Solitary Wave Sol's for the Generalized Fifth Order KdV eqn

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Abstract: In this letter, different kinds of solutions including breather-type soliton and two soliton solutions are obtained for a generalized variable-coefficients fifth-order KdV equation by means of the bilinear method and extended homoclinic test approach. The proposed method can also be applied to solve other types of higher dimensional integrable and non-integrable systems.

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1. Introduction:

The Korteweg-de Vries (KdV) equation has been derived in fields such as shallow water waves, stratified internal waves, ion-acoustic waves, plasma physics and lattice dynamics [1]. When high-order dispersion is considered, the fifth-order KdV (fKdV) equations have been seen in some physical contexts, usually investigated in the exponential asymptotic investigating a generalized variable-coefficient fKdV equation and numerical calculations [2, 3]. Several integrable fKdV, e. g.; the Lax's equation, the Sawada-Kotera (SK) equation and the Kaup-Kupershmidt equation, have been discussed, which have analytic solutions and infinite sets of conservation laws [4-6]. Besides, the higher-order KdV-modified KdV equations with higher-degree nonlinear terms describing gravity waves in the atmosphere have been the periodic and solitary wave solutions of which have been obtained in Li, (2008) [7].

Due to the inhomogeneities of media and non-uniformities of boundaries, the variable-coefficient nonlinear evolution waves, ion-acoustic waves, plasma physics and lattice dynamic equations can be used to describe the real physical backgrounds [8-11].

In this paper, with the aid of symbolic computation [9-11], our interest will be devoted to investigating a generalized variable-coefficient fKdV equation such as the one given

$$u_t + a(t)uu_{xxx} + b(t)u_xu_{xx} + c(t)u^2u_x + d(t)uu_x + e(t)u_{xxx} + l(t)u_{xxxxx} + m(t)u + n(t)u_x = 0, \qquad (1)$$

where $u(x,t)$ is a function of space variable x and time variable t and $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$, $l(t)$, $m(t)$ and $n(t)$ are analytic functions of t. If the parameters are specially chosen, a series of equations can be obtained, which are integrable [4-6] and can be used to describe such physical phenomena as the interaction between a water wave and a floating ice cover and the gravity capillary waves [2, 3].

In case of $d(t)=6$, $e(t)=1$, $l(t)=\xi^2$ and $a(t)=b(t)=c(t)=m(t)=n(t)=0$, Eq. (1) reduces to

$$u_t + 6uu_{xx} + u_{xxx} + \xi^2 u_{xxxxx} = 0, \qquad (2)$$

which has been proposed for the interaction between a water wave and a coating ice cover in river channels and the gravity-capillary waves with the Bond number close to and slightly less than $1/3$, where $u(x,t)$ is the scaled depth, x and t are the scaled space and time coordinates, respectively and ξ is a small parameter [2].

For $a(t)=1$, $b(t)=2$, $d(t)=3$, $e(t)=-\theta$, $l(t)=(2/15)$, $c(t)=m(t)=n(t)=0$, Eq. (1) reduces to

$$u_t + 3uu_x + 2u_xu_{xx} + uu_{xxx} - \theta u_{xxx} + (2/15)u_{xxxxx} = 0, \qquad (3)$$

which has been derived for the classical gravity-capillary water-wave problem, where θ is a scale parameter [3].

In the limiting case $a(t)=b(t)=15$, $c(t)=45$, $l(t)=1$ and $d(t)=e(t)=m(t)=n(t)=0$, Eq. (1) reduces to integrable SK equation of the form

$$u_t + 15uu_{xxx} + 15u_xu_{xx} + 45u^2u_x + u_{xxxxx} = 0, \qquad (4)$$

which has been investigated in [5, 12, 13].

When $d(t)=e(t)=m(t)=n(t)=0$, Eq. (1) reduces to integrable SK equation as

$$u_t + a(t)uu_{xxx} + b(t)u_x u_{xx} + c(t)u^2 u_x + l(t)u_{xxxxx} = 0, \quad (5)$$

some soliton solutions of which have been obtained in [14].

The integrable nonlinear evolution equations (NLEEs) possess several properties, e. g.; N-soliton solutions, Backlünd transformation, Lax's pair and infinite sets of conservation laws [1, 4-6, 9-11]. Since there are choices for the parameters, the variable-coefficient NLEEs can be considered as generalizations of the constant coefficient ones [9-11]. Under certain constraint conditions, the variable-coefficient models may be proved to be integrable and given explicit analytic solutions [15]. The corresponding constraint conditions on Eq. (1) in this paper, which are obtained by the Painlevé analysis [16] and conditions from the variable-coefficient models mapped to the completely integrable constant-coefficient counterparts [14] will be

$$a(t)=b(t)=\frac{15l(t)}{\rho}\exp[\int m(t)dt], \quad (6a)$$

$$c(t)=\frac{45l(t)}{\rho^2}\exp[2\int m(t)dt], \quad (6b)$$

$$d(t)=e(t)=0, \quad (6c)$$

where $\rho \neq 0$ is an arbitrary constant.

It is worth noting that there have been no discussions on Eq. (1) under conditions (6). Considering such insufficiency, we will apply the Hirota bilinear method [13, 17, 18] to investigate the integrability for Eq. (1) and the characteristic-line method [19] to discuss the effect of the variable coefficients in Eq. (1).

The structure of this paper will be organized as follows: In section 2, with symbolic computation, the bilinear form of Eq. (1) is obtained. In order to illustrate the proposed method, we consider a generalized variable-coefficients fifth-order KdV equation and new periodic wave solutions are obtained, which included periodic two solitary solutions, doubly periodic solitary solution. Finally, conclusion and discussion are given in section 3.

2. Soliton solutions of fifth-order KdV equation with variable coefficients

We make the dependent variable transformation

$$u(x,t)=2\rho\exp[-\int m(t)dt]\log[f(x,t)]_{xx}, \quad (7)$$

Where $f(x,t)$ is a real function of x and t. The bilinear equation of Eq. (1) turns out to have the following form

$$[D_x D_t + l(t)D_x^6 + n(t)D_x^2]f(x,t).f(x,t)=0, \quad (8)$$

Where $D_x^m D_t^n$ is the Hirota bilinear derivative operator [18] defined by

$$D_x^m D_t^n f(x,t).g(x,t)=(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'})^m(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'})^n[f(x,t)g(x',t')]_{x'=x,t'=t}, \quad (9)$$

This definition is used to give

$$(D_x^6)f.f=2[f_{6x}f-6f_{5x}f_x+15f_{4x}f_{2x}-10f_{3x}f_{3x}] \quad (10a)$$

$$(D_x D_t)f.f=2[f_{xt}f-f_x f_t], \quad (10b)$$

$$(D_x^2)f.f=2[f_{xx}f-f_x f_x]. \quad (10c)$$

To solve the reduced Eq. (8) by means of the extended homoclinic test function [20-28], we suppose a solution of Eq. (8) as

$$f(x,t)=\exp(k_1 x+c_1 t)+p_1\cos(k_2 x+c_2 t)+q_1\exp(-k_1 x-c_1 t), \quad (11)$$

Where p_1, q_1, c_i, k_i, $(i=1,2)$ are parameters to be determined later.

Substituting Eq. (11) into (8) and equating all coefficients of $\exp[\pm(k_1 x+c_1 t)]$, $\cos(k_2 x+c_2 t)$, $\sin(k_2 x+c_2 t)$ to zero, one gets a set of algebraic equation for p_1, q_1, c_i, k_i, $(i=1,2)$. Solving this set of algebraic equations with the aid of Maple leads to many solutions, from which the following four solutions are chosen as:

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The set of coefficients of solution (11) are given as:

$$k_1=k_1, \qquad k_2=k_2, \quad (12a)$$

$$c_1=-k_1[n(t)+16k_1^4 l(t)], \; c_2=c_2, \quad (12b)$$

$$q_1=q_1, \qquad p_1=0. \quad (12c)$$

According to this set of coefficients, (11) leads to

$$f(x,t)=\exp\{k_1 x-k_1[n(t)+16k_1^4 l(t)]t\} + q_1\exp\{-k_1 x+k_1[n(t)+16k_1^4 l(t)]t\}. \quad (13)$$

Substituting this function into (7) gives a new periodic wave solution of (1) as follows:

$$u(x,t)=8\rho q_1 k_1^2\exp[-\int m(t)dt] / \left\langle \exp\{k_1 x-k_1[n(t)+16k_1^4 l(t)]t\} + q_1\exp\{-k_1 x+k_1[n(t)+16k_1^4 l(t)]t\}\right\rangle^2. \quad (14)$$

Case (2):

For this case, the coefficients of the solution (11) are taken as:

$$k_1=k_1, \qquad k_2=k_2, \quad (15a)$$

$$c_1=-k_1[(k_1^4-10k_1^2k_2^2+5k_2^4)l(t)+n(t)], \quad (15b)$$

$$c_2=-k_2[(k_2^4-10k_1^2k_2^2+5k_1^4)l(t)+n(t)], \quad (15c)$$

$$q_1 = \frac{-p_1^2 k_2^2 (k_1^2 - 3k_2^2)}{4k_1^2 (3k_1^2 - k_2^2)}, \; p_1 = p_1. \tag{15d}$$

These coefficients lead to a form of solution (11) as:

$$f(x,t) = \exp\{k_1 x - k_1[(k_1^4 - 10k_1^2 k_2^2 + 5k_2^4) l(t) + n(t)]t\}$$
$$+ p_1 \cos\{k_2 x - k_2[(k_2^4 - 10k_1^2 k_2^2 + 5k_1^4) l(t) + n(t)]t\}$$
$$+ [\frac{-p_1^2 k_2^2 (k_1^2 - 3k_2^2)}{4k_1^2 (3k_1^2 - k_2^2)}]$$
$$* \exp\{-k_1 x + k_1[(k_1^4 - 10k_1^2 k_2^2 + 5k_2^4) l(t) + n(t)]t\}. \tag{16}$$

Substituting this solution form into (7) admits to a new periodic solitary wave solution of (1).

Case (3):

In this case, the coefficients of solution (11) are represented by

$$k_1 = ik_2, \qquad k_2 = k_2, \tag{17a}$$

$$c_1 = ik_2[16k_2^4 l(t) + n(t)], \; c_2 = -k_2[16k_2^4 l(t) + n(t)], \tag{17b}$$

$$p_1 = p_1, \qquad q_1 = \frac{p_1^2}{4}. \tag{17c}$$

These coefficients are used into (11) to give

$$f(x,t) = \exp\{ik_2 x + ik_2[16k_2^4 l(t) + n(t)]t\}$$
$$+ p_1 \cos\{k_2 x - k_2[16k_2^4 l(t) + n(t)]t\}$$
$$+ \frac{p_1^2}{4} \exp\{-ik_2 x - ik_2[16k_2^4 l(t) + n(t)]t\}. \tag{18}$$

Inserting this equation into (7) admits to a new periodic solitary wave solution of (1).

Case (4):

Finally, we can take a set of coefficients to solution (11) as follows:

$$c_1 = -\sqrt{3}k_2[16k_2^4 l(t) - n(t)], \; c_2 = -k_2[16k_2^4 l(t) + n(t)], \tag{19a}$$

$$k_1 = \sqrt{3}k_2, \qquad k_2 = k_2, \tag{19b}$$

$$p_1 = p_1, \qquad q_1 = 0. \tag{19c}$$

These coefficients lead to solution (11) of the form

$$f(x,t) = \exp\{\sqrt{3}k_2 x - \sqrt{3}k_2[16k_2^4 l(t) - n(t)]t\}$$
$$+ p_1 \cos\{k_2 x - k_2[16k_2^4 l(t) + n(t)]t\}. \tag{20}$$

Using this solution into (7) admits to a new periodic solitary wave solution of (1).

3. Conclusion

Using a bilinear form and the extended homoclinic test approach, we obtain breather-type soliton and two soliton solutions for the generalized variable-coefficients fifth-order KdV equation. The results show that the extended homoclinic test approach is very effective in finding exact solitary wave solutions for nonlinear evolution equation with variable coefficients.

It is worthwhile to mention that the proposed method is reliable and effective and can also be applied to solve other types of higher dimensional integrable and non-integrable systems of nonlinear evolution equations.

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