

## Effect of Openings on the Static and Dynamic Behavior of Quadratic Folded Plate Roofing Systems

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**Abstract:** This paper investigates the effect of openings on the structural behaviour of quadratic folded plate Q.F.P. roofs by using different geometric configurations for the main elements of the system. The impact of such variance on the behaviour of the structure system under both static and dynamic conditions is also investigated. The selected and investigated parameters in this study are the location and size of the openings, as well as the rise of the folded plates (height). Spans of 14 m, 20 m and 26 m were selected for all of the investigated parameters. To meet the goals of this study, a 3-D Finite Element Model (FEM) was adopted to examine the suggested variables. A linear static analysis was performed to analyse the effect of the investigated parameters on the system deflections, moments, tension and compression stress. Q.F.P. slabs with rises varying from 90 cm to 180 cm were studied. The results indicated that the difference in the rise reduced the roof deflection by 72%. Moreover, the behaviour of the folded plate with openings at different locations was improved. The compression stress of the Q.F.P. roofs increased by 120% to 170% when the location of the centre openings on the models length increased from 14 m to 26 m. Folded plate openings at the quarter length of the folded plate further reduced the compression stress by 5% to 56% when the model span changed from 14 m to 26 m. The location of the edge openings had a slight effect on the compressive stress compared to other factors. The opening location did not have a significant effect on the tensile stress of the Q.F.P. slabs. Furthermore, the results indicated that the maximum bending moment for the intermediate beams increased by 69% when the centre openings were located at the beam centre. The maximum bending moment at 0.57 L (intermediate beam length) for the quarter opening location increased by 64% compared to control model. The edge opening location had a slight effect on the diaphragm bending moment and the intermediate beam fixed-end moment. Four different opening sizes were studied, and the effect of the opening size for the 26-m model was found to increase the static and dynamic behaviour by no more than 6%. Three-dimensional dynamic modal analyses were performed, and the effect of different opening locations on the fundamental modes was investigated. The results of the modal analysis showed that the openings location did not significantly affect the fundamental frequencies or fundamental mode shapes. The results obtained from this study emphasise the importance of using an elaborate numerical analysis to address such sensitive models; furthermore, the geometric properties and openings location of each contributing element clearly affected the overall performance of the system. Finally, the model results indicated that the folded plate opening location at the centre of the beam is the most effective parameter of those investigated here, while the edge opening location in the folded plate had the lowest effect on both the static and dynamic behaviour of the investigated system.

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**Key words:** Folded plates; Height; Openings location; Free vibrations.

### 1. INTRODUCTION

The folded plate is one of the most practical structures used in civil engineering applications because it has numerous merits, including its light weight, the ease at which it is formed, its low cost and its high resistance to loads. The folded plate is used in many applications, such as roofs, sandwich plate cores and cooling towers. As the use of folded plates is advantageous compared to flat plates, their behaviour with and without openings under different load conditions must be investigated. There are several methods available for analysing this type of structure (1 to 8). While conventional analysis methods are simple and easy, they have certain limitations related to the generality of application and precision.

Early researchers solved folded plate problems in an approximate manner with the use of the beam

method or a theory that neglected the relative joint displacement (2). However, these two methods could not easily address generalised folded plate problems. Computational approaches and numerical methods for the analysis of folded plates offer more precise solutions than conventional analysis methods. The methods of interest include finite strip methods; one of the earliest studies based on this method, which was introduced by [Cheung 1969], was that of [Golley and Grice 1989], along with the study presented in the same year by [Eterovic and Godoy 1989]. The combined boundary element-transfer matrix method [Ohga et al., 1991] and finite element method (FEM) were the focus of many previous studies [Liu and Huang, 1992; Perry et al., 1992; Duan, 2002]. The FEM is more convenient than the combined boundary element-transfer matrix method because it can be

applied to analyse large complex structures; furthermore, various types of boundary conditions and loadings can be easily implemented. For these reasons, most of the commercial software used for structural analysis uses the FEM method.

This study focused on the Q.F.P. roof because it is one of the most common types of roofs used for roofing systems. The presented parametric study on the effect of opening location on folded plate behaviour was examined using finite element analysis and linear static three-dimensional F.E. analysis. The study analysed the effects of the opening location and the rise of the folded plate on the slab deflection, intermediate beam moments, and stress distribution of the slab's maximum stress.

In some cases, Q.F.P. slabs system should include openings. These openings may be small, such as those needed to accommodate heating, plumbing, and ventilating risers, floor and roof drains, and access hatches, but in some cases, openings may be larger than the code limitation in size and position. The presence of large openings in the slab system reduces the stiffness and increases the deflection; previous studies investigated the effect of the openings on the behaviour of the flat plate structure.

Based on the output of models, design charts for this particular system will be presented to select the

best opening location, and the effects of various parameters on the behaviour of the structural system will be evaluated. Recommendations will be given in accordance with the impact of the opening location on the behaviour of the structural system.

## 2 Finite Element Analyses

### 2.1 Geometry and Dimensions of the Investigated Systems

Q.F.P. panels with a width of 7.8 m were studied, as presented in Figure 1. This width was kept constant throughout the analysis, while the three different spans of 14 m, 20 m, and 26 m were changed to investigate their effect on the behaviour of the structural system. A constant opening size of 3 m×2 m was considered for each panel for openings that were located on the top surface of the folded plate slab at different positions (centre, quarter, and edge). The height of the models varied from 90 to 180 cm. A 3-D F.E. analysis was performed, and the roof was modelled using 3-D quadratic shell elements, which are presented in Figure 2. The intermediate beams, end diaphragms, and columns were modelled as 3-D frame elements, as shown in Figure 2. The length of the openings ranged from 2 m to 8 m, while a constant width of 3 m was used for the different opening locations.

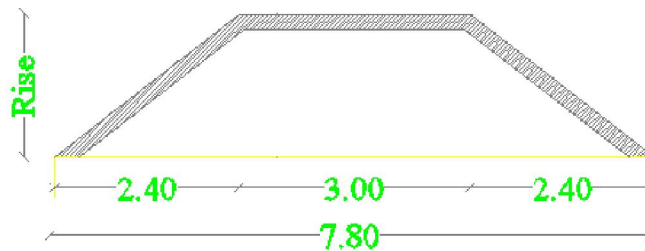


Figure 1: Cross-sectional dimensions of the investigated models

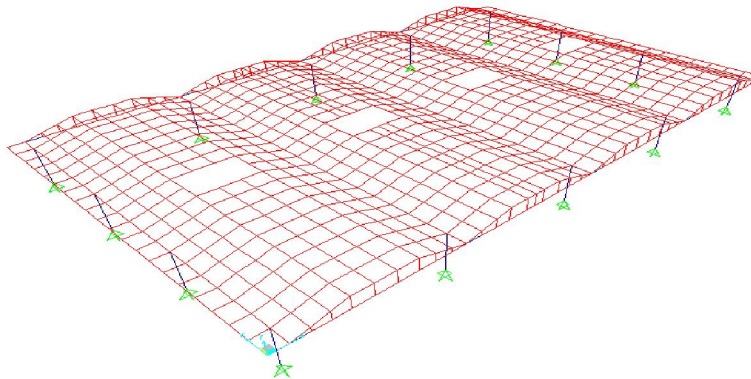


Figure 2: 3-D model of the investigated folded plate panels and centre opening location

### 2.2 Static Analysis of the Q.F.P. System

A 3-D static linear F.E. analysis was performed by applying each of the parameters under

investigation separately. A static load of 150 kg/cm<sup>2</sup> was applied on all of the models to mimic flooring and service loads. Throughout the analysis, the deflections

and stresses of the folded plate were checked and the maximum bending moments at the intermediate beams and diaphragm were reported. The effect of the tested parameters will be presented in detail later.

### 2.2.1 Effect of the Opening Location on the Maximum Deflection of the Q.F.P. System for a Constant Rise

The first investigated parameter was the effect of the folded plate opening location on the maximum deflection at the centre of the folded plate. Spans of 14 m, 20 m, and 26 m were studied. Figure 3 shows the maximum deflection of the folded plate slabs at the central point for the different lengths used versus the opening location for the folded plate. When the openings are located at the centre of the beam, the maximum deflection for the span with a length of 26 m and rise of 0.9 m increased from 47.6 mm to 58.2 mm, with a percentage increase of 22%. When the openings were located at the quarter length of the beam, the maximum deflection of the beam centre was 51.8 mm, with a percentage increase of 9 % compared to control model. The maximum central deflection was 47.6 mm when the openings were located on the edge, which has

a minor effect on the centre deflection point. The measured deflection value when the openings were located at the edge is approximately the same as in the control model.

For the 20-m Q.F.P. span, the central deflection was 15.7 mm when the rise of the folded plate was 0.9 m, as presented in Figure 3. In the most of the investigated cases, the opening location has a significant effect on the extent to which the central deflection increases. For example, when the openings were located at the centre and quarter of the beam, the deflection increased to 19.3 mm and 17 mm, respectively, corresponding to 23% and 8% increases compared to the control case.

The central deflection of the folded plate with a 14-m span was 3.73 mm; this case is considered to be the control model. The effect of the opening location on the central deflection were 4.45 mm (increasing by 19 %), 3.89 mm (increasing by 4 %), and 3.77 mm for the corresponding to centre, quarter, and edge opening locations, respectively.

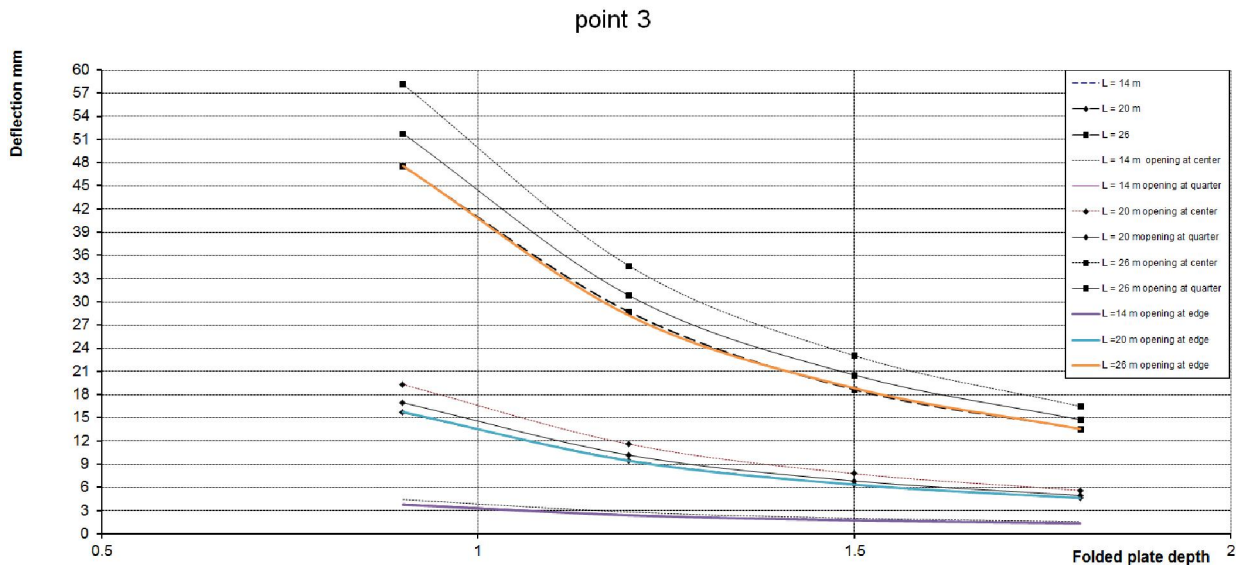


Figure 3: Maximum deflection of the Q.F.P. slabs versus different opening locations for different rises

### 2.2.2 Effect of the Folded Plate Rise on the Maximum Deflection

In this study, the folded plate rise was varied from 90 cm to 180 cm. The folded plate rise can significantly decrease the central deflection, as shown in Figure 3. For the longest investigated span of 26 m, the increase of the folded plate depth (rise) from 0.9 m to 1.8 m with centre opening location reduced the deflection from 58.2 mm to 16.5 mm, while the deflection induced at the quarter opening location decreased from 51.8 mm to 14.8 mm when the folded plate rise increased from 0.9 m to 1.8 m.

For the 20-m model, increasing the folded plate rise from 0.9 m to 1.8 m decreased the central deflection from 19.3 mm to 5.62 mm at the centre opening location, while the central point deflection decreased from 17 mm to 5 mm for the same increase in the folded plate rise when the openings were located at the quarter length of the beam.

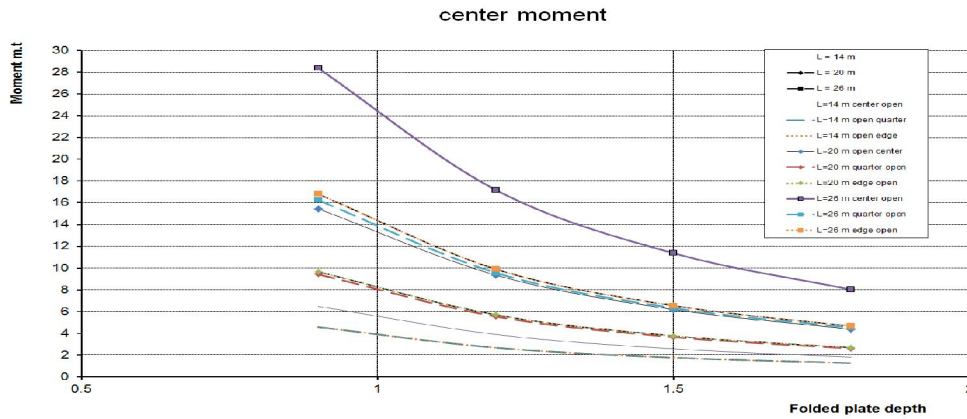
For the 14-m span, the central deflection decreased by 65 % for both the centre and quarter opening locations, as presented in Figure 3. The Q.F.P. slab rise decreases the effect of the opening location on the maximum deflection.

**2.2.3 Effect of the Folded Plate Opening Location on the Intermediate Beam Moment**

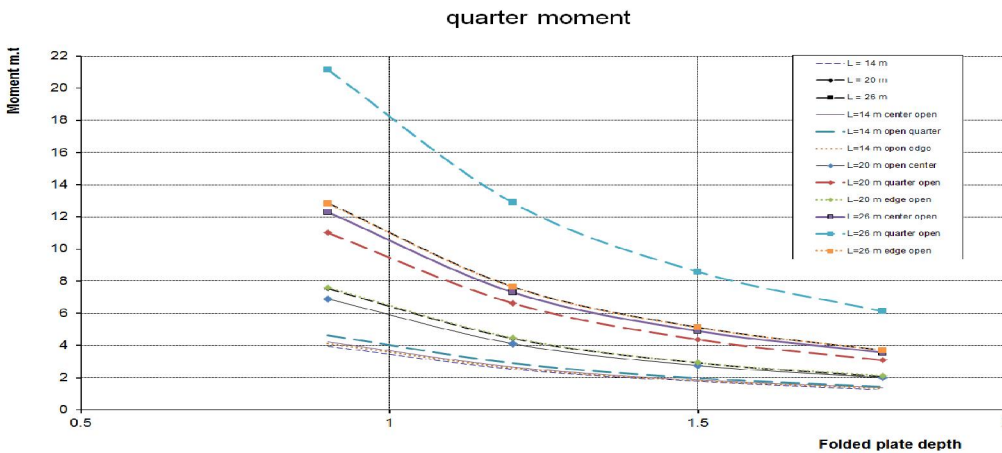
This study considered three opening locations for the Q.F.P. slabs: at the centre, quarter and edge of the beam. These openings can significantly increase the maximum bending moment of the intermediate beam, and the value of the moment depends on the opening positions. For the 26-m span without openings, the maximum bending moment at the centre of the intermediate beam was 16.8 m.t, while the bending moment was 28.37 meter-tons (increasing by 67 %). For the 20 m and 14 m folded plate lengths, the maximum bending moments increased by 60% and 42 %, respectively. The effect of the opening location at the quarter or edge of the beam has no effect on the maximum moment at the centre of the beam, as the

maximum moment is similar to that of the control model, as shown in Figure 4. The maximum moment occurred at the quarter of the intermediate beam at the opening location, and it increased by 64 %, 46% and 30% for the model lengths of 26 m, 20 m and 14 m, respectively, as shown in Figure 5. The edge opening location has a limited significant effect on the intermediate beam bending moment, as shown in Figure 6.

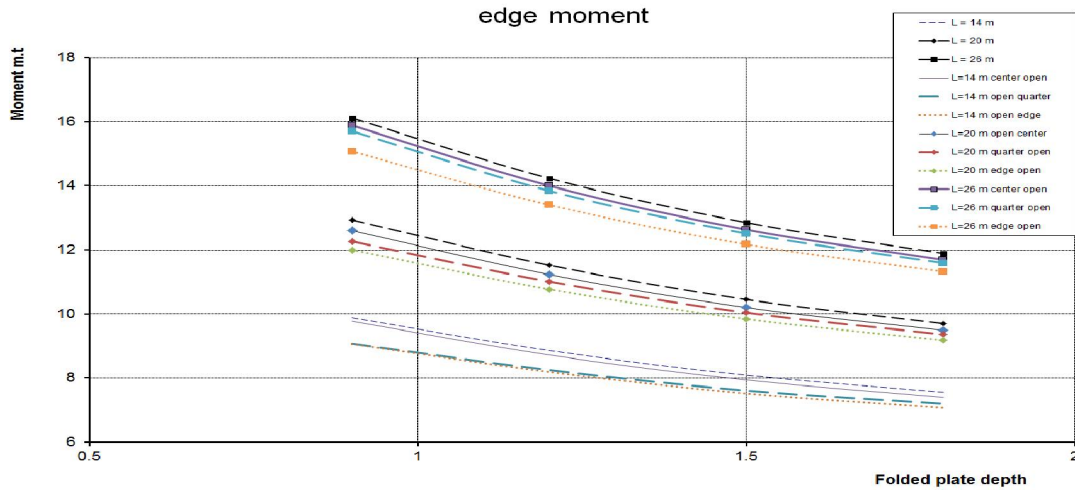
Figures 4-6 present the effects of the opening location and Q.F.P. rise on the bending moment of the intermediate beam. These figures show that the increase of the folded plate rise from 0.9 to 1.8 m results reduces the moment of the intermediate beam at different locations for all of the investigated models.



**Figure 4: Maximum moment of the intermediate beam versus different opening locations**



**Figure 5: Intermediate beam moment at the quarter versus different opening locations**

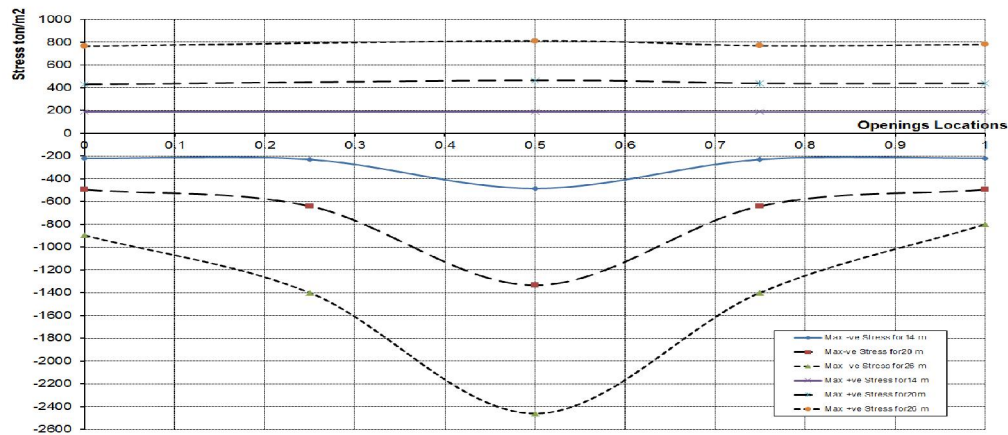


**Figure 6: Fixed end moment of the intermediate beam versus different opening locations**

**2.2.4 Effect of the Opening Location on the Folded Plate Compression and Tension Stresses**

For the minimum investigated rise of 0.9 m, the presence of different opening locations at the centre, quarter and edge of the beam results in a maximum tensile stress and maximum compression stress at the bottom and top of the beam, respectively. Changes in the opening location do not significantly affect the tensile stress, as shown in Figure 7. However, they do significantly affect the compression stresses. At the centre opening location of the 26-m, 20-m and 14-m

spans, the compression stress increased by 174%, 170% and 120 %, respectively, compared to the control beam. For the quarter opening location, the compression stress increased by 56%, 30% and 5 % for the 26-m, 20-m and 14-m spans, respectively. The edge opening location did not noticeably affect the compression stress, which was maximised near the location of the openings. For example, the maximum stress occurred at 0.57 L of the folded plate length for the quarter opening, while it occurred at 0.62 L of folded plate length for the edge opening location.



**Figure 7: Maximum tension and compression stress versus different openings locations**

**2.2.5 Effect of the Opening Location on the Folded Plate Compression and Tension Stresses**

In this study, the length of the openings ranged from 2 m to 8 m, while a constant width of 3 m was used for the different opening locations. The folded plate opening sizes do not significantly increase the static or

dynamic effects. For the longest investigated span of 26 m, the models indicated that the effects of these parameters are less than 6%.

**2.3 FREE VIBRATION ANALYSIS**

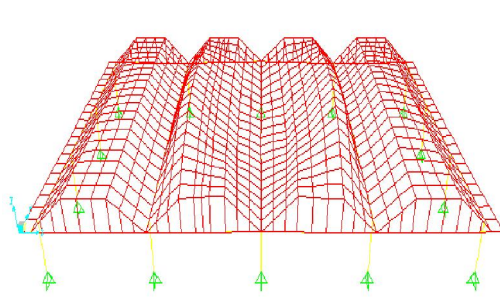
The investigated systems were subjected to a free vibration analysis. An eigenvalue analysis was adopted in the linear dynamic analysis. The arrangement of the mode shapes was consistent for the investigated spans with different parameters. Column sway dominates the first three modes, while the subsequent modes are dominated by roof deformations. Figure 8 displays the roof modes and clearly shows that the dominating mode for the Q.F.P. slab is the intermediate symmetric bending mode. The second mode was anti-symmetric bending, followed by the symmetric bending of the external spans and the alternating modes of anti-symmetric and symmetric plate bending.

In Figure 9, the fundamental frequencies of the system are provided for the Q.F.P. slabs with spans of 14, 20

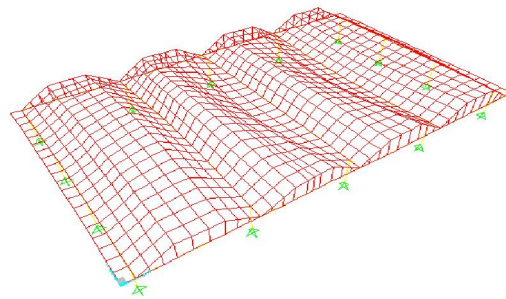
and 26 meters. As shown in the figure, the first three modes were column lateral displacements. The fundamental modes for the folded plate roof began at the fourth mode. The frequency of the three fundamental modes for the 14- and 20-m roof spans were very similar, while the frequency for the 26-m span was 27% lower than these frequencies.

Figure 10 presents the effect of the roof rise on the fundamental frequencies of the system for the 20-m span. Rises of 90 and 180 cm were investigated, and the figure shows that the system with the higher rise has 20% higher fundamental frequencies. This result is likely due to the higher stiffness of the roof in this case. Based on the free

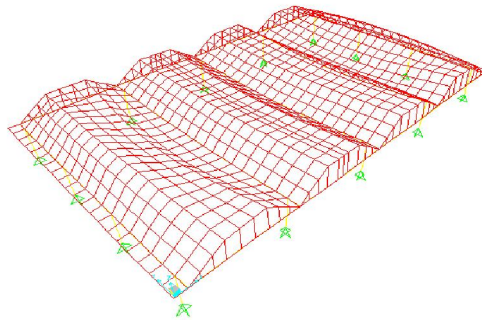
vibration analysis, the change in the opening location has no effect on the roof modes or the fundamental frequencies.



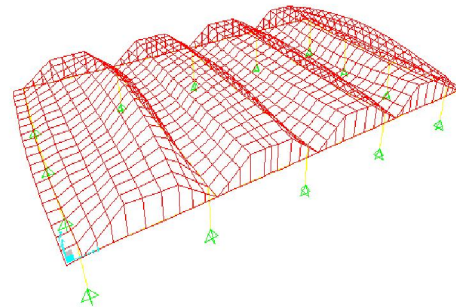
**First mode shape of the Q.F.P. slab**



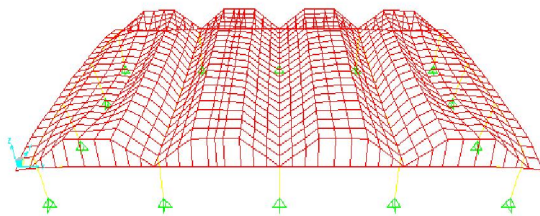
**Second mode shape of the Q.F.P. slab**



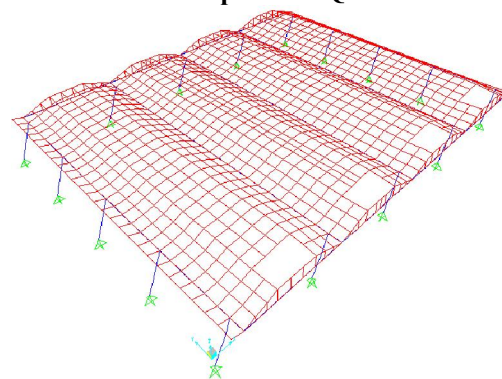
**Third mode shape of the Q.F.P. slab**



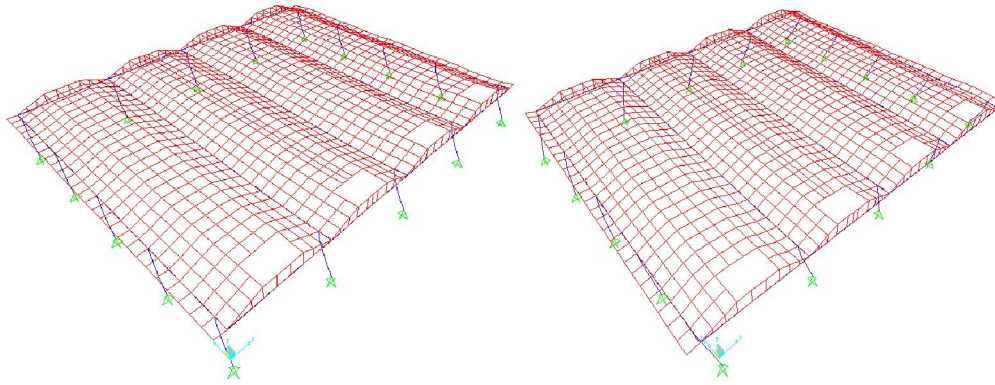
**Fourth mode shape of the Q.F.P. slab**



**Fifth mode shape of the Q.F.P. slab**



**Third mode shape of the Q.F.P. slab with edge openings**



Fourth mode shape of the Q.F.P. slab with edge openings    Fifth mode shape of the Q.F.P. slab with edge openings

Figure 8: Fundamental mode shapes of the Q.F.P. slab

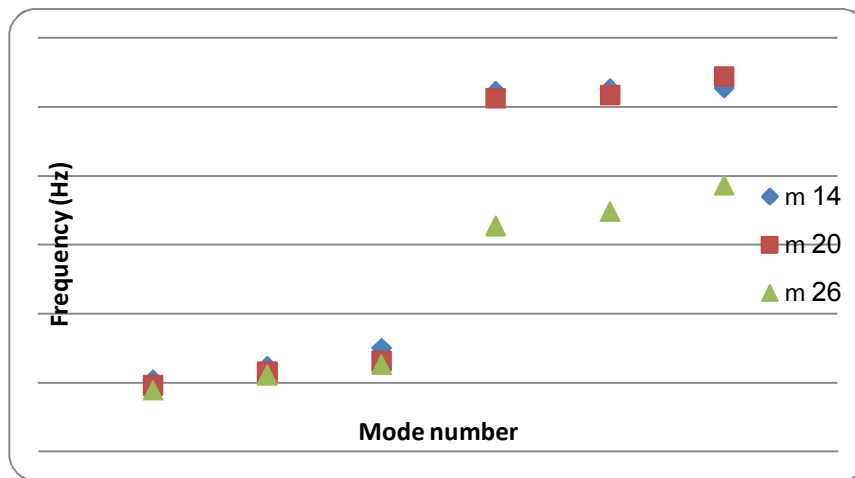


Figure 9: Fundamental mode frequencies for the folded plate roofs with different spans

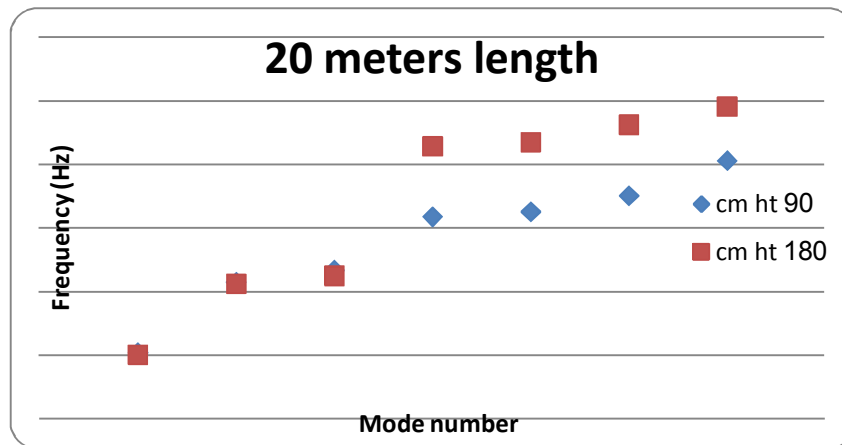


Figure 10: Effect of the folded plate rise on the fundamental frequency under free vibrations

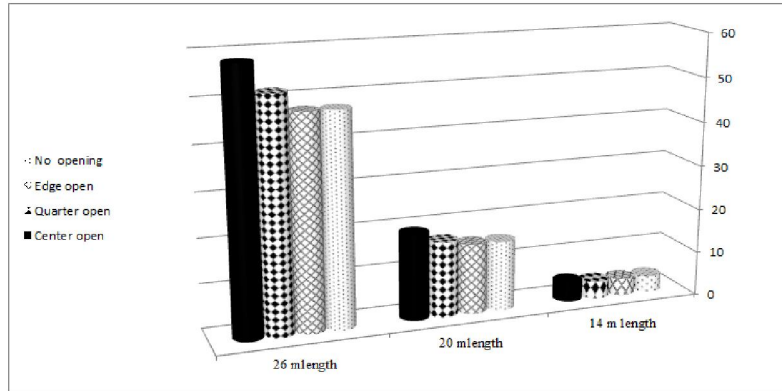
### 3 Discussions of Results

For the models considered in this study, the edge opening location has a limited effect on the Q.F.P. roofs' central deflection, maximum bending moment, edge moment of the intermediate beam and diaphragm bending moment. This effect did not exceed 3%, as shown in Figure 11. The central opening location

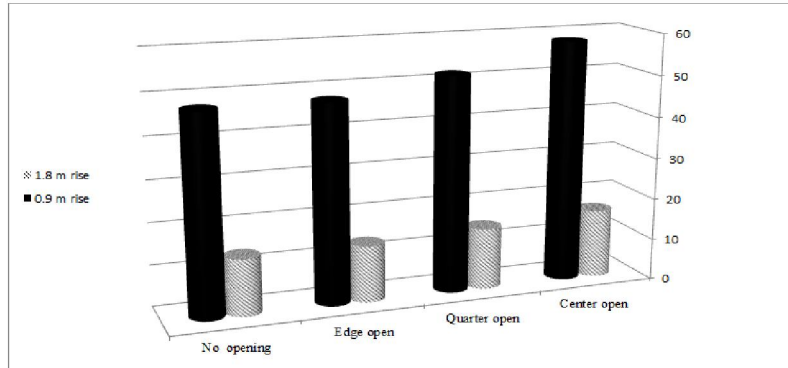
clearly affects the central deflection compared to the deflections induced in both the quarter and edge opening locations. The centre opening location increased the central deflection by 14 % more than the quarter opening for the 14-m and 20-m spans and by 14% more than the quarter opening for the 26-m span.

Doubling the height of the investigated folded plate from 0.9 meters to 1.8 meters is associated with an average deflection reduction of 72%, as shown in Figure 12, which also shows that the degree to which

the central deflection increases for the 20-m and 26-m spans were similar, while the effect of the opening location in the 14-m span was lower.



**Figure 11: Maximum deflection of the Q.F.P. slabs versus different opening locations for different lengths**



**Figure 12: Maximum deflection of the Q.F.P. slabs versus different opening locations for different lengths**

The maximum moment location occurred at the intermediate beam centre in the case of the control and centre opening location models. For the quarter opening location model, the maximum moment occurred at a length of 0.75 (at the opening location) for the intermediate beam length. The location of the maximum moment for the edge opening location model was similar to that of the control model, as presented in Figure 13.

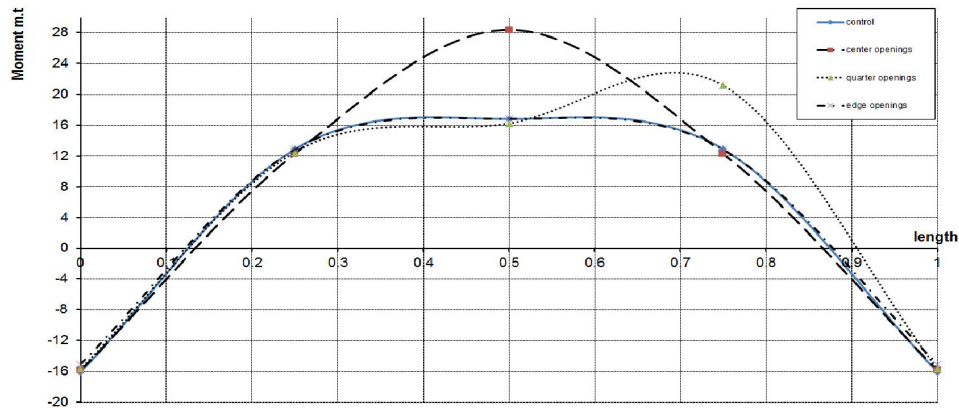
Varying the opening location significantly affected the maximum moment of the intermediate beam. The centre opening location increased the maximum moment by 67%, 60% and 42% for the 26 m, 20 m and 14 m spans, respectively. The maximum moment is located at the quarter of the intermediate beam at the opening location, and it increased by 64 %, 46% and 30% in the 26-m, 20-m and 14-ms, respectively, as shown in Figure 13. The effect of the opening location on the edge moment and diaphragm bending moment did not exceed 3%. At the quarter opening location, the opening location did not appear to affect the maximum moment at the intermediate beam centre or the edge moment.

The effect of the opening location on the maximum tensile stress was minor and did not exceed 4%. However, the opening location significantly affected the compression stress, which increased by 174%, 170%, and 120% for the 26-m, 20-m and 14-m spans, respectively. The quarter opening location increased the compression stress by 56%, 30%, and 5% for the 26-m, 20-m, and 14-m spans, respectively.

The openings size does not significantly affect the static or dynamic behaviour of the beam. Increasing the opening size decreased the total load on the folded plate system (dead or live load). For the dynamic analysis, varying the opening size did not significantly affect the folded plate inertia; thus, effect of the opening size on the dynamic behaviour was not significant.

The opening locations from the centre to the edge of the beam have a limited effect on the fundamental frequency of the Q.F.P. slab. The arrangement of the mode shape patterns was not affected by variations in the opening locations from the centre to the edge of the beam. In other words, the mode shapes were of the same order for all of the investigated spans.





**Figure 13: Intermediate beam moment versus different opening locations**

#### 4 Conclusions

- The proposed approximate static solutions for the fixed boundary conditions at the end diaphragms of the Q.F.P. slab ends (9, 10, and 11) are not reliable for the longer spans, especially for spans higher than 20 meters. Thus, the stiffness should be included in the numerical modeling.
- Increasing the folded plate rise was the most significant factor in improving the static analysis for deflection control, while increasing the roof rise increases the fundamental frequencies of the system.
- The rise effect was most effective in shorter spans.
- Increasing the folded plate rise enhanced the structural behaviour of the Q.F.P. system.
- The edge opening location has a limited effect on the Q.F.P. roofs static and dynamic behaviour.
- Centre and quarter opening locations have an effect on the change in static deflections, or straining actions of the system.
- Spans from 14 to 20 meters for the investigated Q.F.P. slabs had very close fundamental frequencies, this effect widens noticeably on analysing longer spans.
- The centre opening location significantly affected the Q.F.P. slabs deflection greater than quarter opening location by 14%.
- The variation of the opening location has a significant effect on the maximum moment of intermediate beam, while the centre opening location increased the maximum moment by 67%, 60% and 42% which corresponding spans of 26 m, 20 m, 14 m, respectively.
- The opening location has a limited effect on the edge moment and diaphragm bending moment,

which did not increase by more than 5% compared to the control model.

- The quarter opening location did not affect the maximum moment at the intermediate beam centre or the edge moment. The maximum moment is located at the quarter of the intermediate beam at the opening location, and compared to the control model, it increased by 64 %, 46% and 30% for the model lengths of 26 m, 20 m and 14 m, respectively.
- The opening location has no effect on the maximum tensile stress, while it significantly affects the compression stress, which increased by 174%, 170%, 120% for the spans of 26 m, 20 m, 14 m, respectively, compared to the control model.
- The opening locations from the centre to the edge of the beam have a limited effect on the fundamental frequency of the Q.F.P. slab.
- The openings sizes do not have a significant effect on the static and dynamic behaviour of the beams.
- The arrangement of the mode shape patterns was not affected by variations in the opening locations from the centre to the edge of the beam. In other words, the mode shapes were of the same order for all of the investigated spans.

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