# Calculation of Hyperbolic Flow Past a Sphere using Indirect Boundary Element Method

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**Abstract:** In this paper, a steady and hyperbolic flow past a sphere has been calculated using indirect boundary element method (IDBEM), where as in our previous paper, we applied direct boundary element method for this purpose. The velocity distribution for the flow over the boundary of the sphere has been compared with the analytical results.

[Muhammad Mushtaq, Nawazish Ali Shah, & G. Muhammad. **Calculation of Hyperbolic Flow Past a Sphere using Indirect Boundary Element Method.** *Life Sci J* 2012; 9(2):1191-1195]. (ISSN: 1097-8135). http://www.lifesciencesite.com. 177

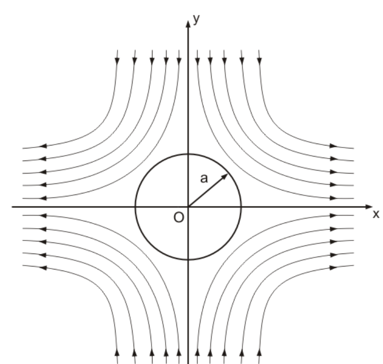
**Keywords:** Indirect boundary element method, hyperbolic flow, Sphere

## Introduction

In recent past, the well-known computational methods such as finite difference method (FDM), finite element method (FEM), and boundary element method (BEM) etc have been applied for the flow field calculations around bodies. Out of these methods, BEM is a modern numerical technique in which only the boundary of the body under consideration is discretized in to different type of elements. BEM is well-suited to problems where domain is exterior to the boundary, as in the case of flow past bodies. The most important features of BEM are the much smaller system of equations and considerable reduction in data, which are essential to run a computer program efficiently. That is why; BEM is more accurate, efficient and economical than other domain methods. The BEM can be classified into two categories i.e. direct and indirect. (see Brebbia and Walker, 1980, Ramsey, 1942, & Milne-Thomson, 1968). Indirect method is a numerical method which depends upon the distribution of singularities, such as sources or doublets over the surface of the body and computes the unknown in the form of singularity strengths. This method is used in different areas like solid and fracture mechanics, fluid dynamics and potential theory etc. [1]. The initial work for potential flow calculations was done by Hess and Smith ([2], [3]). Indirect method is popular due to its simplicity because the discretisation only takes place on the surface of the body.

## Calculation of Hyperbolic Flow Past a Sphere

Let a sphere of radius ‘a’ be taken as stationary and let U be the velocity of a uniform stream flowing in the positive direction of $x$ – axis as shown in figure 1 (see [4] & [11] ).



Hyperbolic flow past a sphere

Figure (1)

Now the stream function in this case is given by

$$\psi = \Omega x (y^2 + z^2) \left\{ 1 - \frac{5}{2}\left(\frac{a}{R_1}\right)^3 + \frac{3}{2}\left(\frac{a}{R_1}\right)^5 \right\}$$

Since $y^2 + z^2 = r^2$

$$\psi = \frac{U x}{\sqrt{4x^2 + y^2 + z^2}} (y^2 + z^2) \left\{ 1 - \frac{5}{2}\left(\frac{a}{R_1}\right)^3 + \frac{3}{2}\left(\frac{a}{R_1}\right)^5 \right\}$$

$$= \frac{U x r^2}{\sqrt{4x^2 + r^2}} \left\{ 1 - \frac{5}{2}\left(\frac{a}{R_1}\right)^3 + \frac{3}{2}\left(\frac{a}{R_1}\right)^5 \right\}$$

Thus $v_x = -\frac{1}{r}\frac{\partial \psi}{\partial r}$

$=$ U

**Error!**

and $v_r = -\frac{1}{r}\frac{\partial \psi}{\partial x}$

$=$ U

**Error!**

**Equation of Indirect Boundary Element Method**

The equation of indirect boundary element method for three – dimensional problems is given by ([7], [8], [9], [10]).

$$-\frac{1}{2}\Phi_i + \phi_\infty + \iint_{S-i} \Phi \frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right) dS = x_i$$

**Discretization of Sphere:**

The surface of the sphere is discretized into quadrilateral elements. The scheme of discretization is as shown in the figure (2).

The indirect boundary element method is applied to calculate the hyperbolic flow solution around the sphere for which the analytical solution is available

Consider the surface of the sphere in one octant to be divided into three quadrilateral elements by joining the centroid of the surface with the mid points of the curves in the coordinate planes as shown in figure (2) ([9], [10], [11]).

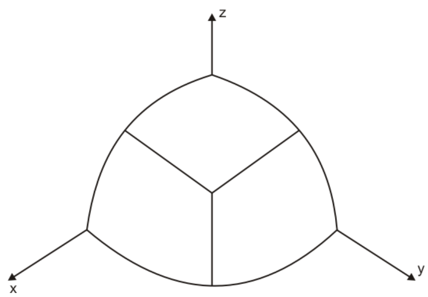



Figure (2)

Figure (3) shows the method for finding the coordinate ($x_p$, $y_p$, $z_p$) of any point P on the surface of the sphere.

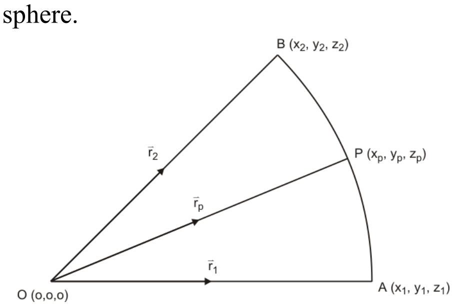



Figure (3)

Then each element is divided further into four elements by joining the centroid of that element with the mid–point of each side of the element. Thus one octant of the surface of the sphere is divided into 12 elements and the whole surface of the body is divided into 96 boundary elements. The above mentioned method is adopted in order to produce a uniform distribution of element over the surface of the body . From above figure, we have the following equation

$$|\vec{r}_p| = 1$$

$$\vec{r}_p \cdot \vec{r}_1 = \vec{r}_p \cdot \vec{r}_2$$

$$(\vec{r}_1 - \vec{r}_2) \cdot \vec{r}_p = 0$$

or in cartesian form

$$x_p^2 + y_p^2 + z_p^2 = 1$$

$$x_p (x_1 - x_2) + y_p (y_1 - y_2) + z_p (z_1 - z_2) = 0$$

$$x_p (y_1 z_2 - z_1 y_2) + y_p (x_2 z_1 - x_1 z_2) + z_p (x_1 y_2 - x_2 y_1) = 0$$

As the body possesses planes of symmetry, this fact may be used in the input to the program and only the non–redundant portion need be specified by input points. The other portions are automatically taken into account. The planes of symmetry are taken to be the coordinate planes of the reference coordinate system. The advantage of the use of symmetry is that it reduces the order of the resulting system of equations and consequently reduces the computing time in running a program. As a sphere is symmetric with respect to all three coordinate planes of the reference coordinate system, only one eighth of the body surface need be specified by the input points, while the other seven–eighth can be accounted for by symmetry.

The sphere is discretised into 96 and 384 boundary elements and the computed velocity distributions are compared with analytical solutions for the sphere using Fortran programming.

The following tables show the comparison of computed and analytical velocity distribution over the surface of sphere using 96 and 384 indirect boundary elements.

**TABLE (1)**

| ELEMENTS | XM | YM | ZM | VELOCITY | EXACT VELOCITY |
|---|---|---|---|---|---|
| 1 | -.177E+00 | -.934E+00 | .177E+00 | .14526E+01 | .25097E+01 |
| 2 | -.522E+00 | -.798E+00 | .157E+00 | .12418E+01 | .38214E+01 |
| 3 | -.798E+00 | -.522E+00 | .157E+00 | .82398E+00 | .34106E+01 |
| 4 | -.934E+00 | -.177E+00 | .177E+00 | .34768E+00 | .32259E+01 |
| 5 | -.934E+00 | .177E+00 | .177E+00 | .34768E+00 | .32259E+01 |
| 6 | -.798E+00 | .522E+00 | .157E+00 | .82398E+00 | .34106E+01 |
| 7 | -.522E+00 | .798E+00 | .157E+00 | .12418E+01 | .38214E+01 |
| 8 | -.177E+00 | .934E+00 | .177E+00 | .14526E+01 | .25097E+01 |
| 9 | .177E+00 | .934E+00 | .177E+00 | .14526E+01 | .25097E+01 |
| 10 | .522E+00 | .798E+00 | .157E+00 | .12418E+01 | .38214E+01 |
| 11 | .798E+00 | .522E+00 | .157E+00 | .82398E+00 | .34106E+01 |
| 12 | .934E+00 | .177E+00 | .177E+00 | .34768E+00 | .32259E+01 |
| 13 | .934E+00 | -.177E+00 | .177E+00 | .34768E+00 | .32259E+01 |
| 14 | .798E+00 | -.522E+00 | .157E+00 | .82398E+00 | .34106E+01 |
| 15 | .522E+00 | -.798E+00 | .157E+00 | .12418E+01 | .38214E+01 |
| 16 | .177E+00 | -.934E+00 | .177E+00 | .14526E+01 | .25097E+01 |
| 17 | -.157E+00 | -.798E+00 | .522E+00 | .14495E+01 | .23718E+01 |
| 18 | -.470E+00 | -.703E+00 | .470E+00 | .13038E+01 | .37699E+01 |
| 19 | -.703E+00 | -.470E+00 | .470E+00 | .96588E+00 | .35883E+01 |
| 20 | -.798E+00 | -.157E+00 | .522E+00 | .82398E+00 | .34106E+01 |
| 21 | -.798E+00 | .157E+00 | .522E+00 | .82398E+00 | .34106E+01 |
| 22 | -.703E+00 | .470E+00 | .470E+00 | .96588E+00 | .35883E+01 |
| 23 | -.470E+00 | .703E+00 | .470E+00 | .13038E+01 | .37699E+01 |
| 24 | -.157E+00 | .798E+00 | .522E+00 | .14495E+01 | .23718E+01 |
| 25 | .157E+00 | .798E+00 | .522E+00 | .14495E+01 | .23718E+01 |
| 26 | .470E+00 | .703E+00 | .470E+00 | .13038E+01 | .37699E+01 |
| 27 | .703E+00 | .470E+00 | .470E+00 | .96588E+00 | .35883E+01 |
| 28 | .798E+00 | .157E+00 | .522E+00 | .82398E+00 | .34106E+01 |
| 29 | .798E+00 | -.157E+00 | .522E+00 | .82398E+00 | .34106E+01 |
| 30 | .703E+00 | -.470E+00 | .470E+00 | .96588E+00 | .35883E+01 |
| 31 | .470E+00 | -.703E+00 | .470E+00 | .13038E+01 | .37699E+01 |
| 32 | .157E+00 | -.798E+00 | .522E+00 | .14495E+01 | .23718E+01 |
| 33 | -.157E+00 | -.522E+00 | .798E+00 | .14495E+01 | .23718E+01 |
| 34 | -.470E+00 | -.470E+00 | .703E+00 | .13038E+01 | .37699E+01 |
| 35 | -.522E+00 | -.157E+00 | .798E+00 | .12418E+01 | .38214E+01 |
| 36 | -.522E+00 | .157E+00 | .798E+00 | .12418E+01 | .38214E+01 |
| 37 | -.470E+00 | .470E+00 | .703E+00 | .13038E+01 | .37699E+01 |
| 38 | -.157E+00 | .522E+00 | .798E+00 | .14495E+01 | .23718E+01 |
| 39 | .157E+00 | .522E+00 | .798E+00 | .14495E+01 | .23718E+01 |
| 40 | .470E+00 | .470E+00 | .703E+00 | .13038E+01 | .37699E+01 |
| 41 | .522E+00 | .157E+00 | .798E+00 | .12418E+01 | .38214E+01 |
| 42 | .522E+00 | -.157E+00 | .798E+00 | .12418E+01 | .38214E+01 |
| 43 | .470E+00 | -.470E+00 | .703E+00 | .13038E+01 | .37699E+01 |
| 44 | .157E+00 | -.522E+00 | .798E+00 | .14495E+01 | .23718E+01 |
| 45 | -.177E+00 | -.177E+00 | .934E+00 | .14526E+01 | .25097E+01 |
| 46 | -.177E+00 | .177E+00 | .934E+00 | .14526E+01 | .25097E+01 |
| 47 | .177E+00 | .177E+00 | .934E+00 | .14526E+01 | .25097E+01 |
| 48 | .177E+00 | -.177E+00 | .934E+00 | .14526E+01 | .25097E+01 |

**TABLE (2)**

| ELEMENTS | XM | YM | ZM | VELOCITY | EXACT VELOCITY |
|---|---|---|---|---|---|
| 1 | -.906E-01 | -.983E+00 | .906E-01 | .14893E+01 | .15503E+01 |
| 8 | -.983E+00 | -.906E-01 | .906E-01 | .17767E+00 | .25889E+01 |
| 16 | -.906E-01 | .983E+00 | .906E-01 | .14893E+01 | .15503E+01 |
| 24 | .983E+00 | .906E-01 | .906E-01 | .17767E+00 | .25889E+01 |
| 32 | .906E-01 | -.983E+00 | .906E-01 | .14893E+01 | .15503E+01 |
| 40 | -.949E+00 | -.830E-01 | .276E+00 | .43321E+00 | .26147E+01 |
| 48 | -.830E-01 | .949E+00 | .276E+00 | .14881E+01 | .15140E+01 |
| 56 | .949E+00 | .830E-01 | .276E+00 | .43322E+00 | .26147E+01 |
| 64 | .830E-01 | -.949E+00 | .276E+00 | .14881E+01 | .15140E+01 |
| 72 | -.875E+00 | -.813E-01 | .459E+00 | .70050E+00 | .27134E+01 |
| 80 | -.813E-01 | .875E+00 | .459E+00 | .14883E+01 | .15045E+01 |
| 88 | .875E+00 | .813E-01 | .459E+00 | .70051E+00 | .27134E+01 |
| 96 | .813E-01 | -.875E+00 | .459E+00 | .14883E+01 | .15045E+01 |
| 104 | -.766E+00 | -.784E-01 | .625E+00 | .94672E+00 | .29172E+01 |
| 112 | -.784E-01 | .766E+00 | .625E+00 | .14886E+01 | .14886E+01 |
| 120 | .766E+00 | .784E-01 | .625E+00 | .94672E+00 | .29172E+01 |
| 128 | .784E-01 | -.766E+00 | .625E+00 | .14886E+01 | .14886E+01 |
| 136 | -.625E+00 | .784E-01 | .766E+00 | .11607E+01 | .31426E+01 |
| 144 | .233E+00 | .605E+00 | .750E+00 | .14499E+01 | .23707E+01 |
| 152 | .570E+00 | -.382E+00 | .716E+00 | .12228E+01 | .31934E+01 |
| 160 | -.433E+00 | -.242E+00 | .859E+00 | .13439E+01 | .31063E+01 |
| 168 | .242E+00 | .433E+00 | .859E+00 | .14479E+01 | .24167E+01 |
| 176 | .813E-01 | -.459E+00 | .875E+00 | .14883E+01 | .15045E+01 |
| 184 | .254E+00 | .254E+00 | .924E+00 | .14465E+01 | .24781E+01 |
| 192 | .906E-01 | -.906E-01 | .983E+00 | .14893E+01 | .15503E+01 |

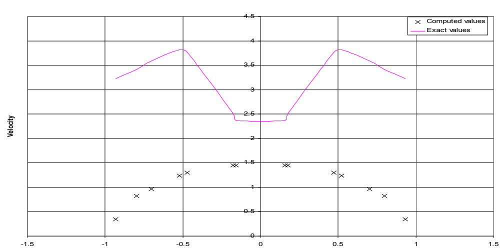



Figure (4): Comparison of computed and analytical velocity distributions over the surface of the sphere using 96 indirect boundary elements.

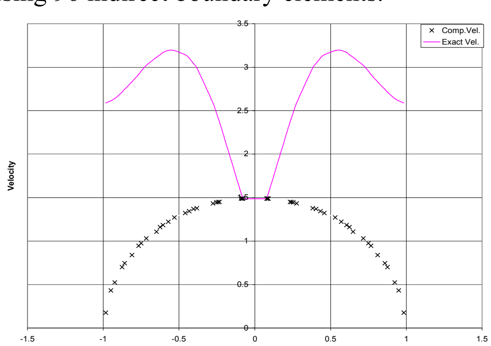



Figure (5): Comparison of computed and analytical velocity distributions over the surface of the sphere using 384 indirect boundary elements.

**Conclusion**

Indirect boundary element method has been applied to calculate the hyperbolic flow past a sphere. The improvement in results gained by taking 384 can be seen from the table (2) and figure (5) and such improvement increases with increase in number of boundary elements. Moreover, the computed results are in good agreement with exact results at the top of a body under consideration where viscous effects are minor.

**Acknowledgement**

We are thankful to the University of Engineering & Technology, Lahore – Pakistan for financial support.

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6/11/2012