Estimations and Prediction from the Inverse Rayleigh Model Based on Lower Record Statistics

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Abstract: This article considers estimation of the unknown parameters for the inverse Rayleigh distribution (*IRD*) based on lower record values. We consider the maximum likelihood (*ML*) and Bayesian inference of the unknown parameters of the model, as well as the reliability and cumulative hazard rate functions. The Bayes estimators are obtained relative to both symmetric (squared error) and asymmetric (linear exponential (*LINEX*)) loss functions. It is noticed that the symmetric and asymmetric Bayes estimators are obtained in closed forms. Bayesian prediction interval of the future record values are obtained as well. Finally, practical examples using real record values are given to illustrate the application of the results.
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1. Introduction

The Rayleigh distribution is a special case of the Weibull distribution, which provides a population model useful in several areas of statistics including life testing and reliability which age with time as its failure rate is a linear function of time. Various applications of this distribution are given in Siddiqui (1962), Polovko (1968), Gross and Clark (1975) and Lee *et al*. (1980). In the life distribution, if the random variable (r.v.) T has Rayleigh distribution, then the r.v. *X=1/T* has an *IRD*. The *IRD* was introduced in literature by Trayer (1964) (see, Mohsin and Shahbaz (2005)) and it has many applications in the area of reliability studies. Voda (1972) mentioned that the distribution of lifetimes of several types of exponential units can be approximated by the *IRD* and discussed some properties of maximum likelihood estimator (*MLE*) of the parameter θ. The probability density function (p.d.f.) of the *IRD* with scale parameter θ is

$$f(x;\,\theta)=\frac{2\theta}{x^3}\,e^{-\frac{\theta}{x^2}},\ x,\theta>0, \qquad (1.1)$$

and a cumulative distribution function (c.d.f.)

$$F(x;\,\theta)=\ e^{-\frac{\theta}{x^2}},\ x>0,\ \theta>0. \qquad (1.2)$$

The reliability, failure rate and the cumulative failure rate (hazard rate) functions of *IRD* are given, respectively, by

$$R(t;\,\theta)=1-F(t;\,\theta)=1-\ e^{-\frac{\theta}{t^2}}, \qquad (1.3)$$

$$h(t;\,\theta)=\frac{f(t;\,\theta)}{R(t;\,\theta)}, \qquad (1.4)$$

$$H(t;\,\theta)=-\ln R(t)=-\ln\left(1-\ e^{-\frac{\theta}{t^2}}\right). \qquad (1.5)$$

Record values are important in many real-life situations involving data relating to weather, sports, economics, and life-tests. The statistical study of record values have been pursued in different directions by several authors; see, Nagaraja (1988), Ahsanullah (1995) and Arnold *et al.* (1998). Some inferential methods based on record values for the Rayleigh and Weibull, generalized Pareto, Lomax, generalized exponential and power function distributions are studied by Balakrishnan and Chan (1993), Sultan and Moshref (2000), Sultan *et al.* (2001), Raqab (2002) and Sultan *et al.* (2002). Moreover, Abd-El-Hakim and Sultan (2001) have obtained the maximum likelihood estimators (MLE's) of Weibull parameters based on record values. Also, Shawky and Bakoban (2010) have derived moments and moment generating functions from *EG* distribution and have made some statistical inferences based on record values.

In this paper, Bayesian and non-Bayesian estimators are derived for scale parameters, reliability and failure rate functions based on lower record values from IR distribution. Soliman *et al.* (2010) discussed the same problem with different prior distribution and another technique.

Now, let $\{X_n, n\geq 1\}$ be an infinite sequence of i.i.d. random variables from an absolutely continuous distribution function F, and probability density function f. Let $X_{i:j}$ denote the i^{th} order statistic of the random sample $X_1, X_2, \ldots, X_j$, and $F_{i:j}$ be its cumulative distribution function. Let $T_k = \min\{X_1, X_2, \ldots, X_k\}$, $k \geq 1$. We say that X_j is a lower record value of this sequence if $T_j < T_{j-1}, j \geq 2$. By definition, X_1 is a record value. Let $L(n) = \min\{j: j > L(n-1), X_j < X_{L(n-1)}\}$, $n \geq 2$ with $L(1) = 1$. Then $X_{L(n)}, n \geq 1$, denotes the sequence of lower record values. From the above definition, the sequence of record statistics can be viewed as order statistics from a sample whose size is determined by the values and the order of occurrence of the observations.

In Bayesian estimation, we consider two types of loss functions. The first is the squared error loss function (quadratic loss) which is classified as a symmetric function and associates equal importance to the losses for overestimation and underestimation of equal magnitude. The second, introduced by Varian (1975), is the LINEX (linear-exponential) loss function which is known asymmetric. These loss functions were widely used by several authors; among of them Rojo (1987), Basu and Ebrahimi (1991), Pandey (1997), Soliman (2000), Nassar and Eissa (2004) and Shawky and Bakoban ((2008) & (2010)).

The quadratic loss for Bayes estimate of a parameter β, say, is the posterior mean assuming that exists, denoted by β_s. The LINEX loss function may be expressed as

$$l(\Delta) \propto e^{c\Delta} - c\Delta - 1, c \neq 0, \qquad (1.6)$$

where $\Delta = \hat{\beta} - \beta$. The sign and magnitude of the shape parameter c reflects the direction and degree of asymmetry, respectively. If $c > 0$, the overestimation is more serious than underestimation, and vice-versa. For c closed to zero, the LINEX loss is approximately squared error loss and therefore almost symmetric.

The posterior expectation of the LINEX loss function Equation (1.6) is

$$E_\beta\left[l(\hat{\beta} - \beta)\right] \propto \exp(c\hat{\beta})\, E_\beta[\exp(-c\beta)] - c\left(\hat{\beta} - E_\beta(\beta)\right) - 1, \qquad (1.7)$$

where $E_\beta(.)$ denotes the posterior expectation with respect to the posterior density of β. By a result of Zellner (1986), the (unique) Bayes estimator of β, denoted by $\hat{\beta}_L$ under the LINEX loss is the value $\hat{\beta}$ which minimizes (1.7), is given by

$$\hat{\beta}_L = -\frac{1}{c}\log\{E_\beta[\exp(-c\beta)]\}, \qquad (1.8)$$

Provided that the expectation $E_\beta[\exp(-c\beta)]$ exists and is finite [Calabria and Pulcini (1996)]. We are interested with maximum likelihood estimation as a classical approach among non-Bayesian methods. The maximum likelihood is based on the information Provided by empirical data. The invariant property was hold to obtain maximum likelihood estimators (MLE's) of reliability and failure rate functions.

In this paper, a discussion of the MLE's is considered in Section 2. In Section 3, Bayesian estimators is obtained. In Section 4, prediction of future records are derived. Numerical illustration and comparisons are presented in Section 5. Finally, conclusions are made in Section 6.

2. Maximum Likelihood Estimation

In this section, the maximum likelihood estimators (MLE's) of $IRD(\theta)$ are derived. We consider the case when θ is unknown. Let $x_1, x_2, \ldots$ be a sequence of i.i.d. random variables from $IRD(\theta)$, the joint density function of first n lower record values $\underline{x} = \left(x_{L(1)}, x_{L(2)}, \ldots, x_{L(n)}\right)$ is given by

$$f_{1,2,\ldots,n}\left(x_{L(1)}, x_{L(2)}, \ldots, x_{L(n)}\right) = \frac{\prod_{i=1}^{n} f(x_{L(i)})}{\prod_{i=1}^{n-1} F(x_{L(i)})}, \qquad (2.1)$$

where $f(.)$ and $F(.)$ are given by (1.1) and (1.2), respectively. Abbreviation $x_{L(i)} = x_i$.

The likelihood function of (2.1), is given by

$$L(\theta|\underline{x}) = u\, \theta^n\, e^{-q\,\theta}, \qquad (2.2)$$

where

$$q = x_n^{-2} \text{ and } u = \prod_{i=1}^{n} \frac{2}{x_i^3}. \qquad (2.3)$$

Then the log-likelihood function, is given by

$$\ell = \ln L(\theta|\underline{x}) = \ln u + n \ln\theta - q\,\theta. \qquad (2.4)$$

It follows, from (2.4), that the MLE of θ is

$$\hat{\theta} = n\, x_n^2\,. \qquad (2.5)$$

For a given t, the MLE of $R(t)$ is obtained by replacing θ by $\hat{\theta}$ in Equation (1.3), then MLE of $H(t) = -\log R(t)$ can be obtained.

3. Bayesian Estimation

The natural family of conjugate prior for θ is a gamma distribution with p.d.f.

$$g(\theta) = \frac{b^a}{\Gamma(a)}\theta^{a-1}e^{-b\theta},\ \theta > 0,\ a, b > 0. \qquad (3.1)$$

Applying Bayes theorem, we obtain, from Equations (2.2) and (3.1), the posterior density of θ as

$$g(\theta|\underline{x}) = \frac{B^A}{\Gamma(A)}\,\theta^{A-1}e^{-B\,\theta},\quad \theta > 0,\ a, b > 0, \qquad (3.2)$$

where $A = a + n$, $B = b + q$ and $q = x_n^{-2}$.

Estimation of θ:

The Bayes estimate $\hat{\theta}_{BS}$ of θ relative to squared error loss function is given by

$$\hat{\theta}_{BS} = \frac{A}{B}. \qquad (3.3)$$

Under LINEX loss function, the Bayes estimate $\hat{\theta}_{BL}$ of θ using Equation (1.8) can be obtained as

$$\hat{\theta}_{BL} = \frac{A}{c} \ln\left(1 + \frac{c}{B}\right). \quad (3.4)$$

Estimation of $R(t)$:

The Bayes estimate $\hat{R}_{BS}(t)$ of $R(t)$ relative to squared error loss function is given by

$$\hat{R}_{BS}(t) = 1 - \left(1 + \frac{t^{-2}}{B}\right)^{-A}. \quad (3.5)$$

Under LINEX loss function, the Bayes estimate of $R(t)$ using Equation (1.8) is

$$\hat{R}_{BL}(t) = 1 - \frac{1}{c} \ln\left\{\sum_{i=0}^{\infty} \frac{c^i}{i!}\left(1 + \frac{i\,t^{-2}}{B}\right)^{-A}\right\}. \quad (3.6)$$

Estimation of $H(t)$:

The Bayes estimate of the cumulative failure rate function $H(t) = -\ln R(t)$ relative to quadratic loss function is

$$\hat{H}_{BS}(t) = \sum_{j=1}^{\infty} \frac{1}{j}\left(1 + \frac{j\,t^{-2}}{B}\right)^{-A}. \quad (3.7)$$

When the LINEX loss function is appropriate, the Bayes estimate of $H(t)$ is

$$\hat{H}_{BL}(t) = \frac{-1}{c}. \ln\left\{\sum_{j=0}^{\infty} (-1)^j \binom{c}{j} \left(1 + \frac{j\,t^{-2}}{B}\right)^{-A}\right\}. \quad (3.8)$$

4. Prediction of the Future Records

In the context of prediction of the future record observations, the prediction intervals provide bounds to contain the results of a future record, which is based on the previous record observed from the same sample.

Let the first n lower record observations $\underline{x} = (x_{L(1)}, x_{L(2)}, \ldots, x_{L(n)})$, then the conditional density function of the s^{th} future lower record $Y = X_{L(s)}$, $1 \le n < s$, for given $x_n = x_{L(n)}$ is given (see Arnold *et al.*, 1998) by

$$f(y|x_n;\,\theta) = \frac{[G(y) - G(x_n)]^{s-n-1}}{\Gamma(s-n)} . \frac{f(y)}{F(x_n)}, \quad 0 < y < x_n < \infty, \quad (4.1)$$

where $G(x) = -\ln F(x) = \theta x^{-2}$, thus, from (1.1) and (1.2), relation (4.1) can be written as

$$f(y|x_n;\,\theta) = \frac{2\,\theta^{s-n}}{y^3\Gamma(s-n)} . [\xi(y)]^{s-n-1} e^{-\theta\xi(y)}, \quad (4.2)$$

where

$$\xi(y) = y^{-2} - x_n^{-2}. \quad (4.3)$$

The Bayes predictive density function of $Y = X_{L(s)}$ given the observed record x_n is given by

$$f^*(y|x_n) = 2\,C1 . \frac{[\xi(y)]^{s-n-1}}{y^3[B + \xi(y)]^{s+a}}, \quad 0 < y < x_n, \quad (4.4)$$

where $C1 = \frac{B^A}{B(s-n, A)}$ and $B(s-n, A)$ is a beta function.

Thus, the Bayesian prediction bounds for $Y = X_s$, given the previous data are obtained by evaluation the following predictive survival function, for some positive λ,

$$f(Y > \lambda|x_n) = \int_{\lambda}^{x_n} f^*(y|x_n) dy = \frac{InBeta(s-n, A, \delta)}{Beta(s-n, A)}, \quad (4.5)$$

where $\delta = \frac{x_n^2 - \lambda^2}{B\lambda^2 x_n^2} = \frac{x_n^2 - \lambda^2}{\lambda^2(bx_n^2 + 1)}$ and $InBeta(z_1, z_2, \delta)$ is the incomplete beta function defined by

$$InBeta(z_1, z_2, \delta) = \int_0^{\delta} \frac{t^{z_1-1}}{(1+t)^{z_1+z_2}} dt.$$

The lower and upper $100\tau\%$ prediction bounds for Y could be found numerically by finding λ from (4.5), using

$$\Pr[LL(x_n) < Y < UL(x_n)] = \tau,$$

where $LL(x)$ and $UL(x)$ are the lower and upper limits, respectively, satisfying

$$\Pr[Y > LL(x_n)|x_n] = \frac{1+\tau}{2}$$

$$\text{and} \quad \Pr[Y > UL(x_n)|x_n] = \frac{1-\tau}{2}. \quad (4.6)$$

As a special important case from (4.5), we predict the first unobserved record value $X_{L(n+1)}$ by putting $s = n+1$, then we get

$$f(x_{n+1} \ge \lambda|x_n) = 1 - (1+\delta)^{-A}. \quad (4.7)$$

From (4.6) and (4.7), the lower and upper $100\,\tau\,\%$ prediction bounds are given, respectively, by

$$LL(x_n) = \frac{x_n}{\{1 + (bx_n^2 + 1)[\left(\frac{1-\tau}{2}\right)^{-\frac{1}{A}} - 1]\}^{\frac{1}{2}}},$$

and

$$UL(x_n) = \frac{x_n}{\{1 + (bx_n^2 + 1)[\left(\frac{1+\tau}{2}\right)^{-\frac{1}{A}} - 1]\}^{\frac{1}{2}}}.$$

5. Illustrative Examples and Simulation Study

To illustrate the estimation and prediction techniques that were shown in the previous sections, we present two data sets.

Example 1 (Real Life Data Set)

This data set is obtained from Proschan (1963) and represents times between successive failures of air conditioning (AC) equipment in a Boeing 720 airplane and they are as follows: 502, 386, 326, 153, 74, 70, 59, 57, 48, 29, 29, 27, 26, 21, 12, we fit the inverse Rayleigh distribution by used Kolmogorov-Simirnov (K-S) test. It is observed that, the K-S distance is 0.21378 with the corresponding P value is 0.43879. For this data set, the Chi- square value is 2.6383. Therefore, it is clear that inverse Rayleigh model fits quite well to the data set. Using our results in Sections 2 and 3, the MLEs $(.)_{ML}$ and the Byes estimators ($(.)_{BS}$, $(.)_{BL}$) of $\theta, R(t)$ and $H(t)$ have been computed and the results are given in Tables 1 and 2. Using the prediction procedure described in Section 4, the 90%, 95% and 99% prediction intervals for the next lower record x_{16} are computed respectively, as follows $(LL(x_{16}), UL(x_{16})) = (3.11395, 3.56848)$, $(3.21959, 3.44610)$ and $(3.30755, 3.35281)$.

Table 1: Estimated values of $\theta, R(t)$ and $H(t)$ with actual Values ($\theta = 2.005$, $a = 2, b = 2, t = 0.75, R(0.75) = 0.97169$ and $H(0.75) = 0.02872$).

Parameters	$(.)_{ML}$	$(.)_{BS}$	$(.)_{BL}$			
			$c = -0.5$	$c = 0.001$	$c = 2$	$c = 3$
θ	2160	8.47059	9.74200	8.46848	5.87703	5.18054
$R(t)$	1	0.99998	0.99998	0.99998	0.99998	0.99998
$H(t)$	0	0.00016	0.00002	0.00002	0.00002	0.00002

Table 2: MSEs of the estimates $\theta, R(t)$ and $H(t)$ when ($\theta = 2.005, a = 2$, $b = 2, t = 0.75, R(0.75) = 0.97169$ and $H(0.75) = 0.02872$).

Parameters	$(.)_{ML}$	$(.)_{BS}$	$(.)_{BL}$			
			$c = -0.5$	$c = 0.001$	$c = 2$	$c = 3$
θ	4.65694×10^6	41.80380	59.86110	41.77660	14.9926	10.08400
$R(t)$	0.00080	0.00080	0.00080	0.00080	0.00080	0.00080
$H(t)$	0.00082	0.00082	0.00082	0.00082	0.00082	0.00082

As shown from Table 2 that the Bayes estimates for all parameters are better than the MLE's estimates.

Example 2 (Simulated Data):

In order to assess the statistical performances of these estimates, a simulation study is conducted. The estimated mean and the mean square errors (MSE's) are computed for each estimator. The random samples are generated as follows:

1. For $\theta = 2.05$, we generate a random samples of sizes n=3, 5, 7, 10 and 15.
2. Using θ, obtained in step (1), with

$$a = 1.2, b = 1, t = 5, R(5) = 0.078728$$

and $H(5) = 2.54176$, the MLEs and the Bayes estimates relative to squared error loss and LINEX loss are computed.
3. Using the prediction procedure described in Section 4, the 95% prediction interval for the next lower records are computed.
4. The above steps are repeated 1000 times and the mean square errors are computed for each method.

Our computational results were computed by using Mathematica 8.0. Estimates, MSE's and prediction intervals are displayed in Tables 3, 4 and 5.

Table 3: Estimated mean values of $\theta, R(t)$ and $H(t)$ with actual Values ($\theta = 2.05$, $a = 1.2, b = 1, t = 5, R(5) = 0.078728$ and $H(5) = 2.54176$).

n	Parameters	$(.)_{ML}$	$(.)_{BS}$	$(.)_{BL}$			
				$c = -0.5$	$c = 0.001$	$c = 2$	$c = 3$
3	θ	2.99361	1.88553	2.17131	1.88507	1.32291	0.17202
	$R(t)$	0.10878	0.07179	0.07202	0.07172	0.07055	0.06998
	$H(t)$	2.39402	2.79622	2.86222	2.79606	2.58279	2.49696
5	θ	2.59565	2.00985	2.21562	2.00949	1.52737	1.38104
	$R(t)$	0.09729	0.07645	0.07668	0.07645	0.07555	0.07511
	$H(t)$	2.42947	2.68977	2.73172	2.68968	2.54457	2.48240
7	θ	2.48476	2.08618	2.24524	2.08589	1.66907	1.53126
	$R(t)$	0.09387	0.07940	0.07959	0.79940	0.07867	0.07831
	$H(t)$	2.42921	2.62383	2.65452	2.62377	2.51381	2.46505
10	θ	2.42726	2.14358	2.26337	2.14336	1.79879	1.67545
	$R(t)$	0.09193	0.08159	0.08173	0.08159	0.08102	0.08074
	$H(t)$	2.43880	2.57999	2.60186	2.57994	2.49924	2.46236
15	θ	1.95219	1.86557	1.92143	1.86546	1.67890	1.60244
	$R(t)$	0.07512	0.07175	0.07182	0.07175	0.07146	0.07131
	$H(t)$	2.38871	2.54430	2.59060	2.56889	2.45236	2.35960

Table 4: MSE of θ, $R(t)$ and $H(t)$ when ($\theta = 2.05, a = 1.2, b = 1, t = 5$, $R(5) = 0.078728$ and $H(5) = 2.54176$).

n	Parameters	$(.)_{ML}$	$(.)_{BS}$	$(.)_{BL}$			
				c= - 0.5	*c*=0.001	*c*= 2	*c*= 3
3	θ	2.50424	0.37600	0.63278	0.37581	0.62311	0.83300
	$R(t)$	0.00662	0.00051	0.00051	0.00051	0.00050	0.00050
	$H(t)$	0.35006	0.16169	0.20043	0.16163	0.09553	0.09437
5	θ	2.19838	0.36540	0.60434	0.36515	0.40672	0.54038
	$R(t)$	0.00258	0.00051	0.00051	0.00051	0.00049	0.00048
	$H(t)$	0.20315	0.10777	0.12248	0.10774	0.08379	0.08630
7	θ	1.17017	0.32847	0.46853	0.32830	0.30776	0.39843
	$R(t)$	0.00142	0.00043	0.00043	0.00043	0.00042	0.00041
	$H(t)$	0.12735	0.07820	0.08598	0.07819	0.06574	0.07793
10	θ	1.14605	0.30293	0.40466	0.30269	0.25988	0.28392
	$R(t)$	0.00141	0.00037	0.00037	0.00037	0.00040	0.00038
	$H(t)$	0.12629	0.07250	0.07452	0.07250	0.06325	0.07788
15	θ	0.19042	0.06424	0.11522	0.06416	0.00052	0.01730
	$R(t)$	0.00025	0.00008	0.00008	0.00008	0.00007	0.00007
	$H(t)$	0.03400	0.00647	0.00432	0.00647	0.01852	0.02631

Table 5: The lower (LL), the upper (UL) and the width of the 95% prediction intervals for the future lower record $X_{L(n+1)}, n = 3, 5, 7, 10$ and 15.

n	Previous Record Values	LL	UL	Width
3	{0.93449, 0.77995, 0.75574}	0.69472	0.70256	0.00784
5	{0.67174, 0.51245, 0.48736, 0.48069, 0.44281}	0.43337	0.43466	0.00129
7	{1.23761, 0.925816, 0.72417, 0.59770, 0.56927, 0.56716, 0.52462}	0.51260	0.51352	0.00092
10	{0.916405, 0.85255, 0.58738, 0.58253, 0.56952, 0.54601, 0.49919, 0.48114, 0.43236, 0.41092}	0.40655	0.40714	0.00059
15	{1.50770, 1.29072, 0.75962, 0.72922, 0.61196, 0.59017, 0.57244, 0.53403, 0.51315, 0.50736, 0.49481, 0.48884, 0.46894, 0.36701, 0.35995}	0.35788	0.35817	0.00029

Tables 1 and 3 show the mean estimates. From Tables 2 and 4, we see that the Bayes estimates for all parameters are better than the MLEs estimates. Table 5 shows the lower and the upper 95% prediction bounds for the next record values ($X_{L(n+1)}$), when n =3, 5, 7, 10 and 15.

6. Conclusion

In this paper we have presented the Bayesian and non-Bayesian estimates of the parameter, reliability function R(t) and cumulative failure rate function H(t) for the lifetime follow the inverse Rayleigh distribution. The estimations are conducted on the MSE of estimated parameters. The MLEs are obtained based on record values. Bayes estimators, under squared error loss and LINEX loss functions, are also derived.

Our observations concerning the results are stated in the following points:

1- Estimation: Tables 1 and 3 show the mean estimates. From Tables 2 and 4, we observe that the Bayes estimates perform better than the MLEs, we also observe that the MSEs decreases as n increases.

2- Prediction: We conclude, from Table 5, that the width of the predictive decreases as n increases.

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