Image Denoising based on Sparse Representation and Non-Negative Matrix Factorization

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Abstract: Image denoising problem can be addressed as an inverse problem. One of the most recent approaches to solve this problem is sparse decomposition over redundant dictionaries. In sparse representation we represent signals as a linear combination of a redundant dictionary atoms. In this paper we propose an algorithm for image denoising based on Non Negative Matrix Factorization (NMF) and sparse representation over redundant dictionary. It trains the initialized dictionary based on training samples constructed from noised image, then it search for the best representation for the source by using the approximate matching pursuit (AMP) which uses the nearest neighbor search to get the best atom to represent that source. During that it alternates between the dictionary update and the sparse coding. We use this algorithm to reconstruct image from denoised one. We will call our algorithm N-NMF.
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1. Introduction

Images always contaminated with noise in the image acquisition process and transmission phases, and denoising is an essential step to improve the image quality by removing the noise without affecting the important image features as much as possible. Commonly, noise removal has been done by using many denoising schemes, from the earlier smoothing filters like adaptive Wiener filter to the frequency domain denoising methods (Gonzalez and Woods 2002) to the lately developed methods which uses multiscale and directional transformations like wavelet, curvelet and ridgelet (Jean-Luc, Candes et al. 2002; Chen and Kegl 2007; Liu and Xu 2008; Sveinsson, Semar et al. 2008)

The success of the wavelet-based models is due to the tendency of images to become sparse in the wavelet domain, which implies that the image can be represented by using a small subset of the wavelet coefficients

One of the WT drawbacks when representing an image with a rich amount of local features is that only one fixed dictionary cannot represent well all this local features and some artifacts will appear in the denoised image.

To overcome this drawback in wavelet transform, a dictionary learning methods had been proposed to learn the dictionary from the data instead of using fixed dictionary. Elad and Aharon (M.Aharon, M.Elad et al. 2006; M.Elad and M.Aharon 2006) proposed sparse redundant representation and K-SVD based denoising algorithm by training a highly over-complete dictionary. Foi et al.(A.Foi, V.Katkovnik et al. 2007) applied a shape-adaptive discrete cosine transform (DCT) to the neighborhood, which can achieve very sparse representation of the image and hence lead to effective denoising. Other techniques uses factorization methods like PCA and SVD to do blockwise analysis in order to conduct image denoising by modeling each pixel and its neighborhood as a vector variable(Zhang, Dong et al. 2010).

2. Sparse signal representation

Sparse representations for signals become one of the hot topics in signal and image processing in recent years. It can represent a given signal $x \in R^n$ as a linear combination of few atoms in an overcomplete dictionary matrix $A \in R^{n\times K}$ that contains K atoms $\{a_i\}_{i=1}^K$ (K > n). The representation of x may be exact x=As or approximate, $x \approx As$, satisfying $\|x - As\|_p \le \varepsilon$, where the vector s is the sparse representation for the vector x.

To find s we need to solve either

$$(P_0)\ \min_s \|s\|_0 \ \text{subject to}\ x = As \qquad (1)$$

Or

$$(P_{0,\varepsilon})\ \min_s \|s\|_0 \ \text{subject to}\ \|x - As\|_2 \le \varepsilon \qquad (2),$$

where $\|\ \|_0$ is the l_0 norm, the number on non-zero elements.

Sparse representation in the presence of noise

Suppose that we want to estimate the source signal z from the observed noised version x

$$x = z + n,$$

where n is a white Gaussian noise. If we assume that the source signal z has a sparse representation over an overcomplete dictionary A, i.e.

$$z = As,$$

where s is the sparse representation of z over the dictionary A, then the problem can be formulated as in equation (2).

In this paper we use an algorithm for solving this problem. Our algorithm likes the known K-SVD algorithm but instead of using the SVD decomposition for dictionary atoms update and the Orthogonal Matching Pursuit (OMP) for sparse representation for the data matrix, we use the Non-Negative Matrix Factorization the dictionary atoms update and the Approximate Matching Pursuit the sparse representation for the data matrix. Also we choose the Gabor dictionary as an initial dictionary instead of the DCT dictionary used on the K-SVD.

2.1 Approximate Matching Pursuit

Given an input signal $x \in R^M$, and a dictionary, $A \in R^{M \times K}$, we want to found a vector of coefficients $s \in R^K$ that minimizes $\|x - As\|_2$. The approximate matching pursuit (AMP) algorithm is described in Algorithm1. This algorithm is similar to the orthogonal matching pursuit algorithm (OMP) except that it addresses the main computational bottleneck for large dictionaries by using nearest neighbor search by allowing any adequately near neighbor to be selected as a component instead of compute a large amount of inner product.

Algorithm 1. Approximate Matching Pursuit

-Input: dictionary $A = [a_1, a_2, ..., a_K] \in R^{M \times K}$,

data $x \in R^M$.

-Initialization: Let $r = x$, $s = 0$, L=∅, err=x'x

- While $err > \varepsilon$ do

- Find any i such that a_i and r are Near Neighbors

-$L = L \cup$ i

-Solve $s_i = \arg\min_{s_i, i \in L} \left\| x - \sum a_i s_i \right\|$

-$r = x - \sum_{i \in L} a_i s_i$

-End while

-Output s

3. Non-Negative Matrix Factorization

Factorization the data into simple, fundamental factors allows humans to identify the most meaningful components of data. Many real-world data are nonnegative and the corresponding hidden components have a physical meaning only when nonnegative. In practice, both nonnegative and sparse decompositions of data are often either desirable or necessary when the underlying components have a physical interpretation. For example, in image denoising, involved variables and parameters may correspond to pixels, and nonnegative sparse decomposition is related to the extraction of relevant parts from the images (Lee and Seung 1999),(Lee and Seung 2001).

The basic NMF problem can be stated as follows:

Given a nonnegative data matrix $X \in R_+^{I \times T}$ (with $x_{it} \geq 0$ or equivalently $X \geq 0$ and a reduced rank J $(J \leq \min(I, T))$, find two nonnegative matrices $A = [a_1, a_2, ..., a_K] \in R_+^{I \times T}$ and $S = B^T = [b_1, b_2, ..., b_J]$ which factorize X as well as possible, that is $X = AS + E = AB^T + E$, where the matrix $E \in R^{I \times J}$ represents approximation error.

4. Image denoising based on N-NMF

In this section, we introduce our N-NMF, which uses Nearest Neighbor search with the Nonnegative Matrix Factorization for image denoising. We choose the Gabor dictionary as an initial dictionary for some reasons. Firstly, choosing Gabor functions allows us to avoid some of the artificial block edge effects of the DCT basis, which used in the K-SVD, since Gabor functions tend to decay smoothly at the edges. Separability is another nice property, since it reduces the computation necessary to perform the matching search(Neff and Zakhor 1995). The other reason is that the Gabor dictionary gives a recovery rate higher than the DCT dictionary(Schnass and Vandergheynst 2008). We generate an initial dictionary A of size $M \times K$ (K>>M) from the Gabor basis functions. For a noised image of size $M \times M$ we generate blocks of size $\sqrt{M} \times \sqrt{M}$ by using a sliding window moving over the image, and then each block is used as a column of the data matrix X, which used after that for learning the dictionary. Then we use the N-NMF algorithms, which alternate between the sparse representation of the data with fixed dictionary and updating the dictionary with fixed representation to get the best dictionary to represent the important component in the image. At the end we use the learned dictionary to reconstruct the source image.

Algorithm 2. N-NMF algorithm.

1. Initialization: Set an overcomplete dictionary $A=[a_1,a_2,...,a_K]\in R^{M\times K}$ from the Gabor wavelet basis (GW) s.t. $\|a_k\|=1$ for all k .
2. Repeat until met the error goal
   - Sparse coding: find the sparse represent--ation $S=[s_1,s_2,...,s_N]$ for data matrix $X=[x_1,x_2,...,x_N]$ based on the fixed dictionary A by using the approximate matching pursuit algorithm (AMP).

     For each column i=1,2,…,N solve

     $$\hat{s}_i=\arg\min_{s_i}\|s_i\|_0 \text{ s.t.} \|x_i-As_i\|\le\varepsilon$$
   - Dictionary update: update the dictionary atoms while fixing the data matrix X and the sparse representation S by using the Nonnegative Matrix Factorization (NMF) for the overall representation error.
3. Reconstruction: reconstruct the denoised image $I_d=A\hat{s}$

5. Experiments and Results

In this work, we used an overcomplete Gabor dictionary as an initial dictionary of size 64x256 generated by using Gabor filter basis of size 8x8, each basis was arranged as an atom in the dictionary. The dictionary was learned by alternating between sparse coding with the currant dictionary and dictionary update with the current sparse representation. For doing that, we use the N-NMF algorithm and the approximate matching pursuit. We applied the algorithm to Lena image (as in Fig. 1), Barbra image and a face from the ORL database of faces (first face in the s1 set as shown in Fig. 2), which can be found in the following link (http://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html).

The results showed that using the overcomplete Gabor dictionary with the nonnegative matrix factorization to learn dictionaries for sparse representation gave a good results. We used that method for image denoising and evaluate our method by calculating the PSNR and compare our results with the K-SVD methods, which showed that our method gave better results over the K-SVD especially with low level noise energy. Also the approximate matching pursuit gave a fast computation compared to the orthogonal matching pursuit used in the K-SVD algorithm.

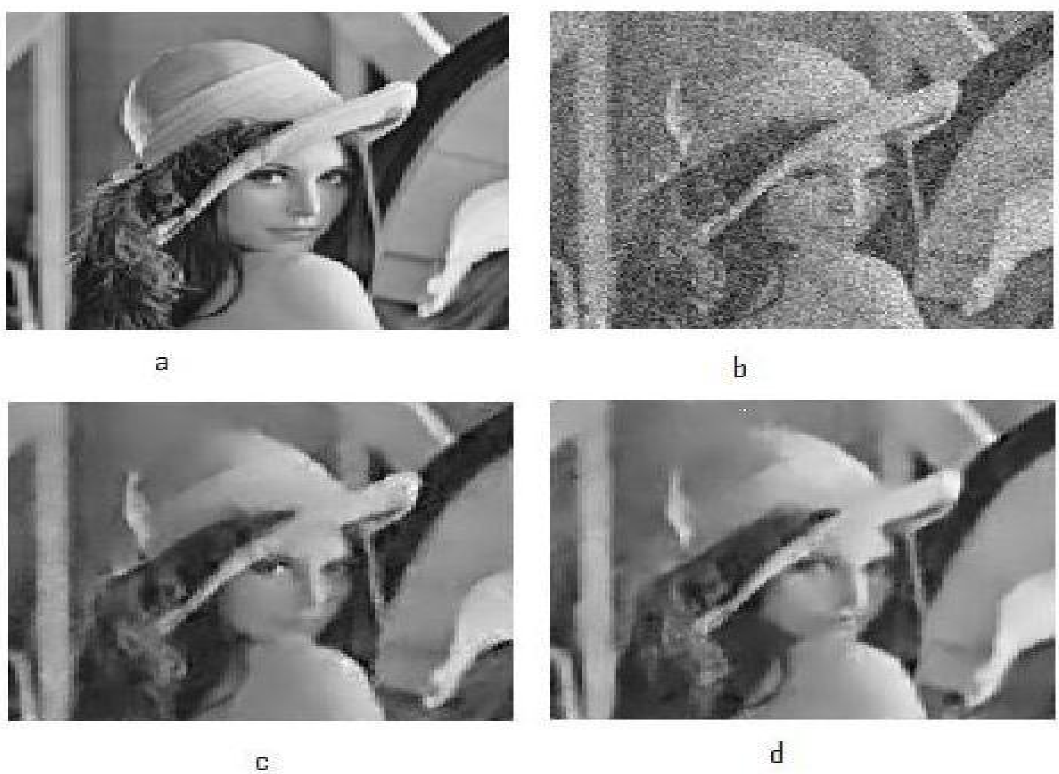

Fig. 1. (a) The original image. (b) The noised image by adding Gaussian noise with sigma=30. (c) The denoised image by using N-NMF algorithm and (d) the denoised image by using K-SVD.

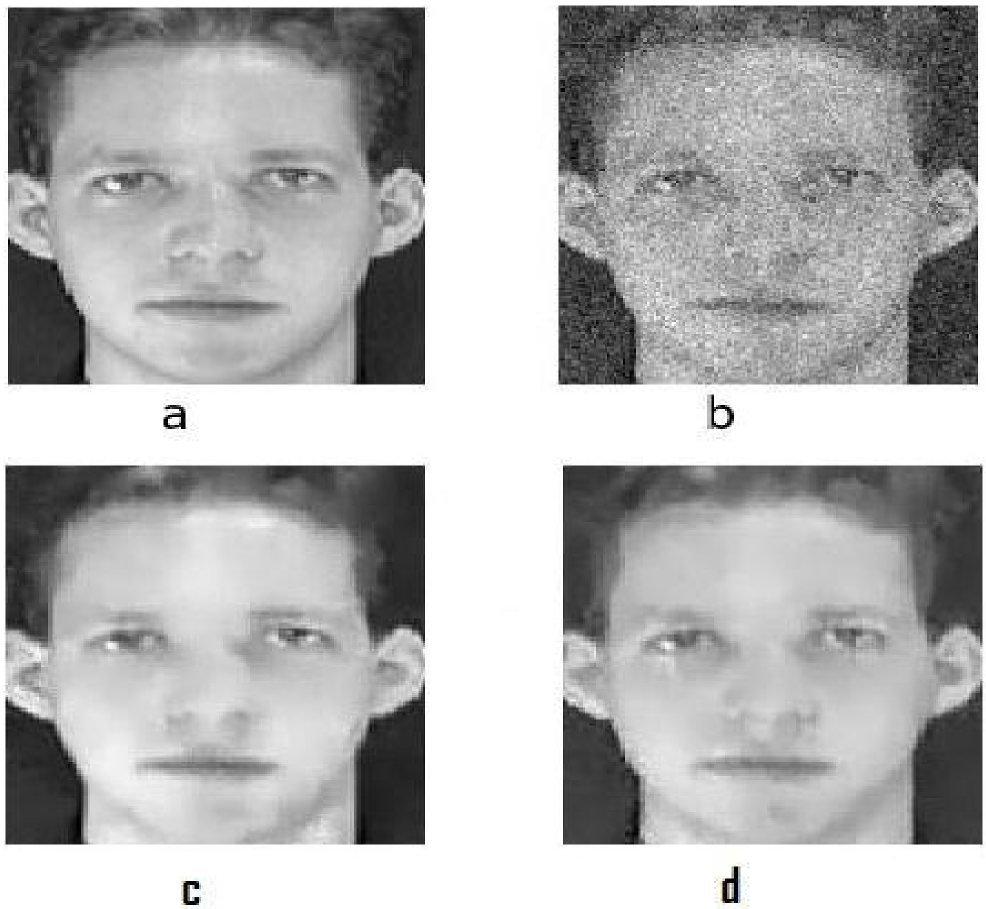

Fig. 2. (a) The original image. (b) The noised image by adding Gaussian noise with sigma=20. (c) The denoised image by using K-SVD and (d) the denoised image by using N-NMF algorithm.

Table 1. The PSNR computed for 3 images with different noise variance level (sigma).

Sigma	Face		Lena		Barbara	
	KSVD	N-NMF	KSVD	N-NMF	KSVD	N-NMF
10	35.5636	38.8481	33.3948	37.0749	32.9430	36.5061
15	33.2755	35.0193	31.1033	32.6425	30.6654	32.0532
25	31.0559	32.2762	28.4547	28.8607	27.7425	27.8119
30	27.2468	27.5401	27.2819	27.5758	26.6532	26.5589

6. Discussion and Conclusion

In this paper, we address the image denoising problem based on sparse coding over an overcomplete dictionary. Based on the fact that both nonnegative and sparse decompositions of data are often either desirable or necessary when the underlying components have a physical interpretation, which implies on real images. We presented an algorithm N-NMF, which used the technique of learning the dictionary to be suitable for representing the important component in the image by using the nonnegative matrix factorization technique for updating the dictionary in the learning process and using approximate matching pursuit algorithm for finding the sparse coding of the data based on the current dictionary. Experimental results show satisfactory recovering of the source image. Future theoretical work on the general behavior of this algorithm is on our further research agenda.

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