-\alpha)D(P_{kj}^*) = \min_{P \in \Psi_{kj}} H(\alpha, P) \qquad (1)$$

The location of the point k can be at any node or edge of the network. If k is located on an edge, $e = (i, h)$ then k is going to split e into two new edges: edge (k, i) with operational probability $q(k, i)$ and length x, and edge (k, h) with operational probability $q(k, h)$ and length $d - x$. For consistency we will let x be the length of the edge between the newly created node k and the vertex of the edge with the smaller index $(i.e., i)$, as shown in Fig. 1. So, the destination node lies on edge (i, h), a distance x from node i.

The edge (i, h) is replaced with a new node labeled k and two new edges (i, k) and (k, h) connecting the endpoints of the original edge. Suppose that k is to be located x units from endpoints i where $0 \le x \le d(i, h) = d$. The probability of no accident on the new edge (i, k) is denoted by $f(x)$ and the probability of no accident on the new edge (k, h) is then $f(d - x)$. A path P_{kj}^l either includes node i or node h. Therefore the hybrid metric for path P_{kj}^l is:

$$\alpha[1 - f(x)Pr\{\bar{A}(P_{ji})\}] + (1-\alpha)[x + D(P_{ji})], \quad \text{if } P_{jk}^l = P_{ji}^l \cup (i, k) \text{ or}$$

$$\alpha[1 - f(d-x)Pr\{\bar{A}(P_{jh})\}] + (1-\alpha)[d - x + D(P_{jh})], \quad \text{if } P_{jk}^l = P_{jh}^l \cup (h, k)$$

where $D(P_{ji}^l)$ and $D(P_{jh}^l)$ are the lengths of paths P_{ji}^l and P_{jh}^l, respectively. Under the assumption that accidents are generated by Poisson Process, $f(x)$ is a convex function and the two functions above are therefore concave functions. The minimum of these functions over all $P \in \Psi_{kj}$ defined in (2) is a piecewise concave function of x. So, in case of location of a new facility on an edge (i, h) the optimal path is found by

$$H(\alpha, P_{jk}^*) = \min \begin{cases} \alpha[1 - f(x)Pr\{\bar{A}(P_{ji}^*)\}] \\ +(1-\alpha)[x + D(P_{ji}^*)] \\ \\ \alpha[1 - f(d-x)Pr\{\bar{A}(P_{jh}^*)\}] \\ +(1-\alpha)[d - x + D(P_{jh}^*)] \end{cases} \qquad (2)$$

Floyd Warshall's algorithm (see, [1]) computes shortest path distances between all pairs of network vertices. The algorithm is modified here to compute the minimum hybrid path between any two vertices in the network.

A realistic assumption regarding $q(i, h)$ is that failures that prohibit the use of the edge for traversal are generated according to a Poisson process with constant rate $\lambda(i, h), (i, h) \in E$, modeling $q(i, h)$ as an exponential function of the physical distance. Since the operational probability $q(i, h)$ is now assumed to be exponentially distributed, the random variable X is defined as the distance until the first failure occurs along an edge.

The failure rate $\lambda(i, h)$ represents the average number of failures per unit length. We represent the relationship between edge length, operational probability and failure rate, using the exponential model introduced by Melachrinoudis and Helander [13], as $q(i, h) = e^{-\lambda(i,h)d(i,h)}$, then $R(P) = e^{-\sum_{(i,h)\in P} \lambda(i,h)d(i,h)}$. We assume that the operational probabilities of the two newly created edges are also functions of the physical displacement of k from node i, as well as the original operational probability $q(i, h)$. Let us define that function as $f(x)$. The probabilities of successful traversal on the newly created edges (i, h) and (k, h) as functions of the physical displacement x, are referred to as $f(x) = q(i, k)$ and $f(d(i, h) - x) = q(k, h)$, respectively. The following four conditions were introduced by Melachrinoudis and Helander [13] with respect to a suitable function $f(x)$:

1) $f(0) = 1$
2) $f(d(i, h)) = q(i, h)$
3) $f(x)$ is monotonically decreasing in $x \in [0, d(i, h)]$ and $q(i, h) \le f(x) \le 1$ and
4) $f(x) * f(d(i, h) - x) = f(d(i, h)) = q(i, h)$

The exponential model satisfies the four conditions. Functions $f(x)$ and $f(d(i, h) - x)$ can be written as

$$f(x) = P[X > x] = e^{-\lambda_e x} \text{ and } f(d - x) = P[X > d - x] = e^{-\lambda_e (d-x)} = q(i, h)e^{\lambda_e x}$$

The Poisson Process provides a specific formula for the probability that no accident occurs while traversing x units over an edge having total length d and accident rate λ. That is, $f(x) = e^{-\lambda x}$ *for* $0 \le x \le d$. Similarly, the probability that an accident occurs is $1 - f(x) = 1 - e^{-\lambda x}$ *for* $0 \le x \le d$. Edge accident-free probabilities can be readily computed using the exponential function, $f(d) = e^{-\lambda d}$ given a road segment of length d and accident rate λ. Similarly, accident probabilities are computed as $1 - f(d) = 1 - e^{-\lambda d}$.

5 Reliable Hybrid Multifacility Locations and Routing Problem

When the storage facility or the disposal site k located on an edge $(i, j) \in E$ then k subdivides the edge into two subedges (i, k) and (k, h) where

$(i,k) \cup (k,h) = (i,h)$ and $(i,k) \cap (k,h) = k$. The safest path P_{jk}^* from node j to node $k \in (i,h)$ can be found by

$$Pr\{\bar{A}(P_{jk}^*)\} = \max[f(x)Pr\{\bar{A}(P_{ji}^*)\}, f(d-x)Pr\{\bar{A}(P_{jh}^*)\}]$$

In this expression, the two terms contained in brackets represent the probability of no accidents on path P_{ji}^* augmented by edge (i,k) and the probability of no-accident on path P_{jh}^* augmented by edge (k,h). Equivalently, the safest path P_{jk}^* can be found by

$$Pr\{\bar{A}(P_{jk}^*)\} = \max[1 - f(x)Pr\{\bar{A}(P_{ji}^*)\}, 1 - f(d-x)Pr\{\bar{A}(P_{jh}^*)\}]$$

Consider the Bernoulli random variable X_{jk}^t, whether or not there is an operational path from j to k for traversal $t = 1, 2, \ldots, w_j$ at a random instance, where $X_{jk}^t = 1$ if an accident occurs on trip t from j to k and $X_{jk}^t = 0$ otherwise. Under the most reliable route policy, the parameter associated with X_{jk}^t is $Pr\{X_{jk}^t = 1\} = Pr\{A(P_{jk}^*)\}$ and $Pr\{X_{jk}^t = 0\} = Pr\{\bar{A}(P_{jk}^*)\}$. If the node weights, w_j for all $j \in V$, reflect the frequency of hazmat shipments from j to the facility, then the random variable N_k defined by

$$N_k = \sum_{j \in V} \sum_{t=1}^{w_j} X_{jk}^t$$

reflects the total number of these trips during which an accident occurs.

The expected total number of accidents occurring over the same planning horizon

$$E[N_k] = E\left[\sum_{j \in V} \sum_{t=1}^{w_j} X_{jk}^t\right] = \sum_{j \in V} w_j Pr\{A(P_{jk}^*)\} \qquad (3)$$

The problem of finding the location of the facility k on $G(V,E)$ to minimize (3) is the reliable 1-median problem which introduced by Helander and Melachrinoudis [10].

Let $S_p \subseteq G(V,E)$ be the set of p points at which storage facilities are to be located, where by a point $k \in G(V,E)$ we mean a point along any edge (i,h) of $G(V,E)$ which may or may not be a vertex of $G(V,E)$

We define the reliability $Pr\{\bar{A}(P_{jk}^*)\}, k \in S_p$ between a vertex j of $G(V,E)$ and a set S_p on $G(V,E)$ by

$$Pr\{\bar{A}(P_{jk}^*)\} = \max_{1 \le l \le p} Pr\{\bar{A}(P_{jl}^*)\}$$

$$Pr\{A(P_{jk}^*)\} = 1 - Pr\{\bar{A}(P_{jk}^*)\}$$

We are looking for the location that minimizes the sum of all weights unreachable nodes, in order to provide a maximum access to network disposal sites. The performance of the network is measured by the number of successful or unsuccessful traversals to demands originating at the nodes. The worst performance could be considered either the maximum number of successful traversals or the minimum number of unsuccessful traversals. We apply this criterion in the definition of the following version of the reliable multifacility problem.

5.1 Multifacility Problem for Hazardous Waste Disposal and Routing

Let $V_k \subseteq V$ be the set of vertices assigned to the disposal site $k \in S_p$:

$$V_k = \{j \in V : Pr\{\bar{A}(P_{jk}^*)\} = \max_{1 \le l \le p} Pr\{\bar{A}(P_{jl}^*)\}\}, \text{ for each } k \in S_p$$

Let X_{jk}^t be a Bernoulli random variable defined in the previous section. The random variable $N(S_p)$ defined by

$$N(S_p) = \sum_{k \in S_p} \sum_{j \in V_k} \sum_{t=1}^{w_j} X_{jk}^t$$

For each set S_p of p points on $G(V,E)$, we define

$$\Omega(S_p) = E[N(S_p)] = E\left[\sum_{k \in S_p} \sum_{j \in V_k} \sum_{t=1}^{w_j} X_{jk}^t\right] = \sum_{k \in S_p} \sum_{j \in V_k} w_j \, Pr\{A(P_{jk}^*)\}$$

If S_p^* on $G(V,E)$ such that

$$\Omega(S_p^*) = \max_{\{S_p : S_p \subset G(V,E), |S_p| = p\}} \Omega(S_p) \qquad (4)$$

then S_p is called a reliable multifacility problem for hazardous materials location and routing of $G(V,E)$. The sets $V_k, 1 \le k \le p$ constitute a partition of V, *i.e.*, $\bigcup_{1 \le k \le p} V_k = V$.

This model is simultaneously finding:

- the set S_p of p destinations $k \in G(V,E)$.
- the assignment of shipping origins to destinations, denoted by the sets V_k for each $k \in S_p$ and
- the routing from each origin j to the assigned destination k.

These routes will be shortest distance paths in case of ignoring the population impact by the hazmat accidents *i.e.*, $\alpha = 0$, otherwise the chosen routes will compromise population impact and transportation cost.

For traditional p-median problem, Hakimi [8] has shown that there exists a set of p nodes $V_p^* \subseteq V$ that is optimal. A vertex optimality condition similar to Hakimi's [8] follows:

Lemma 1 Under the Safest Path Designation policy, there exists at least one p- node subset of V that solves (4).

5.2 Hybrid Path Designation in Case of Multifacilty Location

Then under the hybrid path designation policy, the hybrid path designation model is mathematically stated as

$$\min_{\{S_p:\, S_p \subseteq G(V,E), |S_p|=p\}} \sum_{k \in S_p} \sum_{j \in V_k} w_j H(\alpha, P^*_{jk}) \qquad (5)$$

where $H(\alpha, P^*_{jk})$ is defined in (1). As stated previously, weights w_j may reflect the combined effect of frequency of transport shipments and virulence of the materials being transported, in a surrogate objective function. If the disposal facility $k \in S_p$ lies on edge (i,h) a distance x from node i. The hybrid metric for optimal path P^*_{jk} is defined in (2).

For the reliable multifacility problem, there exists at least one p-node V^*_p subset of V that is optimal. A vertex optimality condition is similar to Hakimi [8].

Lemma 2 Under the hybrid path designation policy, there exists at least one p-node subset of V that solves (5).

For a fixed value of α, the method with hybrid path designation defined by (5) can be solved by methods similar to those used for the traditional p-median problem. The solution should be accompanied with appropriate sensitivity analysis on the parameter α within its range, $0 \le \alpha \le 1$.

6 Adaptation Floyd Warshall's Algorithm

We propose an extension of Floyd Warshall's algorithm which is a shortest path algorithm to find a minimum accident probability path. The Floyd Warshall's algorithm is used to determine the shortest path distances between every pair of nodes in a network. This is known as the all-pairs shortest path problem.

Let $A = [H(u,v)]$ be $n \times n$ weight matrix and let $P = [pred(u,v)]$ be $n \times n$ matrix, where $pred(u,v) = u$. There are n iterations during the execution of the algorithm. Iteration k begins with two $n \times n$ matrices.

In this section, we present an algorithm for finding the optimal node location on undirected network with unreliable links, *i.e.*, an optimal location is one that minimizes the objective function stated by expression (2). Our interest in a node location, as opposed to a location on an existing edge, assumes that the mathematical conditions leading to vertex optimality, presented in the last section, hold. The algorithm presented in this section is polynomial with respect to worse-case running times. The algorithm is $O(n^3)$ and is basically an adaptation of the Floyd Warshall's algorithm for finding all pairwise paths in a graph.

Algorithm

Given are: $G(V,E), d(i,j), q(i,j), (i,j) \in E, \alpha$
Begin
    for all node pairs $(i,j) \in V \times V$ **do**
        $Pr\{\bar{A}(i,j)\} := 0;\ D(i,j) := 0;\ H(i,j) := \infty;\ pred(i,j) := 0;$
    for all $i \in V$ **do**
        $Pr\{\bar{A}(i,i)\} := 1;\ D(i,i) := 0;\ H(i,i) := \infty;\ pred(i,i) := 0;$
    for each edge $(i,j) \in E$ **do**
        $Pr\{\bar{A}(i,j)\} := q(i,j);\ D(i,j) := d(i,j);$
        $H(i,j) := \alpha[1 - q(i,j)] + (1-\alpha)d(i,j);$
        $pred(i,j) := i;$
    for each $k := 1\ to\ n$ **do**
        for each $(i,j) \in V \times V$ **do**
            $if\ H(i,j) > \alpha[1 - Pr\{\bar{A}(i,k)\}Pr\{\bar{A}(k,j)\}] + (1-\alpha)[D(i,k) + D(k,j)]$
        then
        begin
            $Pr\{\bar{A}(i,j)\} := Pr\{\bar{A}(i,k)\}Pr\{\bar{A}(k,j)\};$
            $D(i,j) := D(i,k) + D(k,j);$
            $H(i,j) := \alpha[1 - Pr\{\bar{A}(i,k)\}Pr\{\bar{A}(k,j)\}] + (1-\alpha)[D(i,k) + D(k,j)];$
            $pred(i,j) := pred(i,k);$
        end
end

7 Computational Experiences

The following example problem is provided to demonstrate the procedure presented in the previous section. The example problem network consists of 33 nodes and 54 arcs. The nodes weights are given in Table 1. Table 2 displays the edges, edge lengths in miles, accident rates in accident per million miles and accident free probability. For accident rates, USA average rates were used from Harwood et al. [9], after adjusting them for local road

The adapted Floyd Warshall's algorithm is coded in C and implemented for different values of α. The designated routes corresponding to the solutions and the optimal locations for disposal facilities are illustrated in Figure 3. The optimal disposal facility may stay without change but the optimal spanning tree corresponding to designated routes changes.

Published accident rates can be used for various types of roadways, see for example Harwood et al. [9]. They report accident rates that are generally very small, e.g. of the order 10^{-6} accidents per mile. Rates of this order allow for linear approximation $f(d) \approx 1 - \lambda d$ instead of the exact expression $f(d) = 1 - e^{-\lambda d}$.

Table 1: Node weights of the network in Figure 2

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
w_i	8	0	1	1	2	0	6	1	1	1	1	3	2	1	1	0	1
i	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	
w_i	1	1	3	0	1	1	2	1	6	5	0	5	6	0	1	4	

Table 2: Edges attributes of the network in Figure 2

Edge (i,h)	Distance $d(i,h)$	Accident rate $\lambda(i,h)$	Accident probability $\lambda(i,h)d(i,h) \times 10^{-6}$	Edge accident-free probability $1 - e^{-\lambda(i.h)d(i,h)} \approx 1 - \lambda(i,h)d(i,h) \times 10^{-6}$
(1,2)	26	4.0	1.04×10^{-4}	0.999896
(1,8)	22	7.0	1.54×10^{-4}	0.999846
(1,24)	21	4.0	8.40×10^{-5}	0.999916
(2,3)	17	4.0	6.80×10^{-5}	0.999932
(2,5)	18	5.0	9.00×10^{-5}	0.99991
(3,4)	41	5.0	2.05×10^{-4}	0.999795
(3,5)	17	5.0	8.50×10^{-5}	0.999915
(5,6)	30	5.0	1.50×10^{-4}	0.99985
(6,7)	11	8.0	8.80×10^{-5}	0.999912
(6,8)	13	7.5	9.75×10^{-5}	0.9999025
(7,9)	15	7.5	1.125×10^{-4}	0.9998875
(7,10)	12	7.5	9.00×10^{-5}	0.99991
(7,13)	22	8.0	1.76×10^{-4}	0.999824
(7,14)	18	8.0	1.44×10^{-4}	0.999856
(8,9)	14	7.0	9.80×10^{-5}	0.999902
(9,10)	8	7.0	5.60×10^{-5}	0.999944
(9,23)	17	5.0	8.50×10^{-5}	0.999915
(9,24)	18	3.0	5.40×10^{-5}	0.999946
(10,11)	11	7.0	7.70×10^{-5}	0.999923
(10,23)	16	4.0	6.40×10^{-5}	0.999936
(11,12)	5	4.0	2.00×10^{-5}	0.99998
(11,18)	12	7.0	8.40×10^{-5}	0.999916
(11,20)	14	6.0	8.40×10^{-5}	0.999916
(13,14)	19	4.0	7.60×10^{-5}	0.999924
(13,17)	17	5.0	8.50×10^{-5}	0.999915
(13,18)	7	7.0	4.90×10^{-5}	0.999951
(14,15)	4	7.0	2.80×10^{-5}	0.999972
(14,16)	55	9.0	4.95×10^{-4}	0.99951
(15,16)	32	7.0	2.24×10^{-4}	0.999776

(16,17)	24	4.0	9.60×10^{-5}	0.999904
(17,21)	26	4.0	1.04×10^{-4}	0.999896
(18,19)	6	4.0	2.40×10^{-5}	0.999976
(18,20)	5	5.0	2.50×10^{-5}	0.999975
(19,21)	18	4.0	7.20×10^{-5}	0.999928
(20,21)	10	6.0	6.00×10^{-5}	0.99994
(21,22)	8	4.0	3.20×10^{-5}	0.999968
(21,27)	14	6.0	8.40×10^{-5}	0.999916
(22,23)	16	4.0	6.40×10^{-5}	0.999936
(22,26)	19	5.0	9.50×10^{-5}	0.999905
(23,24)	7	4.0	2.80×10^{-5}	0.999972
(23,26)	23	5.0	1.15×10^{-4}	0.999885
(24,25)	26	3.0	7.80×10^{-5}	0.999922
(25,26)	9	2.0	1.80×10^{-5}	0.999982
(26,27)	25	3.0	7.50×10^{-5}	0.999925
(26,29)	54	5.0	2.70×10^{-4}	0.99973
(26,30)	50	3.0	1.50×10^{-4}	0.99985
(27,28)	52	6.0	3.12×10^{-4}	0.999688
(28,29)	19	2.0	3.80×10^{-5}	0.999962
(28,33)	35	6.0	2.10×10^{-4}	0.99979
(29,30)	15	3.0	4.50×10^{-5}	0.999955
(30,31)	42	2.0	8.40×10^{-5}	0.999916
(31,32)	10	6.0	6.00×10^{-5}	0.99994
(32,29)	43	5.0	2.15×10^{-4}	0.999785
(32,33)	22	6.0	1.32×10^{-4}	0.999868

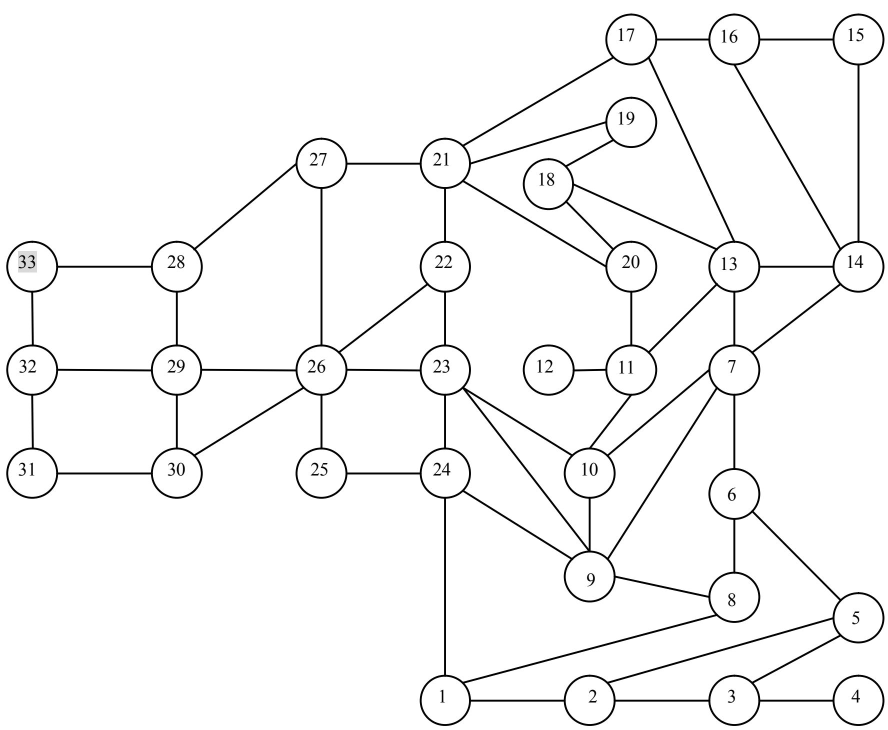

Figure 2: A hazardous wastes transportation network. Edges and nodes attributes are given in Tables 1 and 2

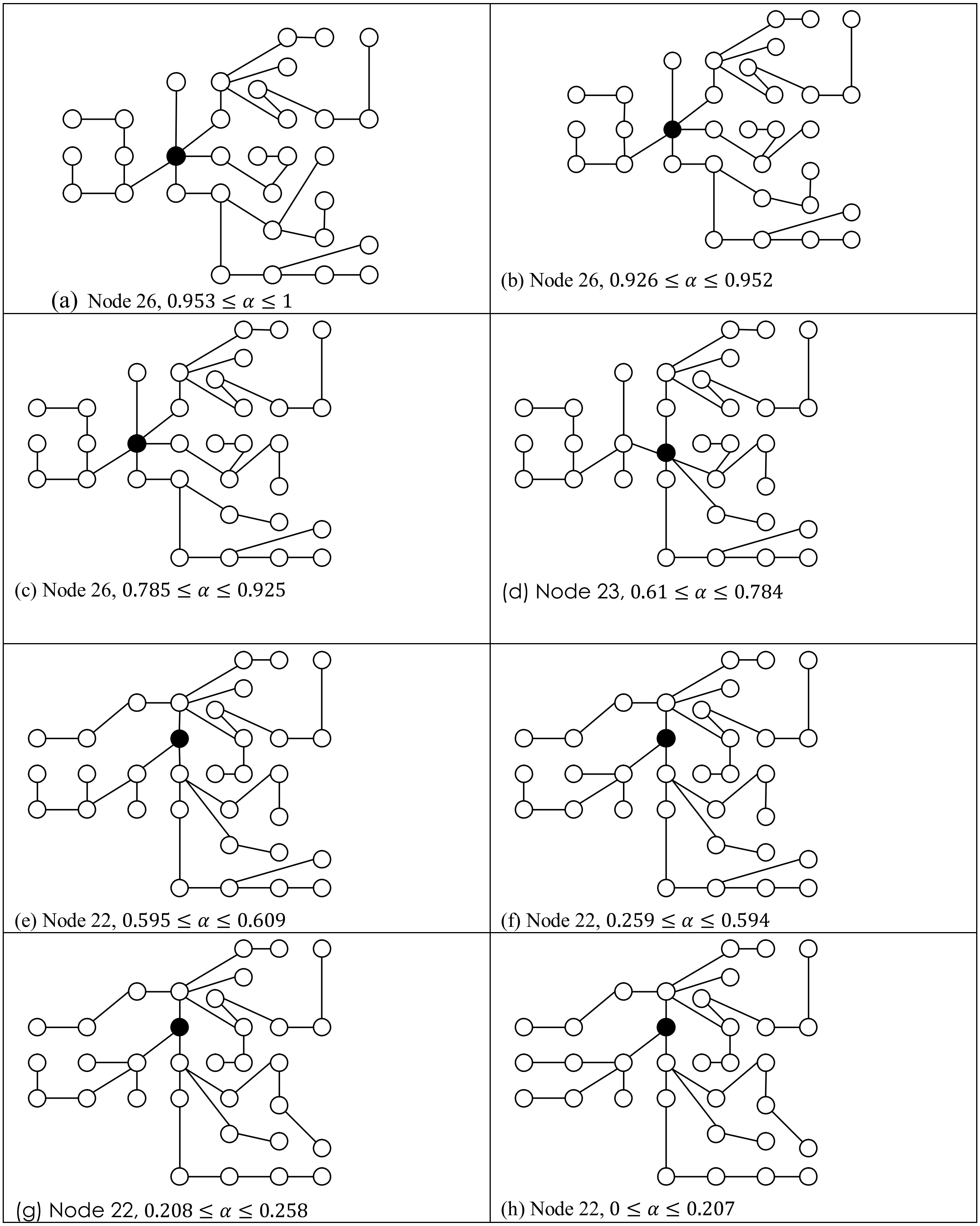

Figure 3: a, b, c, d, e, f, g, and h give the optimal facility locations and the optimal spanning trees representing routing for different values of α

8. Conclusions

In this paper a hybrid path metric designation model for locating disposal or treatment facility and routing hazardous wastes through the underlying transportation network has been presented. The model determines the location of hazardous waste facilities and the routes from given hazardous waste generation sites to the selected disposal facilities. The path reliability measures the expected number of accidents over a given planning horizon. So, reliability refers to the probability of a hazmat transport vehicle completing a journey from an origin to a destination. Two different location modeling frameworks were introduced: (1) The hybrid path designation model for locating a single facility and routing hazardous wastes to it and (2) reliable hybrid multifacility location and routing problem. A numerical example is presented to illustrate the applicability of the hybrid path designation model.

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